ME 193B / 292B: Feedback Control of Legged Robots
HW #4

Problem 1. Two Link downhill Walker

In this problem, we will setup a simulation for our first walking robot. We will use no control and perform the simulation of a passive dynamic walker, where the robot walks down a gentle slope - see Figure 1. To achieve this, do the following:

(a) Create `two_link_dynamics.m` file to be called by `ode45`. This file takes as input time and the state vector and computes the time-derivative of the state using the dynamical model given below. In particular, let the generalized coordinates, generalized velocities, and state vector be,

$$q = \begin{bmatrix}\theta \\ \phi\end{bmatrix}, \quad \dot{q} = \begin{bmatrix}\dot{\theta} \\ \dot{\phi}\end{bmatrix}, \quad x = \begin{bmatrix}q \\ \dot{q}\end{bmatrix}.$$

The continuous-time dynamics of the system is given by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix}0 \\ 0\end{bmatrix},$$

where,

$$D(q) = \begin{bmatrix}1+2\beta(1-\cos\phi) & -\beta(1-\cos\phi) \\ \beta(1-\cos\phi) & -\beta\end{bmatrix}, \quad C(q,\dot{q})\dot{q} = \begin{bmatrix}-\beta\sin\phi(\dot{\phi}^2 - 2\dot{\theta}\dot{\phi}) \\ \beta\dot{\theta}^2\sin\phi\end{bmatrix},$$

$$G(q) = \begin{bmatrix}(\beta g/l)\{\sin(\theta-\phi-\gamma) - \sin(\theta-\gamma)\} - (g/l)\sin(\theta-\gamma) \\ (\beta g/l)\sin(\theta-\phi-\gamma)\end{bmatrix}.$$

Assume $\beta = m/M = 0.01$, $\gamma = 0.01\ rad$, and $g/l = 1$.

(b) Create `two_link_impactdynamics.m` file to be called at an impact event. This file takes the pre-impact state $\mathbf{x}^-$ as input and outputs the post-impact state $\mathbf{x}^+$. We will do the re-labelling as part of this file so that the output state vector has the re-labelled generalized

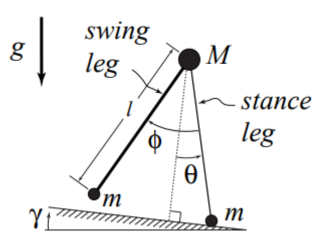

Figure 1: The Two-Link downhill Walker. The legs are symmetric with length l with a hip mass M and foot mass m. The ground slope is γ. The stance leg makes an angle θ with respect to the ground vertical and the swing leg makes an angle ϕ with respect to the stance leg.

positions and velocities. We will assume the impact map with the re-labelling is given by the following transformation:

$$\begin{bmatrix}\theta\\ \phi\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}^{+} = \begin{bmatrix}-1 & 0 & 0 & 0\\ -2 & 0 & 0 & 0\\ 0 & 0 & \cos(2\theta) & 0\\ 0 & 0 & \cos(2\theta)(1-cos(2\theta)) & 0\end{bmatrix}\begin{bmatrix}\theta\\ \phi\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}^{-}$$

Note that the above equation also reflects a change of names for the stance and swing legs (i.e., re-labeling is included.)

(c) Create `two_link_event.m` file to be called by `ode45` to detect when a impact with the ground happens. We will ignore foot scuffing as well as ground penetration of the swing foot and define the collision to occur when the following geometric condition is met:

$$\phi(t) - 2\theta(t) = 0.$$

Write your function so that you only detect $-ve$ to $+ve$ crossings. Moreover, you will need to stop integration when the event is detected.

(d) Create `simulate_two_link_walker.m` that setups up a single step of walking simulation using all the functions you created above. You can use the following initial condition to start your simulation:

$$\mathbf{x_0} = \begin{bmatrix}0.2065\\ 0.4130\\ -0.2052\\ -0.0172\end{bmatrix}.$$

Once this works, modify this function to simulate for $N = 10$ steps. (You can use the provided `animate_two_link_walker.m` to animate your simulated data.)

Problem 2. Two Link downhill Walker Poincaré map

We will numerically compute the Poincare map for the two link downhill walker system. Consider the Poincare section defined by

$$\mathcal{S} := \{\mathbf{x} \mid \phi - 2\theta = 0\}. \tag{1}$$

Write a MATLAB function `x1` = `TwoLinkPoincare` (`x0`) that takes in a point `x0` on the Poincaré section and returns the point on the Poincaré section after one complete cycle (i.e. at the next intersection of the solution with the Poincaré section). Inside the function, you should change θ to ensure $\mathbf{x}_0 \in \mathcal{S}$. Compute the linearized Poincaré map about the fixed point given by $\mathbf{x}_0$. Comment on the stability of this periodic downhill walking gait.

Instructions

1. You may submit either a typeset or handwritten solution. In either case, submit a **PDF** version of your solutions on bCourses, with the naming convention: `firstName_lastName_HW.pdf`.
2. Start each problem on a separate page.
3. You may choose to use a symbolic math package such as the `Symbolic Math Toolbox` (`https://www.mathworks.com/help/symbolic/index.html`) in `MATLAB` or `Mathematica`.
4. Do include all your code, if any.

5. Please submit a single pdf of your HW. (If typset on a computer, please save to pdf. If handwritten, please scan to pdf.)
6. **Honor Code.** You are to do your own work. Discussing the homework with a friend is fine. Sharing results or MATLAB code is not.
