ME 193B / 292B: Feedback Control of Legged Robots
HW #7

Problem 1. Optimizing a Periodic Orbit for a Three-Link Walker

In this problem, you will implement your first optimizer to compute a periodic walking gait for the three-link walker we saw in earlier HWs. We will use the model of the three-link walker given to you in HW#5.

- **Recall:** Recall from HW#5 that q_1, q_2, q_3 were the relative angles for the system while $\theta_1, \theta_2, \theta_3$ were the absolute angles. Moreover, the generalized coordinates for the system was $q := \begin{bmatrix} x & y & q_1 & q_2 & q_3 \end{bmatrix}^T$ where (x, y) was the Cartesian position of the hip. Defining the state of the system as $s := \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$, we computed the dynamics as $\dot{s} = f(s) + g(s)u$. The generalized coordinates post-impact (after Relabelling) were obtained as $q^+ = Rq^-$, and $\dot{q}^+ = R\Delta_{\dot{q}}(q^-, \dot{q}^-)$. Finally, we assumed impact occurred at $\theta_1 = \theta_1^d := \pi/8$.
- **Initial Conditions and Outputs:** In HW#5 we used the magical initial conditions and *scissor-symmetric* gait defined through the initial condition x_0 and output y:

$$x_0 = \begin{bmatrix} -0.3827 \\ 0.9239 \\ 2.2253 \\ 3.0107 \\ 0.5236 \\ 0.8653 \\ 0.3584 \\ -1.0957 \\ -2.3078 \\ 2.0323 \end{bmatrix}, \qquad y := \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := \begin{bmatrix} \theta_3 - \theta_3^d \\ \theta_2 + \theta_1 \end{bmatrix}. \tag{1}$$

While this was good to control our first walking gait, it was a contrived example. We want to have the freedom to design more fluid walking gaits (no one walks with a scissor-symmetric gait) at different walking speeds.

In this HW, we will use a constrained nonlinear optimization to compute outputs as well as initial conditions. We will construct a parametrized family of outputs given by,

$$y := \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := \begin{bmatrix} \theta_3 - h_{d,1}(\theta_1, \alpha) \\ \theta_2 - h_{d,2}(\theta_1, b) \end{bmatrix}, \tag{2}$$

where

$$h_{d,1}(\theta_1, \alpha) = \alpha_0 + \alpha_1\theta_1 + \alpha_2(\theta_1)^2 + \alpha_3(\theta_1)^3, \tag{3}$$

$$h_{d,2}(\theta_1, \beta) = -\theta_1 + \left\{\beta_0 + \beta_1\theta_1 + \beta_2(\theta_1)^2 + \beta_3(\theta_1)^3\right\}(\theta_1 + \theta_1^d)(\theta_1 - \theta_1^d). \tag{4}$$

The rather particular form of $h_{d,2}$ was arrived at by imposing $h_{d,2}(\theta_1^d, \beta) = -h_{d,2}(-\theta_1^d, \beta) = -\theta_1^d$, which is the condition needed for the swing leg end to have height zero at impact.

- **Optimization Problem:** Your goal is to setup a optimization problem to determine a *periodic* walking gait for the 3-link walker that respects the unilateral ground contact constraints, the friction cone constraints, and walking at or above a desired average speed. To achieve this, we will identify the optimization variables, the cost function, and the constraints. We will use this and solve the problem through MATLAB's `fmincon` function.
- **Optimization Variables:** Optimize for the output parameter vectors a, b and the state at start of the step $q_0, \dot{q}_0$. You can determine the hip states from $q_0, \dot{q}_0$ and assume the horizontal hip position starts at the origin, i.e., $x = 0$. To seed the optimization, you can use values for $q_0, \dot{q}_0$ from HW#3 and the following values of α, β:

$$\begin{array}{cccc} \alpha_0 = \pi/6, & \alpha_1 = 0, & \alpha_2 = 0, & \alpha_3 = 0, \\ \beta_0 = 0, & \beta_1 = 0, & \beta_2 = 0, & \beta_3 = 0. \end{array} \tag{5}$$

- **Cost / Objective Function:** Minimize the cost that represents the electric motor energy used per distance traveled, given by,

$$J := \frac{1}{x(T_I)} \int_0^{T_I} ||u(t)||^2 dt, \tag{6}$$

where T_I is the impact time represent the time taken for one step, $x(T_I)$ is the distance traveled by the hip in one step, and $||u(t)||^2$ is the norm of the input vector evaluated at time t. Minimizing this cost function tends to reduce peak torque demands over a step.
- **Constraints:** Enforce the following constraints in your optimization:
  (a) Unilateral ground constraints, $F^v_{st}(t) \geq 0$.
  (b) Friction cone constraints, $\left|\frac{F^h_{st}(t)}{F^v_{st}(t)}\right| \leq \mu_s$. Here, $F^v_{st}(t)$ and $F^h_{st}(t)$ represent the vertical and horizontal stance forces at time t respectively. Use $\mu_s = 0.75$.
  (c) Desired average speed constraints, $\frac{x(T_I)}{T_I} \geq v^d$. Use the desired speed $v_d = 0.7$ m/s.
  (d) Periodicity constraints, $q_0 = Rq^-$, $\dot{q}_0 = R\Delta_{\dot{q}}(q^-, \dot{q}^-)$, i.e., the configuration and velocities of the 3-link walker at the start of the step and at the end of the step post-impact and post-relabeling should be identical.
- **Control:** Use either the finite-time input-output linearizing controller from HW#5 or the standard input-output linearizing controller,

$$u = -(L_g L_f y)^{-1}(-L_f^2 y + v), \text{where} \tag{7}$$

$$v = -k_p y - k_d \dot{y}. \tag{8}$$

Using the above information, solve the constrained nonlinear optimization problem using `fmincon`. For the obtained periodic orbit, submit plots of

(a) θ_1 vs $\dot{\theta}_1$,
(b) u_1 and u_2 vs time (on the same plot).
(c) F_{st} vs time (both components of F_{st} on the same plot).

Be prepared for a lot of frustration.

Problem 2. Walking at different Speeds Using the optimization setup of **Problem 1.**, find the fastest periodic walking gait you can optimize (by increasing v^d). Report the fastest speed you can find. Also find and report the slowest walking gait you can optimize (by decreasing v^d). For both cases, submit the following plots:

(a) θ_1 vs $\dot{\theta}_1$,
(b) u_1 and u_2 vs time (on the same plot).
(c) F_{st} vs time (both components of F_{st} on the same plot).

Instructions

1. You may submit either a typeset or handwritten solution. In either case, submit a **PDF** version of your solutions on bCourses, with the naming convention: `firstName_lastName_HW7.pdf`.
2. Start each problem on a separate page.
3. You may choose to use a symbolic math package such as the `Symbolic Math Toolbox` (`https://www.mathworks.com/help/symbolic/index.html`) in `MATLAB` or `Mathematica`.
4. Do include all your code, if any.
5. Please submit a single pdf of your HW. (If typset on a computer, please save to pdf. If handwritten, please scan to pdf.)
6. **Honor Code.** You are to do your own work. Discussing the homework with a friend is fine. Sharing results or MATLAB code is not.