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`% Lagrangian dynamics of three link robot (HW01)`

```
clc
close all
clear all
```

System Constants

```
syms g L_torso L_leg1 L_leg2 m_torso m_leg1 m_leg2...
      I_torso I_leg1 I_leg2 real
```

Configuration Variables (Figure 1a)

```
syms x y q1 q2 q3 dx dy dq1 dq2 dq3 d2x d2y d2q1 d2q2 d2q3 real
% Generalized Coordinates
q = [x; y; q1; q2; q3] ;
% Generalized Velocities
dq = [dx; dy; dq1; dq2; dq3] ;
% Generalized Acceleration
d2q = [d2x; d2y; d2q1; d2q2; d2q3] ;
```

Define numerical values / vectors (cases (i) and (ii))

```
qi = [0.5, 0.5*sqrt(3), 150*pi/180, 120*pi/180, 30*pi/180]';
dqi = [-0.8049, -0.4430, 0.0938, 0.9150, 0.9298]';
qii = [0.3420, 0.9397, 170*pi/180, 20*pi/180, 30*pi/180]';
dqii = [-0.1225, -0.2369, 0.5310, 0.5904, 0.6263]';

L_torso_num = 1/2; L_leg1_num = 1; L_leg2_num = 1;
m_torso_num = 10; m_leg1_num = 5; m_leg2_num = 5;
I_torso_num = 1; I_leg1_num = 1/2; I_leg2_num = 1/2;
g_num = 9.81;
```

1(a)

Positions

```
p_torso = [L_torso.*sin(q3)/2 + x; L_torso.*cos(q3)/2 + y] ;
p_leg1 = [L_leg1.*sin(q1 + q3)/2 + x; L_leg1.*cos(q1 + q3)/2 + y] ;
p_leg2 = [L_leg2.*sin(q2 + q3)/2 + x; L_leg2.*cos(q2 + q3)/2 + y] ;

P_sym = [p_torso, p_leg1, p_leg2] ;

Pi = subs(P_sym, q, qi);
Pi = subs(Pi, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
    [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
    m_leg1_num, m_leg2_num]);
Pi = vpa(Pi,4) % m

Pii = subs(P_sym, q, qii);
Pii = subs(Pii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
    [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
    m_leg1_num, m_leg2_num]);
Pii = vpa(Pii,4) % m

Pi =

[ 0.625,    0.5,    0.75]
[ 1.083, 0.366, 0.433]
```

```

Pii =

[ 0.467,  0.171,  0.725]
[ 1.156,  0.4699,  1.261]

```

1(b)

Velocities

```

dp_torso = jacobian(p_torso, q) * dq ;
dp_leg1 = jacobian(p_leg1, q) * dq ;
dp_leg2 = jacobian(p_leg2, q) * dq ;

V_sym = [dp_torso, dp_leg1, dp_leg2] ;

Vi = subs(V_sym, q, qi);
Vi = subs(Vi, dq, dq_i);
Vi = subs(Vi, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
          [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
           m_leg1_num, m_leg2_num]);
Vi = vpa(Vi,4) % m/s

Vii = subs(V_sym, q, qii);
Vii = subs(Vii, dq, dq_ii);
Vii = subs(Vii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
           [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
            m_leg1_num, m_leg2_num]);
Vii = vpa(Vii,4) % m/s

Vi =

[ -0.6036, -1.317, -1.604]
[ -0.5592, -0.443, -0.9042]

Vii =

[ 0.0131, -0.6663, 0.2685]
[ -0.3152, -0.03899, -0.7029]

```

1(c)

Kinetic Energy

```

KE_torso = 1/2 * dp_torso' * m_torso * dp_torso + 1/2 * I_torso *...
          ( jacobian(q3, q) * dq )^2 ;
KE_leg1 = 1/2 * dp_leg1' * m_leg1 * dp_leg1 + 1/2 * I_leg1 *...
          ( jacobian(q3 + q1, q) * dq )^2 ;

```

```

KE_leg2 = 1/2 * dp_leg2' * m_leg2 * dp_leg2 + 1/2 * I_leg2 * ...
    ( jacobian(q3 + q2, q) * dq )^2 ;

KE_sym = KE_torso + KE_leg1 + KE_leg2 ;

KE_i = subs(KE_sym, q, qi);
KE_i = subs(KE_i, dq, dq_i);
KE_i = subs(KE_i, [L_torso, L_leg1, L_leg2, m_torso, m_leg1,
    m_leg2,...
    I_torso, I_leg1, I_leg2], [L_torso_num, L_leg1_num, L_leg2_num,...
    m_torso_num, m_leg1_num, m_leg2_num, I_torso_num, I_leg1_num,...
    I_leg2_num]);
KE_i = vpa(KE_i,4) % Joules

KE_ii = subs(KE_sym, q, qii);
KE_ii = subs(KE_ii, dq, dqii);
KE_ii = subs(KE_ii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1,
    m_leg2,...
    I_torso, I_leg1, I_leg2], [L_torso_num, L_leg1_num, L_leg2_num,...
    m_torso_num, m_leg1_num, m_leg2_num, I_torso_num, I_leg1_num,...
    I_leg2_num]);
KE_ii = vpa(KE_ii,4) % Joules

KE_i =

18.23

KE_ii =

3.928

```

1(d)

Potential Energy

```

PE_torso = m_torso * g * [0 1] * p_torso ;
PE_leg1 = m_leg1 * g * [0 1] * p_leg1 ;
PE_leg2 = m_leg2 * g * [0 1] * p_leg2 ;

PE_sym = PE_torso + PE_leg1 + PE_leg2;

PE_i = subs(PE_sym, q, qi);
PE_i = subs(PE_i, [m_torso, m_leg1, m_leg2, L_torso, L_leg1, L_leg2,
    g],[m_torso_num,...
    m_leg1_num, m_leg2_num, L_torso_num, L_leg1_num, L_leg2_num,
    g_num]);
PE_i = vpa(PE_i,4) % Joules

PE_ii = subs(PE_sym, q, qii);
PE_ii = subs(PE_ii, [m_torso, m_leg1, m_leg2, L_torso, L_leg1, L_leg2,
    g],[m_torso_num,...

```

```

        m_leg1_num, m_leg2_num, L_torso_num, L_leg1_num, L_leg2_num,
        g_num]);
PE_ii = vpa(PE_ii,4) % Joules

PE_i =

145.4

PE_ii =

198.3

```

1(e)

Lagrangian

```
L = KE_sym - PE_sym ;
```

Equations of Motion (LHS)

```

% State vector
x = [q;
     dq] ;
% Time-derivative of State
dx = [dq ;
      d2q] ;

EOM = jacobian(jacobian(L, dq), x) * dx - jacobian(L, q)' ;
EOM = simplify(EOM);

% Derive the dynamics in terms of the Robot Manipulator Dynamics
% D(q) d2q + C(q, dq) dq + G(q) = B(q) u

% set actuated coordinates
q_act = [q1; q2] ;

[D, C, G, B] = LagrangianDynamics(KE_sym, PE_sym, q, dq, q_act);

% Check if both sets of equations give the same results:
% The following expression below should give you zero.
simplify(D*d2q + C*dq + G - EOM)

ans =

0
0
0
0

```

Numerical Matrices

D matrix

```

case (i)

Di = subs(D, q, qi);
Di = subs(Di, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2,...
               I_torso, I_leg1, I_leg2], [L_torso_num, L_leg1_num, L_leg2_num,...
               m_torso_num, m_leg1_num, m_leg2_num, I_torso_num, I_leg1_num,...
               I_leg2_num]);
Di = vpa(Di,4)

% case (ii)
Dii = subs(D, q, qii);
Dii = subs(Dii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2,...
                  I_torso, I_leg1, I_leg2], [L_torso_num, L_leg1_num, L_leg2_num,...
                  m_torso_num, m_leg1_num, m_leg2_num, I_torso_num, I_leg1_num,...
                  I_leg2_num]);
Dii = vpa(Dii,4)

Di =

[ 20.0,    0, -2.5, -2.165, -2.5]
[    0, 20.0,    0, -1.25, -2.5]
[ -2.5,    0, 1.75,    0, 1.75]
[ -2.165, -1.25,    0, 1.75, 1.75]
[ -2.5, -2.5, 1.75, 1.75, 5.125]

Dii =

[ 20.0,    0, -2.349, 1.607, 1.423]
[    0, 20.0, 0.8551, -1.915, -2.31]
[ -2.349, 0.8551, 1.75,    0, 1.75]
[ 1.607, -1.915,    0, 1.75, 1.75]
[ 1.423, -2.31, 1.75, 1.75, 5.125]

```

C matrix

```

case (i)

Ci = subs(C, [q, dq], [qi dq]);
Ci = subs(Ci, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
           [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
           m_leg1_num, m_leg2_num]);
Ci = vpa(Ci,4);
display(Ci)

```

```

% case (ii)
Cii = subs(C, [q, dq], [qii dqii]);
Cii = subs(Cii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2],...
    [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
    m_leg1_num, m_leg2_num]);
Cii = vpa(Cii,4);
display(Cii)

Ci =

[ 0, 0,      0, -2.306, -3.468]
[ 0, 0, 2.559,  3.994,  4.54]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]

Cii =

[ 0, 0, 0.9895, -2.33, -2.123]
[ 0, 0, 2.719, -1.955, -0.5924]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]

```

G matrix

```

case (i)

Gi = subs(G, [q, dq], [qi dq]);
Gi = subs(Gi, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2,
    g],...
    [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
    m_leg1_num, m_leg2_num, g_num]);
Gi = vpa(Gi,4);
display(Gi)

% case (ii)
Gii = subs(G, [q, dq], [qii dqii]);
Gii = subs(Gii, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, m_leg2,
    g],...
    [L_torso_num, L_leg1_num, L_leg2_num, m_torso_num,...
    m_leg1_num, m_leg2_num, g_num]);
Gii = vpa(Gii,4);
display(Gii)

Gi =

      0
    196.2

```

```
      0
    -12.26
    -24.53
```

```
Gii =
```

```
      0
    196.2
     8.388
    -18.79
    -22.66
```

B matrix

```
case (i)
```

```
Bi = B;
display(Bi)
```

```
% case (ii)
Bii = B;
display(Bii)
```

```
Bi =
```

```
[ 0, 0]
[ 0, 0]
[ 1, 0]
[ 0, 1]
[ 0, 0]
```

```
Bii =
```

```
[ 0, 0]
[ 0, 0]
[ 1, 0]
[ 0, 1]
[ 0, 0]
```

Problem 2 - Change of Coordinates

2(a) (Handwritten portion included at end of document)

Work shown on paper

```
T = [1 0 0 0 0; 0 1 0 0 0; 0 0 1 0 1; 0 0 0 1 1 ; 0 0 0 0 1];
```

```
d = [0;0;-pi;-pi;0];
```

2(b) (Handwritten portion included at end of document)

Work shown on paper

```
dT = T;  
dd = zeros(5,1);
```

```
ddT = T;  
ddd = dd;
```

2(c)

```
qi_tild = T*qi + d  
dqi_tild = dT*dqi + dd
```

```
qii_tild = T*qii + d  
dqii_tild = dT*dqii + dd
```

```
qi_tild =
```

```
0.5000  
0.8660  
0  
-0.5236  
0.5236
```

```
dqi_tild =
```

```
-0.8049  
-0.4430  
1.0236  
1.8448  
0.9298
```

```
qii_tild =
```

```
0.3420  
0.9397  
0.3491  
-2.2689  
0.5236
```

```
dqii_tild =
```

```
-0.1225
```

```
-0.2369
1.1573
1.2167
0.6263
```

2(d)

Configuration Variables

```
syms x y th1 th2 th3 dx dy dth1 dth2 dth3 d2x d2y d2th d2th d2th real
% Generalized Coordinates
q_tild = [x; y; th1; th2; th3] ;
% Generalized Velocities
dq_tild = [dx; dy; dth1; dth2; dth3] ;
% Generalized Acceleration
d2q_tild = [d2x; d2y; d2th; d2th; d2th] ;
```

Kinematics

Positions

```
p_torso_tild = [ L_torso.*sin(th3)/2 + x; L_torso.*cos(th3)/2 + y] ;
p_leg1_tild = [ - L_leg1.*sin(th1)/2 + x; - L_leg1.*cos(th1)/2 + y] ;
p_leg2_tild = [ - L_leg2.*sin(th2)/2 + x; - L_leg2.*cos(th2)/2 + y] ;

% Velocities
dp_torso_tild = jacobian(p_torso_tild, q_tild) * dq_tild ;
dp_leg1_tild = jacobian(p_leg1_tild, q_tild) * dq_tild ;
dp_leg2_tild = jacobian(p_leg2_tild, q_tild) * dq_tild ;
```

Kinetic Energy

```
KE_torso_tild = 1/2 * dp_torso_tild' * m_torso * dp_torso_tild +
1/2*...
I_torso * ( jacobian(th3, q_tild) * dq_tild )^2 ;
KE_leg1_tild = 1/2 * dp_leg1_tild' * m_leg1 * dp_leg1_tild + 1/2 *
I_leg1 *...
( jacobian(th1, q_tild) * dq_tild )^2 ;
KE_leg2_tild = 1/2 * dp_leg2_tild' * m_leg2 * dp_leg2_tild + 1/2 *
I_leg2 *...
( jacobian(th2, q_tild) * dq_tild )^2 ;

KE_sym_tild = KE_torso_tild + KE_leg1_tild + KE_leg2_tild ;
```

Potential Energy

```
PE_torso_tild = m_torso * g * [0 1] * p_torso_tild ;
PE_leg1_tild = m_leg1 * g * [0 1] * p_leg1_tild ;
PE_leg2_tild = m_leg2 * g * [0 1] * p_leg2_tild ;
PE_sym_tild = PE_torso_tild + PE_leg1_tild + PE_leg2_tild ;
```

Lagrangian

```
L_tild = KE_sym_tild - PE_sym_tild ;
```

Equations of Motion (LHS)

Variables to find d/dt (partial L / partial dq) State vector

```
x_tild = [q_tild;  
          dq_tild] ;  
% Time-derivative of State  
dx_tild = [dq_tild ;  
          d2q_tild] ;  
  
EOM_tild = jacobian(jacobian(L_tild, dq_tild), x_tild) * dx_tild -  
           jacobian(L_tild, q_tild)' ;  
EOM_tild = simplify(EOM_tild) ;
```

Derive the dynamics in terms of the Robot Manipulator Dynamics

$D(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = B(q) u$

```
% set actuated coordinates  
q_act_tild = [pi + th1 - th3; pi + th2 - th3] ;  
  
[D_tild, C_tild, G_tild, B_tild] = LagrangianDynamics(KE_sym_tild,  
PE_sym_tild, q_tild,...  
dq_tild, q_act_tild) ;  
  
% Check if both sets of equations give the same results:  
% Check LHS of both sets to ensure they are the same  
% The following expression below should give you zero.  
simplify(D_tild*d2q_tild + C_tild*dq_tild + G_tild - EOM_tild)  
  
ans =  
  
0  
0  
0  
0  
0  
0
```

Numerical Matrices

D_tild matrix

case (i)

```

Di_tild = subs(D_tild, q_tild, qi_tild);
Di_tild = subs(Di_tild, dq_tild, dq_i_tild);
Di_tild = subs(Di_tild, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num]);
Di_tild = vpa(Di_tild,4)

% case (ii)
Dii_tild = subs(D_tild, q_tild, qii_tild);
Dii_tild = subs(Dii_tild, dq_tild, dqii_tild);
Dii_tild = subs(Dii_tild, [L_torso, L_leg1, L_leg2, m_torso,
    m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num]);
Dii_tild = vpa(Dii_tild,4)

Di_tild =

[ 20.0,      0, -2.5, -2.165, 2.165]
[      0, 20.0,      0, -1.25, -1.25]
[ -2.5,      0, 1.75,      0,      0]
[ -2.165, -1.25,      0, 1.75,      0]
[ 2.165, -1.25,      0,      0, 1.625]

Dii_tild =

[ 20.0,      0, -2.349, 1.607, 2.165]
[      0, 20.0, 0.8551, -1.915, -1.25]
[ -2.349, 0.8551, 1.75,      0,      0]
[ 1.607, -1.915,      0, 1.75,      0]
[ 2.165, -1.25,      0,      0, 1.625]

```

C_tild matrix

```

case (i)

Ci_tild = subs(C_tild, q_tild, qi_tild);
Ci_tild = subs(Ci_tild, dq_tild, dq_i_tild);
Ci_tild = subs(Ci_tild, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num]);
Ci_tild = vpa(Ci_tild,4)

% case (ii)
Cii_tild = subs(C_tild, q_tild, qii_tild);
Cii_tild = subs(Cii_tild, dq_tild, dqii_tild);
Cii_tild = subs(Cii_tild, [L_torso, L_leg1, L_leg2, m_torso,
    m_leg1, ...

```

```

        m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
        L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
        I_leg1_num, I_leg2_num]);
Cii_tild = vpa(Cii_tild,4)

Ci_tild =

[ 0, 0,      0, -2.306, -1.162]
[ 0, 0, 2.559,  3.994, -2.013]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]

Cii_tild =

[ 0, 0, 0.9895, -2.33, -0.7829]
[ 0, 0, 2.719, -1.955, -1.356]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]

```

G_tild matrix

```

case (i)

Gi_tild = subs(G_tild, q_tild, qi_tild);
Gi_tild = subs(Gi_tild, dq_tild, dq_i_tild);
Gi_tild = subs(Gi_tild, [L_torso, L_leg1, L_leg2, m_torso, m_leg1, ...
        m_leg2, I_torso, I_leg1, I_leg2, g], [L_torso_num, L_leg1_num,...
        L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
        I_leg1_num, I_leg2_num, g_num]);
Gi_tild = vpa(Gi_tild,4)

% case (ii)
Gii_tild = subs(G_tild, q_tild, qii_tild);
Gii_tild = subs(Gii_tild, dq_tild, dqii_tild);
Gii_tild = subs(Gii_tild, [L_torso, L_leg1, L_leg2, m_torso,
        m_leg1, ...
        m_leg2, I_torso, I_leg1, I_leg2, g], [L_torso_num, L_leg1_num,...
        L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
        I_leg1_num, I_leg2_num, g_num]);
Gii_tild = vpa(Gii_tild,4)

Gi_tild =

      0
    196.2
      0
   -12.26
   -12.26

```

```
Gii_tild =
```

```
    0
  196.2
   8.388
 -18.79
 -12.26
```

B_tild matrix

```
case (i)
```

```
Bi_tild = B_tild
```

```
% case (ii)
```

```
Bii_tild = B_tild
```

```
Bi_tild =
```

```
[ 0,  0]
[ 0,  0]
[ 1,  0]
[ 0,  1]
[-1, -1]
```

```
Bii_tild =
```

```
[ 0,  0]
[ 0,  0]
[ 1,  0]
[ 0,  1]
[-1, -1]
```

2(e) (Handwritten portion included at end of document)

```
% Linear algebra is on paper, final expression for D_tild_alt is
% implimented below
D_tild_alt = inv(T')*D*inv(T);

% Double check that the two versions of D_tild are equivalent by
% inputting
% numeric values into D_tild_alt for case (i). Then compare to the
% vector obtained from computing Lagrangian dynamics.

Di_tild_alt = subs(D_tild_alt, q, qi);
```

```

Di_tild_alt = subs(Di_tild_alt, dq, dqi);
Di_tild_alt = subs(Di_tild_alt, [L_torso, L_leg1, L_leg2, m_torso,
    m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num]);
Di_tild_alt = vpa(Di_tild_alt,4)

% This vector should output zeros
disp(Di_tild_alt - Di_tild)

Di_tild_alt =

[ 20.0,      0, -2.5, -2.165, 2.165]
[      0, 20.0,      0, -1.25, -1.25]
[ -2.5,      0, 1.75,      0,      0]
[ -2.165, -1.25,      0, 1.75,      0]
[ 2.165, -1.25,      0,      0, 1.625]

[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]

```

2(f) (Handwritten portion included at end of document)

```

% Linear algebra is on paper, final expressions for C_tild_alt,
% G_tild_alt,
% and B_tild_alt are implemented below

C_tild_alt = inv(T')*C*inv(T) ;
G_tild_alt = inv(T')*G ;
B_tild_alt = inv(T')*B ;

% Double check that the two versions of B_tild_alt, G_tild_alt,
% and C_tild_alt are equivalent by inputting
% numeric values into each matrix for case (i). Then compare to the
% vectors obtained from computing Lagrangian dynamics.

Ci_tild_alt = subs(C_tild_alt, q, qi);
Ci_tild_alt = subs(Ci_tild_alt, dq, dqi);
Ci_tild_alt = subs(Ci_tild_alt, [L_torso, L_leg1, L_leg2, m_torso,
    m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2 ], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num]);
Ci_tild_alt = vpa(Ci_tild_alt,4)

```

```

Gi_tild_alt = subs(G_tild_alt, q, qi);
Gi_tild_alt = subs(Gi_tild_alt, dq, dqi);
Gi_tild_alt = subs(Gi_tild_alt, [L_torso, L_leg1, L_leg2, m_torso,
    m_leg1, ...
    m_leg2, I_torso, I_leg1, I_leg2, g], [L_torso_num, L_leg1_num,...
    L_leg2_num, m_torso_num, m_leg1_num, m_leg2_num, I_torso_num,...
    I_leg1_num, I_leg2_num, g_num]);
Gi_tild_alt = vpa(Gi_tild_alt,4)

Bi_tild_alt = B_tild_alt

% These vectors should output all zeroes
display(Ci_tild - Ci_tild_alt)
display(Gi_tild - Gi_tild_alt)
display(Bi_tild - Bi_tild_alt)

Ci_tild_alt =

[ 0, 0,      0, -2.306, -1.162]
[ 0, 0, 2.559,  3.994, -2.013]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]
[ 0, 0,      0,      0,      0]

Gi_tild_alt =

      0
    196.2
      0
   -12.26
   -12.26

Bi_tild_alt =

[ 0, 0]
[ 0, 0]
[ 1, 0]
[ 0, 1]
[-1, -1]

ans =

[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0]

ans =

```

```
0
0
0
0
0
```

```
ans =
```

```
[ 0, 0]
[ 0, 0]
[ 0, 0]
[ 0, 0]
[ 0, 0]
```

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