ME 292 B | HW # 1

2(e). Start with given equation,

$$\cancel{\tfrac{1}{2}}\dot{q}^T D(q)\dot{q} = \cancel{\tfrac{1}{2}}\dot{\tilde{q}}^T \tilde{D}(\tilde{q})\dot{\tilde{q}}$$

- plug in expression for $\dot{\tilde{q}}$,

$$\dot{q}^T D(q)\dot{q} = (T\dot{q})^T \tilde{D}(\tilde{q})(T\dot{q})$$

- Distribute transpose,

$$\dot{q}^T \underline{D(q)}\dot{q} = \dot{q}^T \underline{T^T \tilde{D}(\tilde{q})T}\dot{q}$$

- Isolate terms inbetween $\dot{q}^T$ and $\dot{q}$ for each side of the equation

$$D(q) = T^T \tilde{D}(\tilde{q})T$$

- Multiply by inverse of $T^T$ and $T$

$$(T^T)^{-1}D(q)T^{-1} = \cancel{(T^T)^{-1}T^T}\tilde{D}(\tilde{q})\cancel{TT^{-1}}$$

$$\therefore \boxed{\tilde{D}(\tilde{q}) = (T^T)^{-1}\cdot D(q)\cdot T^{-1}}$$

2(F). Start by rearranging equation (2) as,

$$D(q)\ddot{q} = B(q)u - C(q,\dot{q})\dot{q} - G(q)$$

- Next, introduce equation (4) with results from 2(e) plugged in,

$$((T^T)^{-1}\cdot D(q)\cdot T^{-1})\ddot{\tilde{q}} + \tilde{C}\dot{\tilde{q}} + \tilde{G} = \tilde{B}u$$

- introduce expressions for $\ddot{\tilde{q}}$ and $\dot{\tilde{q}}$,

$$((T^T)^{-1}\cdot D(q)\cdot \cancel{T^{-1}})(\cancel{T}\ddot{q}) + \tilde{C}(T\dot{q}) + \tilde{G} = \tilde{B}u$$

- Rearrange,

$$(T^T)^{-1}\cdot D(q)\ddot{q} = -\tilde{C}(T\dot{q}) - \tilde{G} + \tilde{B}u$$

- Multiply by $T^T$,

$$D(q)\ddot{q} = T^T(-\tilde{C}(T\dot{q}) - \tilde{G} + \tilde{B}u)$$

- Continued...