2(a). From comparing the geometries of Figure 1a and Figure 1b,

$q(1) = \tilde{q}(1) = x$, $q(2) = \tilde{q}(2) = y$, $q(5) = \tilde{q}(5) = \theta_3 = q_3$

However, $q(3) \neq \tilde{q}(3)$ and $q(4) \neq \tilde{q}(4)$, but we can relate these variables by,

$\theta_1 = q_1 + q_3 - \pi$ and $\theta_2 = q_2 + q_3 - \pi$

- To incorporate all of these equations, we can write a configuration variable transformation as,

$\tilde{q} = Tq + d$ or

$$\underbrace{\begin{bmatrix} x \\ y \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}}_{\tilde{q}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} x \\ y \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{q} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -\pi \\ -\pi \\ 0 \end{bmatrix}}_{d}$$

* Note that T is invertible and $q = T^{-1}\tilde{q} - T^{-1}d$

2(b). To find mapping between $\dot{q}$ and $\dot{\tilde{q}}$, differentiate the equations from 2(a) and find that,

$\dot{q}(1) = \dot{\tilde{q}}(1) = \dot{x}$, $\dot{q}(2) = \dot{\tilde{q}}(2) = \dot{y}$, $\dot{q}(5) = \dot{\tilde{q}}(5) = \dot{\theta}_3 = \dot{q}_3$

and, $\dot{\theta}_1 = \dot{q}_1 + \dot{q}_3$, $\dot{\theta}_2 = \dot{q}_2 + \dot{q}_3$

$\therefore$ $\dot{\tilde{q}} = T\dot{q} + d$ where T is the same as 2(a) and $d = [0; 0; 0, 0, 0]^T$

- Now differentiate again to find,

$\ddot{q}(1) = \ddot{\tilde{q}}(1) = \ddot{x}$, $\ddot{q}(2) = \ddot{\tilde{q}}(2) = \ddot{y}$, $\ddot{q}(5) = \ddot{\tilde{q}}(5) = \ddot{\theta}_3 = \ddot{q}_3$

$\ddot{\theta}_1 = \ddot{q}_1 + \ddot{q}_3$, $\ddot{\theta}_2 = \ddot{q}_2 + \ddot{q}_3$

$\therefore$ $\ddot{\tilde{q}} = T\ddot{q} + d$ where T is the same as 2(a) and $d = [0, 0, 0, 0, 0]^T$