OM Homework Set 1

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1 Homework 1: Linear Algebra

1.1

Verify the following identities:

1. $\|\mathbf{a}+\mathbf{b}\|^2 + \|\mathbf{a}-\mathbf{b}\|^2 = 2(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2)$
2. $(\mathbf{a}+\mathbf{b})^T(\mathbf{a}-\mathbf{b})^T = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2$
3. $\|\mathbf{a}+\mathbf{b}\| \geq \|\mathbf{a}-\mathbf{b}\|, \forall a, b \in \mathbb{R}^n$

1.2

A matrix $\mathbf{B}$ is symmetric if $\mathbf{B} = \mathbf{B}^T$. Prove that for any square matrix $\mathbf{B}$, $\mathbf{B}+\mathbf{B}^T$ is symmetric and that if $\mathbf{A}$ is invertible, then $(\mathbf{A}^{-1})^T = (\mathbf{A}^T)^{-1}$.

1.3

For a finite dimensional vector space, prove that the L1 and L2 norms are equivalent. Specifically, there exist constants $C_1, C_2 \in \mathbb{R}$ such that $0 < \mathbf{C}_1 \leq \mathbf{C}_2$ and

$$C_1 \|\mathbf{x}\|_2 \leq \|\mathbf{x}\|_1 \leq C_2 \|\mathbf{x}\|_2 \, \forall \mathbf{x}$$

2 Homework 2: Linear Programming

2.1

Solve the following problem graphically

$$\begin{aligned} \text{minimize} \quad & z = 5x_1 + 2x_2 \\ \text{subject to} \quad & 6x_1 + x_2 \geq 6, \\ & 4x_1 + 3x_2 \geq 12, \\ & x_1 + 2x_2 \geq 4 \end{aligned}$$

You may use software to plot, but make sure the vertices, regions are labelled and annotated.

2.2

Consider the LP

$$\begin{aligned} \text{minimize} \quad & z = c_1x_1 + c_2x_2 + c_3x_3 \\ \text{subject to} \quad & x_1 + x_2 \geq 1, \\ & x_1 + 2x_2 \leq 3, \\ & x_1 \geq 0; x_2 \leq 0 \end{aligned}$$

Give the optimal value and the optimal set for the following values of the c

i $c = (-1, 0, 1)$

ii $c = (0, 1, 0)$

iii $c = (0, 0, -1)$

You may use software to plot, but make sure the vertices, regions are labelled and annotated.

2.3

(A Transportation Problem) Suppose that a distributor has two canneries labeled 1 and 2, and three warehouses labeled a, b, and c in different geographical locations. The canneries can fill 250 and 450 cases of tins per day, respectively. Each of the warehouses can sell 200 cases per day. The distributor wishes to determine the number of cases to be shipped from the two canneries to the three warehouses so that each warehouse obtains as many cases as it can sell daily at the minimum total transportation cost. The availability of cases at the canneries and the demands which must be met exactly at each warehouse are summarized in the table below:

Cases Available	
Cannery	Cases
1	250
2	450
Total	700

Cases Demanded	
Warehouse	Cases
a	200
b	200
c	200
Total	600

The Shipping cost in $/case is given below:

Cannery	Warehouse a	Warehouse b	Warehouse c
1	3.4	2.2	2.9
2	3.4	2.4	2.5

Formulate the problem as an LP.

3 Homework 3: Graph Theory, Computational Complexity

3.1

Consider the complete graph with $n = 7$ vertices. Calculate the number of 5-length paths from vertex 4 to vertex 7.

3.2

Let f and g be unbounded monotonically increasing functions on $\mathbb{R}$. Does the following implication hold? Justify.

$$f \in \mathbf{O}(g) \Rightarrow \log f \in \mathbf{O}(\log g)$$

3.3

Verify the following identity:

$$\log(n!) \in \Theta(n \log n)$$

4 Submission Instructions

Your submission is expected to be a **pdf** file with title **rollnumber-hw1.pdf** containing the in-order scanned images of your hand-written answers (and plots).

- Submission Deadline: Jan 27$^{\text{th}}$, 2020, time: 23:55
- No deadline extensions will be provided.
- Final submission needs to be uploaded on Moodle.
- Any form of plagiarism will be penalized.