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Design and Analysis of Algorithms (COEN 279)
Homework 1

Question 1 (5 points). *Express the function $n^3/1000 + 100n^2 - 100n + 3$ in Θ-notation.*

Question 2 (10 points). *Consider the following two programs:*

```
Alg1(n)
  for i = 1 to n
    for j = 1 to 2^n
      Print(j)
```

and

```
Alg2(n)
  for i = 1 to n
    for j = 1 to 2^i
      Print(j)
```

For each of these programs give the asymptotic runtime as $\Theta(f(n))$ for some function f and justify your work.

Question 3 (15 points). *Consider sorting n numbers stored in array $A[1:n]$ by first finding the smallest element of $A[1:n]$ and exchanging it with the element in $A[1]$. Then find the smallest element of $A[2:n]$, and exchange it with $A[2]$. Then find the smallest element of $A[3:n]$, and exchange it with $A[3]$. Continue in this manner for the first $n-1$ elements of A. Write pseudocode for this algorithm, which is known as* **selection sort**. *What loop invariant does this algorithm maintain? Why does it need to run for only the first $n-1$ elements, rather than for all n elements? Give the worst-case running time of selection sort in Θ-notation.*

Question 4 (10 points). *Solve the following recurrence relations and give a Θ bound for each of them.*

(a) $T(n) = 2T(n/3) + 1$

(b) $T(n) = 5T(n/4) + n$

(c) $T(n) = 9T(n/3) + n^2$

(d) $T(n) = 8T(n/2) + n^3$

(e) $T(n) = 49T(n/25) + n^{3/2} \log n$

Question 5 (10 points). *Given a sorted array of distinct integers $A[1:n]$, you want to find out whether there is an index i for which $A[i] = i$. Devise an optimal divide-and-conquer algorithm and derive it's worst-case running time.*

Question 6 (20 points). *The n^{th} Hadamard matrix is a $2^n \times 2^n$ matrix of some importance. It can be defined recursively by*

$$H_0 = [1], \qquad H_{n+1} = \begin{bmatrix} H_n & H_n \\ H_n & -H_n \end{bmatrix}.$$

Give an algorithm that given a 2^n-dimensional (column) vector v computes $H_n v$ using $O(n \log n)$ operations.