1(b).

For $y > l_0$, $\ddot{y} = -g \rightarrow \int_{y_0}^{y} \ddot{y}\, dy = \int_0^t -g\, dt$

- Integrating we get, $\dot{y} = -gt + \dot{y}_0$
- Assuming $\dot{y}_0 = 0$ and energy conservation, apex could be found when $t = 0$ (defining X_0 for Poincare' surface)

$\therefore$ $\dot{y}$ @ apex $= 0$

- Integrating again we get, $y = -\frac{gt^2}{2} + \dot{y}_0 t + y_0$
- Using same assumptions as before, we find y @ apex from setting $t = 0$

$\therefore$ y @ apex = constant = y (initial height)

$$\boxed{\therefore\ P(x) = \begin{bmatrix} y \\ 0 \end{bmatrix}}$$

1(c).

There is one fixed point at $x^* = \begin{bmatrix} y \\ 0 \end{bmatrix}$

- This is truly a fixed point since, $P(x^*) = x^*$ (with energy conservation)
- For linear approximation, $P(x)$ is already linear so this is not applicable.
- x^* is stable iff $\left| \lambda_i \left(\frac{\partial P}{\partial x}\Big|_{x^*} \right) \right| < 1$
- $\frac{\partial P}{\partial x}\Big|_{x^*} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
- The eigenvalues of $\frac{\partial P}{\partial x}\Big|_{x^*}$ are 0 and 1

$\boxed{\therefore \text{ The fixed point is stable in the sense of Lyapunov}}$