

Advanced Digital Signal Processing (ADSP) Lab - Python

Lab Manual

Course Code: EEE G613

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▼ Experiment No. - 8

Weiner Filter for Filtering Application

Let $d(n)$ be an AR(1) process with an autocorrelation sequence

$$r_d(k) = \alpha^{|k|}$$

With $0 < \alpha < 1$, and suppose that $d(n)$ is observed in the presence of uncorrelated white noise, $v(n)$, that has a variance of $\sigma_v^2 = 1$

$$x(n) = d(n) + v(n)$$

Note: Wherever required perform mathematical derivations in your observation book using those expressions write Matlab code.

a) Consider for $\alpha = 0.8$, 1st order, 2nd order and 5th order filter. Write a Matlab Program to find $w(z)$ (using which $w(e^{j\omega})$). Plot the magnitude vs frequency response for these filter orders.

Calculate the SNR for these filter orders. Write down your observations

b) For a 3rd order filter consider $\alpha = 0.1, 0.2, 0.3 \dots 0.9$. Calculate mean-square error for each α value. Plot mean square error vs α . Write down your observations.

Hint: Check example 7.2.1 from Monsoon H. Hayes book.

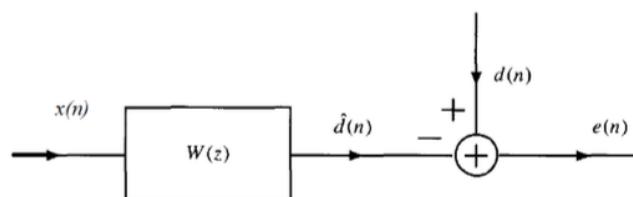


Fig.1 Illustration of the general wiener filtering problem.

▼ Python Code-

```
#import libraries
import numpy as np
import matplotlib.pyplot as plt
```

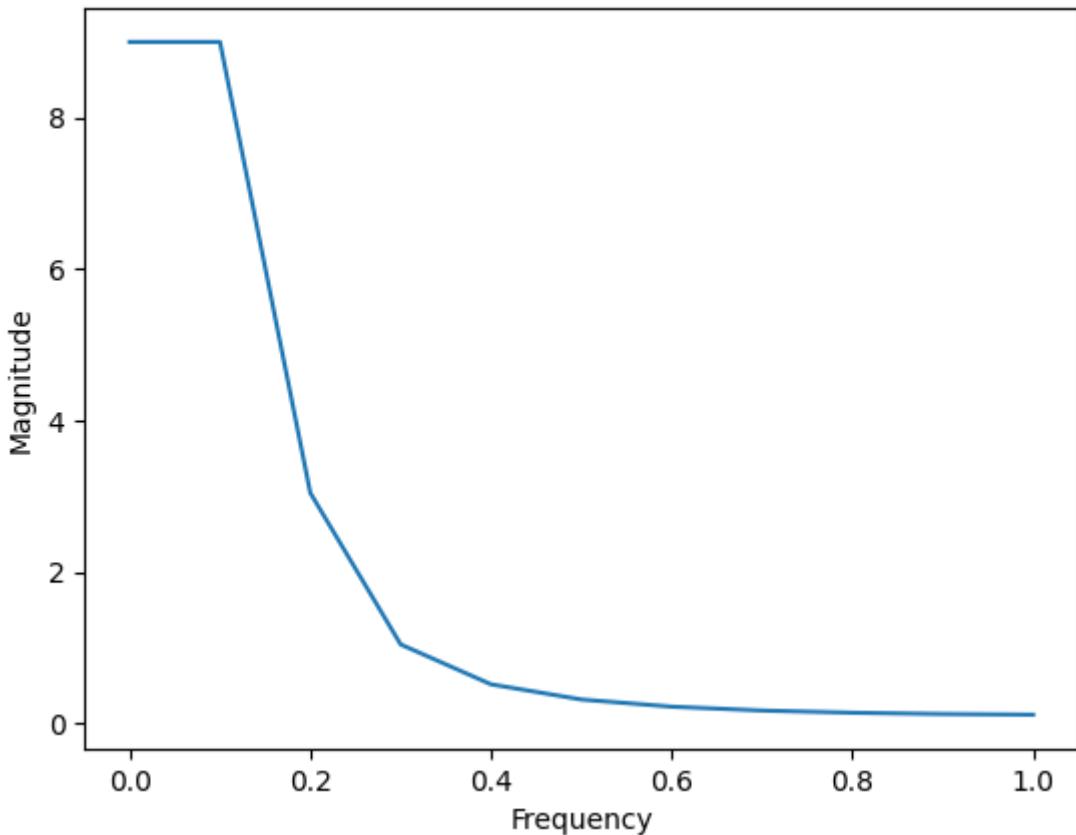
```

# For 1st order
a = 0.8
s = 1
f = np.arange(0, 1.1, 0.1)
w = 2 * np.pi * f
Rx = np.array([[1 + s ** 2, a], [a, 1 + s ** 2]])
Rdx = np.array([1, a])
W1 = np.linalg.inv(Rx).dot(Rdx)
signalp = np.dot(W1.T, np.dot(np.array([[1, a], [a, 1]]), W1))
noisep = np.dot(W1.T, W1)
SNR1 = 10 * np.log10(signalp / noisep)
psd = [(1 - a ** 2) / ((1 + a ** 2) - 2 * a * np.cos(0))]
for i in range(1, 11):
    psd.append((1 - a ** 2) / ((1 + a ** 2) - 2 * a * np.cos((i - 1) / 10 * np.pi)))
print('Weiner Coefficients for 1st order filter = ', W1)
print('SNR for 1st order filter = ', SNR1)
plt.figure()
plt.plot(f, psd)
plt.title('PSD of 1st order Weiner Filter')
plt.xlabel('Frequency')
plt.ylabel('Magnitude')

```

Weiner Coefficients for 1st order filter = [0.4047619 0.23809524]
 SNR for 1st order filter = 2.3025185815993248
 Text(0, 0.5, 'Magnitude')

PSD of 1st order Weiner Filter



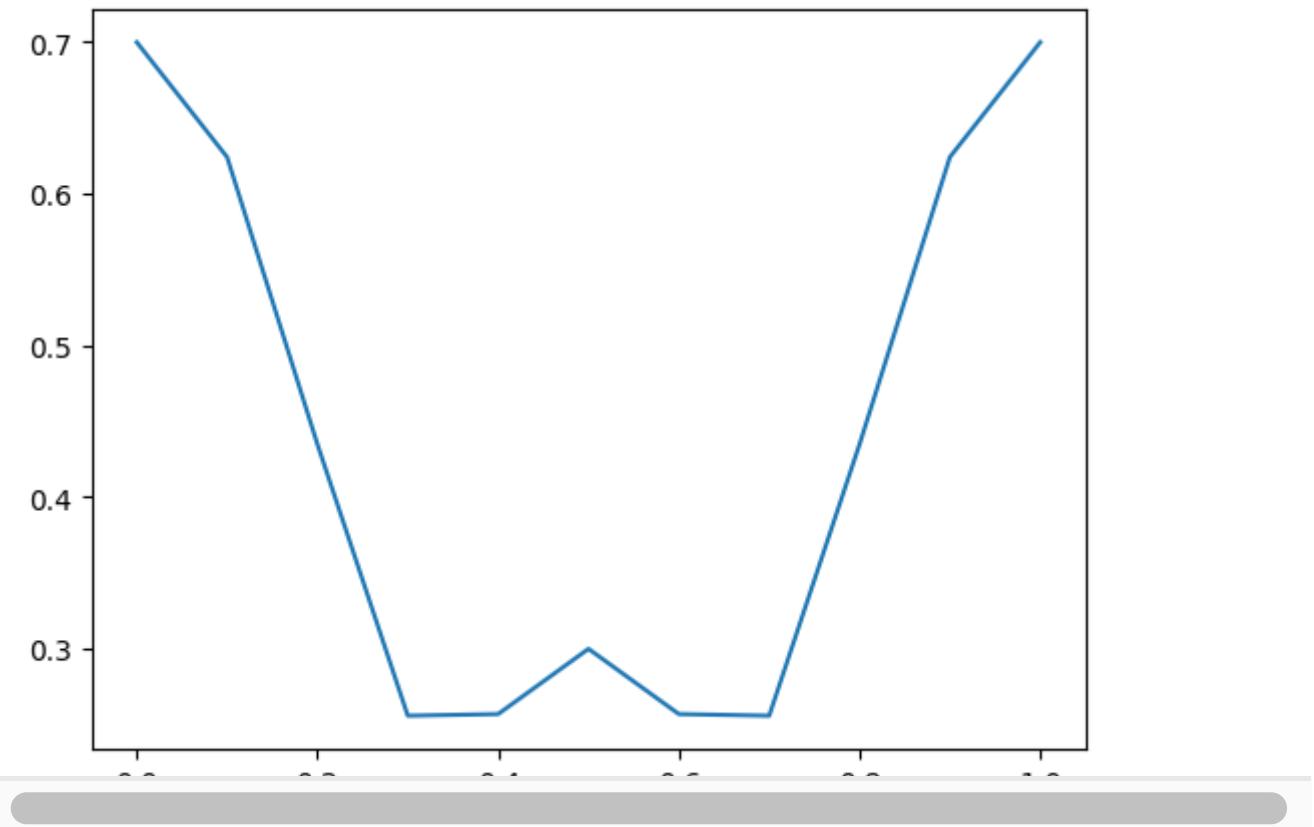
```

# For 2nd order
Rx = np.array([[1 + s ** 2, a, a ** 2], [a, 1 + s ** 2, a], [a ** 2, a, 1 + s ** 2]])
Rdx = np.array([1, a, a ** 2])
W2 = np.linalg.inv(Rx).dot(Rdx)
signalp = np.dot(W2.T, np.dot(np.array([[1, a, a ** 2], [a, 1, a], [a ** 2, a, 1]])))
noisep = np.dot(W2.T, W2)
SNR2 = 10 * np.log10(signalp / noisep)
Wequ2 = 0.3824 + (0.2 * np.exp(-1j * w)) + (0.1176 * (np.exp(-1j * w) ** 2))
print('Weiner Coefficients for 2nd order filter = ', W2)
print('SNR for 2nd order filter = ', SNR2)
plt.figure()
plt.plot(f, np.abs(Wequ2))
plt.title('PSD of 2nd order Weiner Filter')

```

Weiner Coefficients for 2nd order filter = [0.38235294 0.2 0.11764706
 SNR for 2nd order filter = 3.196683179158083
 Text(0.5, 1.0, 'PSD of 2nd order Weiner Filter')

PSD of 2nd order Weiner Filter

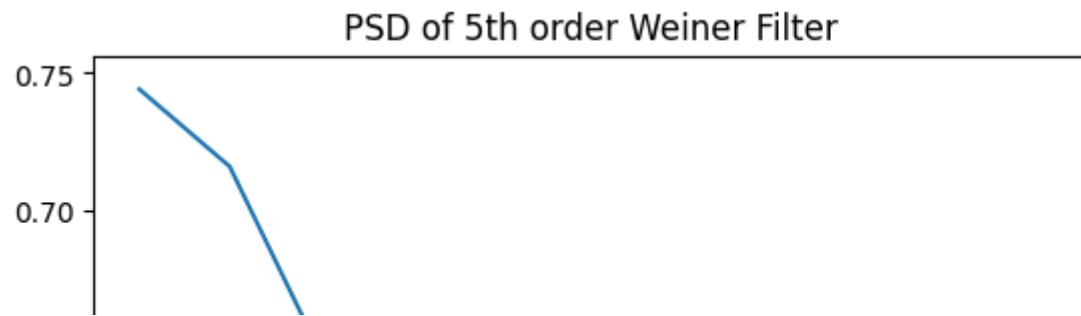


```

# For 5th order
n = 5
k = np.arange(1, n + 2)
a = 0.8
sigma = 1
Rx = np.array([[1 + s ** 2, a, a ** 2, a ** 3, a ** 4, a ** 5],
               [a, 1 + s ** 2, a, a ** 2, a ** 3, a ** 4],
               [a ** 2, a, 1 + s ** 2, a, a ** 2, a ** 3],
               [a ** 3, a ** 2, a, 1 + s ** 2, a, a ** 2],
               [a ** 4, a ** 3, a ** 2, a, 1 + s ** 2, a],
               [a ** 5, a ** 4, a ** 3, a ** 2, a, 1 + s ** 2]])
Rdx = np.array([1, a, a ** 2, a ** 3, a ** 4, a ** 5])
W5 = np.linalg.inv(Rx).dot(Rdx)
print('Weiner Coefficients for 5th order filter=', W5)
signalp = np.dot(W5.T, np.dot(np.array([[1, a, a ** 2, a ** 3, a ** 4, a ** 5],
                                         [a, 1, a, a ** 2, a ** 3, a ** 4],
                                         [a ** 2, a, 1, a, a ** 2, a ** 3],
                                         [a ** 3, a ** 2, a, 1, a, a ** 2],
                                         [a ** 4, a ** 3, a ** 2, a, 1, a],
                                         [a ** 5, a ** 4, a ** 3, a ** 2, a, 1]]), '
noisep = np.dot(W5.T, W5)
SNR5 = 10 * np.log10(signalp / noisep)
print('SNR for 5th order filter =', SNR5)
f = np.arange(0, 1.1, 0.1)
rd = a ** (k - 1)
rv = np.concatenate(([sigma], np.zeros(n)))
rx = rd + rv
R = np.zeros((n + 1, n + 1))
for i in range(n + 1):
    for j in range(n + 1):
        R[i, j] = rx[np.abs(i - j)]
w = np.linalg.inv(R).dot(rd)
winereqn = w[0]
for b in range(n):
    winereqn += w[b + 1] * np.exp((-b * 1j) * f * np.pi)
plt.figure()
plt.plot(f, np.abs(winereqn))
plt.title('PSD of 5th order Weiner Filter')
plt.show()

```

Weiner Coefficients for 5th order filter [0.37511445 0.18769456 0.09412196 0.1
SNR for 5th order filter = 3.665383527068688



```
# For a 3rd order filter consider  $\alpha = 0.1, 0.2, 0.3 \dots 0.9$ . Calculate mean-square error and SNR

#import libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import toeplitz

n = 3
a = np.arange(0, 1.1, 0.1)
mse = []
SNR = []
```