# THE TRIPLE CROSS PRODUCT $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}})$

Note that the vector $\vec{\mathbf{G}} = \vec{\mathbf{B}} \times \vec{\mathbf{C}}$ is perpendicular to the plane on which vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ lie. Thus, taking the cross product of vector $\vec{\mathbf{G}}$ with an arbitrary third vector, say $\vec{\mathbf{A}}$, the result will be a vector perpendicular to $\vec{\mathbf{G}}$ and thus lying in the plane of vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$. Therefore, one can express the vector $\vec{\mathbf{F}} = \vec{\mathbf{A}} \times \vec{\mathbf{G}}$ as a linear combination of the vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$, i.e.,

$$\vec{\mathbf{F}} = m\vec{\mathbf{B}} + n\vec{\mathbf{C}}$$

Taking the scalar product of the both sides of this expression with vector $\vec{\mathbf{A}}$, and noting that $\vec{\mathbf{A}} \cdot \vec{\mathbf{F}} = 0$ one obtains

$$m\,(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) + n\,(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}) = 0$$

For this equality to be valid for any $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$, one is tempted to write

$$m = \lambda\,(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}), \quad n = -\lambda\,(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})$$

in which the unknown proportionality constant $\lambda$ has been introduced so as serve for the above solutions to hold true with no loss in generality.

Thus, one has

$$\begin{aligned} \vec{\mathbf{F}} &= \vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) \\ &= \lambda \left\{ (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\,\vec{\mathbf{B}} - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\,\vec{\mathbf{C}} \right\} \end{aligned}$$

Selecting arbitrarily $\vec{\mathbf{A}} = \hat{\mathbf{k}}$, $\vec{\mathbf{B}} = \hat{\mathbf{j}}$, and $\vec{\mathbf{C}} = \hat{\mathbf{k}}$, for instance, and substituting in the above equality, one obtains $\lambda = 1$.

Hence, one eventually obtains the vector identity

$$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\,\vec{\mathbf{B}} - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\,\vec{\mathbf{C}}$$