Instructor Name: Siddhartha Nath
Email: snath@scu.edu

Design and Analysis of Algorithms (COEN 279)
Homework 3 – 100 points

Note: For all algorithm design questions, you must (i) explain your algorithm in English, (ii) provide pseudocode (if modifications to algorithms discussed in the class are required), (iii) prove correctness, and (iv) explain and provide running time complexity to receive full credit.

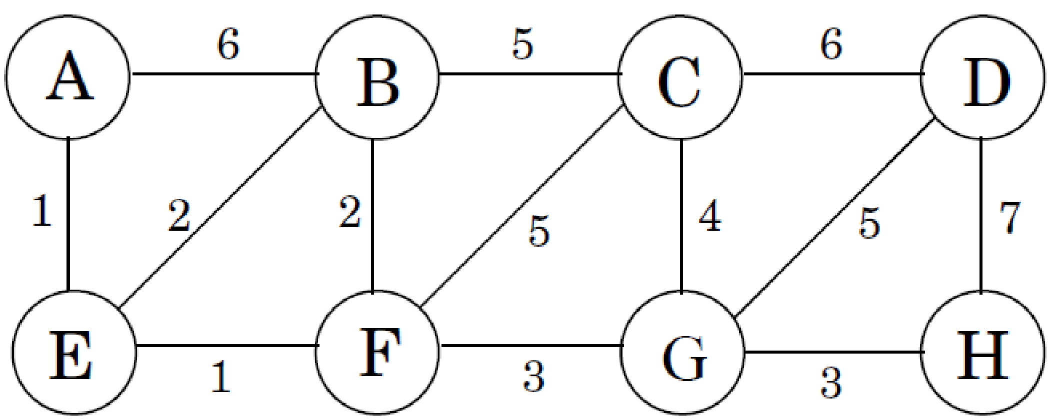
Question 1 (15 points). *Give an algorithm that takes as input a directed graph with positive edge lengths, and returns the length of the shortest cycle in the graph (if the graph is acyclic, it should say so). Your algorithm should take time at most* $O(|V|^3)$.

Question 2 (15 points). *Suppose that we are given a weighted, directed graph* $G = (V; E)$ *in which edges that leave the source vertex* s *may have negative weights, all other edge weights are non-negative, and there are no negative-weight cycles. Prove that Djikstra's algorithm correctly finds the shortest paths from* s *in this graph.*

Question 3 (20 points). *There is a network of roads* $G = (V; E)$ *connecting a set of cities* V. *Each road in* E *has an associated length* l_e. *There is a proposal to add* **one** *new road to this network, and there is a list* E' *of pairs of cities between which the new road can be built. Each such potential road* $e' \in E'$ *has an associated length. As a designer for the public works department you are asked to determine the road* $e' \in E'$ *whose addition to the existing network* G *would result in the maximum decrease in the driving distance between two fixed cities* s *and* t *in the network. Give an efficient algorithm for solving this problem.*

Question 4 (15 points). *Suppose you are given a connected graph* $G = (V; E)$, *with each edge having a unique weight (all edge weights are distinct). Prove that* G *has a unique minimum spanning tree.*

Question 5 (15 points). *Consider the following graph.*



(a) What is the cost of its minimum spanning tree (MST)?

(b) How many minimum spanning trees does it have?

(c) *Suppose Kruskal's algorithm is run on this graph, in what order are the edges added to the MST? For each edge sequence, give a cut that justifies its addition.*

Question 6 (20 points). *Prove the following two properties of the Huffman encoding scheme.*

(a) *If some character occurs with frequency more than* $2/5$*, then there is guaranteed to be a codeword of length 1.*

(b) *If all characters occur with frequency less than* $1/3$*, then there is guaranteed to be no codeword of length 1.*