ME 193B / 292B: Feedback Control of Legged Robots
HW #2

Problems

Problem 1 **Dynamics of Systems with Constraints**

The dynamics derived in HW #1 represents a freely falling three-link robot with two motor inputs at the hip. To derive the dynamics of a walking robot, we need to add an additional constraint while formulating the dynamics, which is enforced through an external (constraint) force. For walking, we assume the stance foot of the robot to be fixed w.r.t. the ground. This is enforced through the ground reaction forces acting on the stance foot. Such a constraint can be written as,

$$p_{st}(q) \equiv c, \tag{1}$$

where $p_{st} \in \mathbb{R}^2$ is the position of the stance foot and $c \in \mathbb{R}^2$ is a constant. For instance, if the stance foot is constrained to be on the ground at the origin, then $c = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. This type of constraint (a constraint only on the configuration position variables) is called a *holonomic constraint*. Then, the external forces at the stance foot, $F_{st} \in \mathbb{R}^2$ enter the system dynamics as,

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)u + J_{st}(q)^T F_{st}, \tag{2}$$

where $J_{st} = \frac{\partial p_{st}}{\partial q} \in \mathbb{R}^{2\times 5}$ is the *Jacobian* of the constraint (position of the stance foot here). In this problem, we will derive the expression for this external force that enforces the above holonomic constraint.

(a) Symbolically compute the expression of the position of the stance foot p_{st} as a function of the configuration variables. Compute the value of the stance foot position for the two numerical configurations (given in Problem 1 of HW #1.)

(b) Compute the expression of the Jacobian of the position of the stance foot J_{st} as a function of the configuration variables. Compute its value for the two numerical configurations.

(c) Compute the expression of the time derivative of the Jacobian of the stance foot $\dot{J}_{st}$ as a function of the configuration variables and velocities. Report its value for the given two configurations and velocities.

(d) Compute the external force required to enforce the constraint (1) for the given two configurations and velocities. Assume the motor inputs to be 0, i.e., $u = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$.

Problem 2 Impact Map

Derive the expression for the function that computes (a) the post-impact velocity given the pre-impact velocity and (b) the impact impulse at the swing leg. Assume a rigid plastic impact (i.e. coefficient of restitution is zero) and that the swing foot does not slip or bounce on impact.

Report the values of the post-impact velocities and the impact impulse given the pre-impact state

$$(q^-, \dot{q}^-) = \left(\begin{bmatrix} 0.3827\ m \\ 0.9239\ m \\ 3.0107\ rad \\ 2.2253\ rad \\ 0.5236\ rad \end{bmatrix}, \begin{bmatrix} 1.4782\ m/s \\ -0.6123\ m/s \\ 1.6\ rad/s \\ -1.6\ rad/s \\ 0\ rad/s \end{bmatrix} \right).$$