

Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT	
	Stamp/Signature of the Invigilator

MID-SEMESTER EXAMINATION	SEMESTER (Autumn-2015)
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Roll Number										Section		Name	
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Subject Number	M	A	1	0	0	0	1			Subject Name	Mathematics I
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Department/Centre/School	
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Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. Use last page(s) of the answer-script for rough work. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you.** After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.
11. Please see overleaf for more instructions

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5	6	7	Total
Marks Obtained								
Marks Obtained (in words)				Signature of the Examiner				Signature of the Scrutineer

Instructions and Guidelines to the Students appearing in the Examination

1. The question-cum-answer booklet has 28 pages and 7 questions.
2. All questions are compulsory.
3. Answer each question in the space provided below to that question only. Otherwise it will not be checked.
4. No additional answer sheet will be provided.
5. Use the space for rough work given in the booklet only.
6. After the completion of the examination do not leave the examination hall until the invigilator collects the booklet.

- 1(a). For what value of α , does $\frac{(\sin 2x + \alpha \sin x)}{x^3}$ tend to a finite limit as $x \rightarrow 0$? [2 M]
- 1(b). Show that between any two roots of $e^x \cos x - 1 = 0$, there exists at least one root of $e^x \sin x - 1 = 0$. [2 M]
- 1(c). Using Lagrange's Mean Value Theorem, prove the inequality $e^x > 1 + x, x > 0$. [1 M]

2(a). Find the points of inflexion of the curve $(1+x^2)y=1-x$. [2M]

2(b). Establish the inequality $a^x > x^a$ for $x \geq a \geq e$. [2M]

3(a). Find all asymptote(s) of the curve $y = \sqrt{x^2 + 4x + 1} - x$. [2M]

3(b). Find the curvature of the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sqrt{2}, b)$. [2M]

4(a). Discuss the continuity of the following function at the origin:

[2M]

$$f(x, y) = \begin{cases} \frac{x^4 - x^2 y^6}{(x^2 + y^6)^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

4(b). Discuss the differentiability of the following function at the origin:

[2M]

$$f(x, y) = \begin{cases} \sqrt{xy} & \text{for } xy \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

5(a). Discuss the continuity of the first order partial derivatives of the function

$$f(x, y) = \begin{cases} x^3 \sin\left(\frac{1}{y^2}\right) & \text{if } y \neq 0 \\ 0 & \text{if } y = 0 \end{cases}$$

at the origin?

[3M]

5(b). Find $\frac{\partial z}{\partial x}$ at $(0, 0, 0)$ if $x^3 + z^3 + x e^{yz} + z \cos x = 0$.

[1M]

6(a). Let the function $f(x, y) = x^3 - x^2y + y^3$ be approximated by the 2nd degree Taylor's polynomial about the point $(2, 3)$.

Find δ such that the absolute value of the remainder R_2 (involving third order partial derivatives) is less than 0.019 whenever $|x - 2| < \delta, |y - 3| < \delta$. [3M]

6(b). Let $u = x^2 f\left(\frac{y}{x}\right) - g\left(\frac{y}{x}\right)$ where f and g are continuous functions and possess continuous partial derivatives of 2nd order. Using Euler's theorem for homogeneous function, find α such that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2x^\alpha f\left(\frac{y}{x}\right).$$

[2M]

7. Find the points of local maximum, local minimum and saddle points for the function

$$f(x, y) = (x^2 - y^2) e^{(-x^2 - y^2)/2} \quad [4 \text{ M}]$$