Advanced Digital Signal Processing (ADSP) Lab - Python Lab Manual

> **Course Code:** EEE G613
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Experiment No. - 5

1. Signal Modeling using Prony's approximation Method

Design a linear lowpass filter having cutoff frequency of $\pi/2$. The frequency response of the *ideal lowpass filter* is given below

$$X(e^{j\omega}) = \begin{cases} e^{-jn_d\omega}; |\omega| < \frac{\pi}{2} \\ 0; otherwise \end{cases}$$

where n_d is the filter delay. Take $n_d = 5$.

Note: wherever required you solve the equations in observation book and use those expressions to write Matlab programs. Solving equations in Matlab requires symbolic math so we do not go for it.

a) Design a filter for p=1 and q=1 see how it matches with x(n). Plot the impulse responses take for e.g. n=1 to 20

b) Design another filter with p=5 and q=5 see how it matches with x(n). Plot the impulse responses

c) Compare Prony's and Pade's approximation (results from last lab) for p=5 and q=5. Which method is better? Give reasons.

Python Code- Part (a):

```
#import libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import lfilter
import time

start_time = time.time()
# Part a
I_a = np.zeros(21)
for n in range(21):
  if n != 5:
    I_a[n] = np.sin((n - 5) * (np.pi / 2)) / ((n - 5) * np.pi)
```

```
  else:
    I_a[n] = 0.5

n_a = len(I_a)
p_a = 1
q_a = 1
I_a = np.append(I_a, np.zeros(2 * p_a))
Rxx_a = np.zeros((p_a + 1, p_a + 1))
for l in range(p_a + 1):
  for k in range(p_a + 1):
    rxx = []
    for i in range(q_a + 1, n_a):
      if i - l > 0 and i - k > 0:
        rxx.append(I_a[i - l] * I_a[i - k])
    Rxx_a[k, l] = sum(rxx)

rxk_a = Rxx_a[1:, 0]
a_a = -np.linalg.solve(Rxx_a[1:, 1:], rxk_a)
b_a = np.zeros(q_a + 1)

for i in range(q_a + 1):
  aa = [0]
  for k in range(p_a):
    if i + 1 - k > 0:
      aa.append(a_a[k] * I_a[i + 1 - k])
    b_a[i] = I_a[i] + sum(aa)

j_a = len(I_a) - 1
o_a = np.zeros(j_a)
o_a = np.append(1, o_a)
a_a = np.append(1, a_a)
h_a = lfilter(b_a, a_a, o_a)
error_a = I_a[:n_a] - h_a[:n_a]
```

Python Code- Part (b):

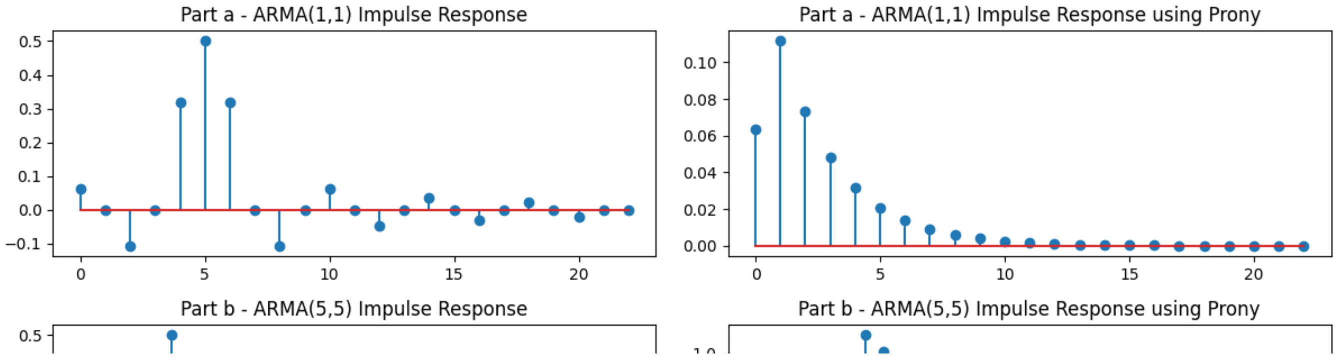
```
# Part b
I_b = np.zeros(21)
for n in range(21):
  if n != 5:
    I_b[n] = np.sin((n - 5) * (np.pi / 2)) / ((n - 5) * np.pi)
  else:
    I_b[n] = 0.5

n_b = len(I_b)
p_b = 5
q_b = 5
I_b = np.append(I_b, np.zeros(2 * p_b))
Rxx_b = np.zeros((p_b + 1, p_b + 1))
for l in range(p_b + 1):
  for k in range(p_b + 1):
    rxx = []
    for i in range(q_b + 1, n_b):
      if i - l > 0 and i - k > 0:
        rxx.append(I_b[i - l] * I_b[i - k])
    Rxx_b[k, l] = sum(rxx)

rxk_b = Rxx_b[1:, 0]
a_b = -np.linalg.solve(Rxx_b[1:, 1:], rxk_b)
b_b = np.zeros(q_b + 1)
for i in range(q_b + 1):
  aa = [0]
  for k in range(p_b):
    if i + 1 - k > 0:
      aa.append(a_b[k] * I_b[i + 1 - k])
  b_b[i] = I_b[i] + sum(aa)

j_b = len(I_b) - 1
o_b = np.zeros(j_b)
o_b = np.append(1, o_b)
a_b = np.append(1, a_b)
h_b = lfilter(b_b, a_b, o_b)
error_b = I_b[:n_b] - h_b[:n_b]
```

Python Code- Part (c):

```
# Part c - Prony
I_c = np.zeros(21)
for n in range(21):
  if n != 5:
    I_c[n] = np.sin((n - 5) * (np.pi / 2)) / ((n - 5) * np.pi)
  else:
    I_c[n] = 0.5

n_c = len(I_c)
p_c = 5
q_c = 5
I_c = np.append(I_c, np.zeros(2 * p_c))
Rxx_c = np.zeros((p_c + 1, p_c + 1))
for l in range(p_c + 1):
  for k in range(p_c + 1):
    rxx = []
    for i in range(q_c + 1, n_c):
      if i - l > 0 and i - k > 0:
        rxx.append(I_c[i - l] * I_c[i - k])
    Rxx_c[k, l] = sum(rxx)

rxk_c = Rxx_c[1:, 0]
a_c = -np.linalg.solve(Rxx_c[1:, 1:], rxk_c)
b_c = np.zeros(q_c + 1)
for i in range(q_c + 1):
  aa = [0]
  for k in range(p_c):
    if i + 1 - k > 0:
      aa.append(a_c[k] * I_c[i + 1 - k])
  b_c[i] = I_c[i] + sum(aa)

j_c = len(I_c) - 1
o_c = np.zeros(j_c)
o_c = np.append(1, o_c)
a_c = np.append(1, a_c)
h1_c = lfilter(b_c, a_c, o_c)

# Part c - Pade
p_c = 5
q_c = 5
x_c = np.zeros(21)
for n in range(21):
  if n != 5:
    x_c[n] = np.sin((n - 5) * (np.pi / 2)) / ((n - 5) * np.pi)
  else:
    x_c[n] = 0.5

x_c = np.append(x_c, np.zeros(2 * (p_c + 1)))
N = len(x_c) + 2 * (p_c + 1) - 2
x_c_pad = np.pad(x_c, (p_c, p_c), 'constant')
X_c = np.zeros((N - p_c, p_c + 1))
for i in range(p_c + 1):
  X_c[:, i] = x_c_pad[p_c - i:N - i]

Xq = X_c[q_c + 1:q_c + p_c + 2, 1:p_c + 1]
xqplus1 = X_c[q_c + 1:q_c + p_c + 2, 0]
# Solve the linear system using least squares (lstsq) instead of solve
apb, residuals, _, _ = np.linalg.lstsq(Xq, xqplus1, rcond=None)
a_c_pade = np.insert(apb, 0, 1)
b_c_pade = X_c[0:q_c + 1, 0:p_c + 1].dot(a_c_pade)
h2_c = lfilter(b_c_pade, a_c_pade, x_c)
error_c_pade = x_c[:n_c] - h2_c[:n_c]

# Calculate elapsed runtime
elapsed_time = time.time() - start_time
print(f"\nElapsed Runtime: {elapsed_time:.4f} seconds\n")
```

```
    Elapsed Runtime: 0.0473 seconds
```

```
# Plot the impulse responses and errors
plt.figure(figsize=(12, 8))
plt.subplot(3, 2, 1)
plt.stem(I_a)
plt.title("Part a – ARMA(1,1) Impulse Response")
plt.subplot(3, 2, 2)
plt.stem(h_a)
plt.title("Part a – ARMA(1,1) Impulse Response using Prony")
plt.subplot(3, 2, 3)
plt.stem(I_b)
plt.title("Part b – ARMA(5,5) Impulse Response")
plt.subplot(3, 2, 4)
plt.stem(h_b)
plt.title("Part b – ARMA(5,5) Impulse Response using Prony")
plt.subplot(3, 2, 5)
plt.stem(I_c)
plt.title("Part c – Pade Impulse Response (5,5)")
plt.subplot(3, 2, 6)
plt.stem(h1_c)
plt.title("Part c – Impulse Response using Prony (5,5)")
plt.tight_layout()
plt.show()
plt.figure(figsize=(12, 4))
plt.subplot(1, 2, 1)
plt.stem(error_a)
plt.title("Part a – ARMA(1,1) Error")
plt.subplot(1, 2, 2)
plt.stem(error_b)
plt.title("Part b – ARMA(5,5) Error")
plt.tight_layout()
plt.show()
plt.figure(figsize=(12, 4))
plt.tight_layout()
plt.show()
```


Part a - ARMA(1,1) Impulse Response
Part a - ARMA(1,1) Impulse Response using Prony
Part b - ARMA(5,5) Impulse Response
Part b - ARMA(5,5) Impulse Response using Prony