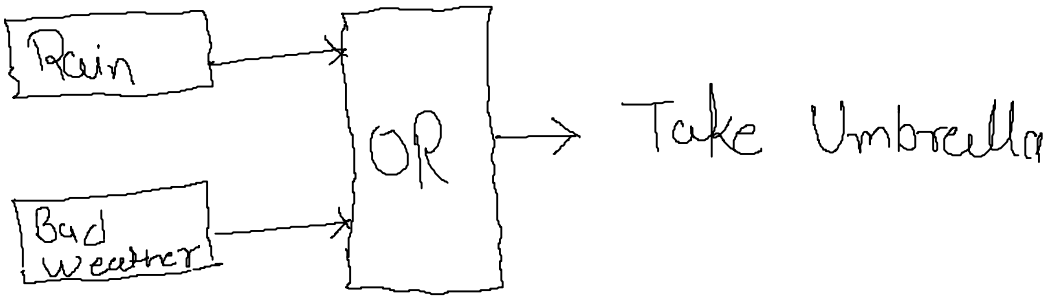


Boolean Algebra

Saturday, August 08, 2020
8:51 AM



| R | W | U |
|---|---|---|
| F | F | F |
| T | F | T |
| F | T | T |
| T | T | T |

BA-1.1

Saturday, August 08, 2020
8:54 AM

AND

| A | B | R |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

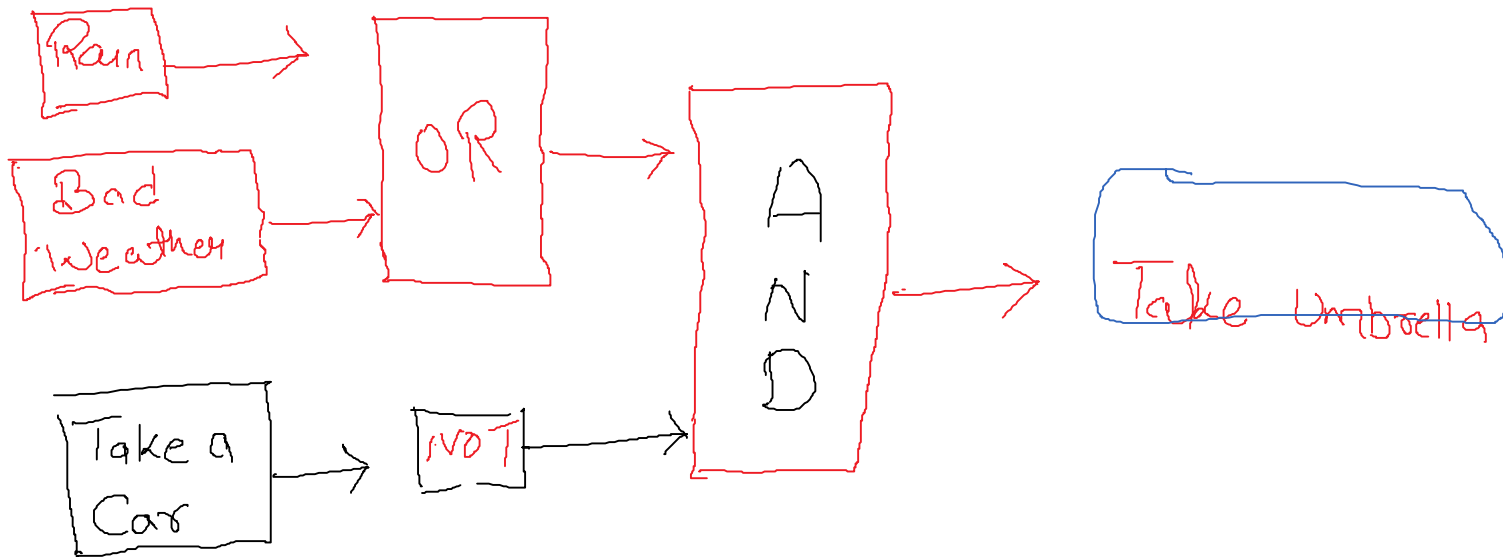
OR

| A | B | R |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NOT

| A | R |
|---|---|
| 0 | 1 |
| 1 | 0 |

→ NOT = Inverse
 \wedge AND
 \vee OR



* If I don't take Car then I will take the Umbrella, If it is Rain or weather forecast is bad.

$$\text{Take Umbrella} = [\text{NOT}(\text{take car}) \text{ AND } ((\text{Bad Weather}) \text{ OR } (\text{Rain}))]$$

$$U = \neg C \wedge (W \vee R)$$

$$\wedge = \text{AND}, \quad \vee = \text{OR}, \quad \neg = \text{NOT}$$

| R W C | | | $x_1 = R \vee W$ | | | $x_2 = \neg C$ | $U = x_1 \wedge x_2$ |
|--|---|---|------------------|---|---|----------------|----------------------|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|--|---|---|---|--|---|
| ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ○ | | ○ |
| ✓ | ✓ | ○ | ✓ | | ✓ | ✓ | ○ | | ○ |
| ✓ | ✓ | ✓ | ○ | | ✓ | ✓ | ○ | | ○ |
| ✓ | ✓ | ✓ | ○ | | ✓ | ✓ | ○ | | ○ |

RULE

Saturday, August 08, 2020
9:16 AM

$$\left. \begin{array}{l} \neg (\neg A) = A \\ [A \wedge \neg A = 0] \\ A \vee \neg A = 1 \end{array} \right\} \text{NOT operator}$$

$$\begin{array}{l} A = 1 \\ \neg A = 0 \\ 1 \wedge 0 = 0 \\ 1 \vee 0 = 1 \end{array}$$

$$\text{Associative} \left\{ \begin{array}{l} (A \wedge B) \wedge C = A \wedge (B \wedge C) \\ (A \vee B) \vee C = A \vee (B \vee C) \end{array} \right.$$

$$\text{Commutative} \left\{ \begin{array}{l} A \wedge B = B \wedge A \\ A \vee B = B \vee A \end{array} \right.$$

$$\text{Distributive} \left\{ \begin{array}{l} A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C) \\ A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) \end{array} \right.$$

de Morgan's theorem

$$\neg (A \vee B) = \neg A \wedge \neg B$$

$$\neg (A \wedge B) = \neg A \vee \neg B$$

Mathematical Induction

Saturday, August 08, 2020
9:24 AM

Basic = Result is true for $n=0$ or 1

Induction Hypothesis = assume that result is True for $n=k$

Induction Step = Prove that result is true for $n=k+1$

Final Step of Induction Step \rightarrow Actual proof

Prove the following series by Principal of M.I.

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Basic Let $n=1$

$$\begin{aligned} \text{L.H.S.} &= 1 \\ &= \frac{1(1+1)}{2} \\ &= 1 \end{aligned}$$

Induction Hypothesis = we assume that result is true for $n=k$

$$1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$$

Induction step:- we have to prove that the result is true for $n=(k+1)$

$$= \frac{(k+1)(k+1+1)}{2}$$

L.H.S.

$$\text{L.H.S.} = 1 + 2 + 3 + \dots + k + k+1$$

$$= \frac{(k+1)(k+1+1)}{2}$$

$$= \frac{k(k+1)}{2}$$

from hypothesis
stop

$$\frac{k(k+1)}{2} + k+1$$

$$k(k+1) + 2k+1$$

$$k^2 + k + 2k + 1$$

$$\frac{k^2 + 3k + 1}{2}$$

$$\frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)(k+1+1)}{2}$$

$$\text{L.H.S.} = \text{R.H.S.}$$