# 13-Identification of Breakpoints in Time Series

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#### 1 Text needs to be written

The example is adapted from the help pages of R package "strucchange", see Zeileis, A. et al. (2002), Journal of Statistical Software, 7(2), https://www.jstatsoft.org/v07/i02/

```
library(strucchange)

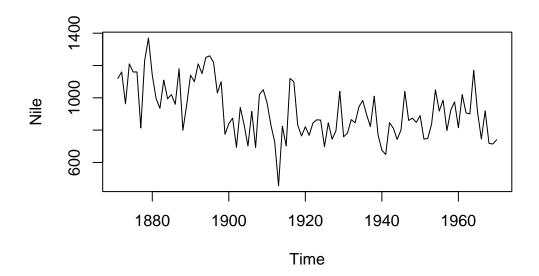
Lade nötiges Paket: zoo

Attache Paket: 'zoo'

Die folgenden Objekte sind maskiert von 'package:base':
    as.Date, as.Date.numeric

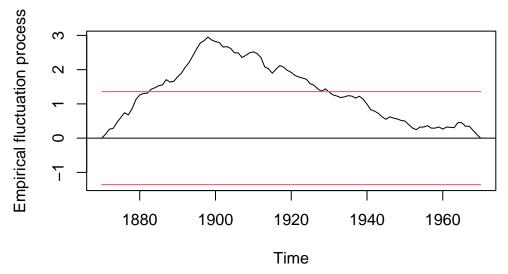
Lade nötiges Paket: sandwich

data("Nile")
plot(Nile)
```



## OLS-CUMSUM test for structural breaks in the time series
## are there periods with different discharge?
ocus <- efp(Nile ~ 1, type = "OLS-CUSUM")
plot(ocus)</pre>

## **OLS-based CUSUM test**



sctest(ocus)

OLS-based CUSUM test

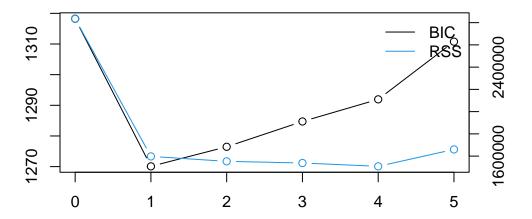
data: ocus S0 = 2.9518, p-value = 5.409e-08

```
## identify time of structural break (with respect to mean value)
bp.nile <- breakpoints(Nile ~ 1)</pre>
summary(bp.nile)
     Optimal (m+1)-segment partition:
Call:
breakpoints.formula(formula = Nile ~ 1)
Breakpoints at observation number:
           28
m = 1
m = 2
           28
                    83
m = 3
           28
                 68 83
           28 45 68 83
m = 4
m = 5 15 30 45 68 83
Corresponding to breakdates:
             1898
m = 1
m = 2
             1898
                            1953
m = 3
             1898
                       1938 1953
m = 4
             1898 1915 1938 1953
m = 5 1885 1900 1915 1938 1953
Fit:
                    2
                            3
                                    4
RSS 2835157 1597457 1552924 1538097 1507888 1659994
       1318
            1270
                       1276
                               1285
                                       1292
                                               1311
```

## the BIC also chooses one breakpoint

plot(bp.nile)

### **BIC and Residual Sum of Squares**



## Number of breakpoints

```
## fit null hypothesis model and model with 1 breakpoint
fm0 <- lm(Nile ~ 1)
fm1 <- lm(Nile ~ breakfactor(bp.nile, breaks = 1))
plot(Nile)
lines(ts(fitted(fm0), start = 1871), col = 3)
lines(ts(fitted(fm1), start = 1871), col = 4)
lines(bp.nile)

## confidence interval
ci.nile <- confint(bp.nile)
ci.nile</pre>
```

Confidence intervals for breakpoints of optimal 2-segment partition:

#### Call:

confint.breakpointsfull(object = bp.nile)

Breakpoints at observation number:

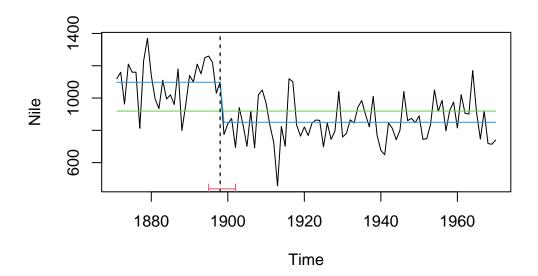
```
2.5~\% breakpoints 97.5~\%
```

1 25 28 32

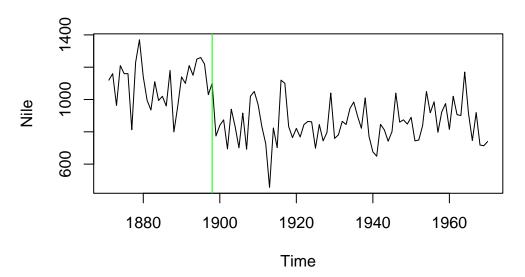
Corresponding to breakdates:

```
2.5 % breakpoints 97.5 %
1 1895 1898 1902
```

#### lines(ci.nile)



```
## mark breakoint using a simpler (and less fancy) method
plot(Nile)
dat <- data.frame(time = time(Nile), Q = as.vector(Nile))
abline(v=dat$time[bp.nile$breakpoints], col="green")</pre>
```



## ANOVA test whether the two models are significantly different anova(fm0, fm1)

Analysis of Variance Table

Model 1: Nile ~ 1

Model 2: Nile ~ breakfactor(bp.nile, breaks = 1)

```
Res.Df RSS Df Sum of Sq F Pr(>F)

1 99 2835157

2 98 1597457 1 1237700 75.93 7.439e-14 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

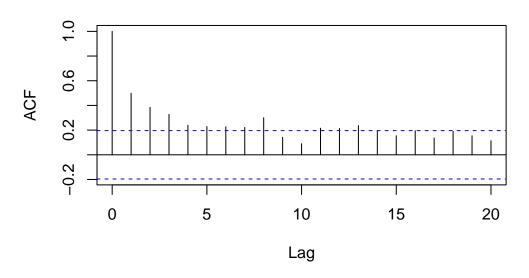
## alternative: AIC-based model comparison.

## The model with lower AIC is better
AIC(fm0,fm1)
```

df AIC fm0 2 1313.031 fm1 3 1257.663

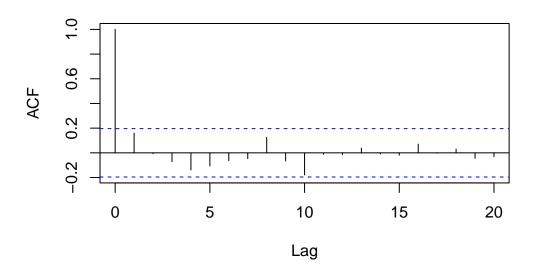
## some tests for quality and assumptions of the fitted model
acf(residuals(fm0))

## Series residuals(fm0)



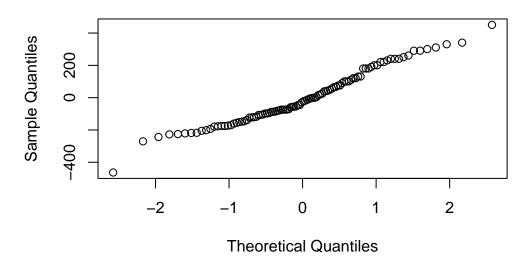
acf(residuals(fm1))

# Series residuals(fm1)



qqnorm(residuals(fm0))

## Normal Q-Q Plot



qqnorm(residuals(fm1))

# Normal Q-Q Plot

