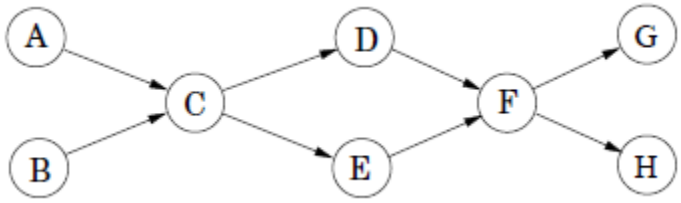
Instructor Name: Siddhartha Nath
Email: snath@scu.edu

Santa Clara University
School of Engineering
Computer Science and Engineering

Design and Analysis of Algorithms (COEN 279)
Homework 2 – 100 points

Note: For all algorithm design question, you must (i) explain your algorithm in English, (ii) provide pseudocode, (iii) prove correctness, and (iv) explain and provide running time complexity to receive full credit.

Question 1 (10 points). *Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that appears first in alphabetical order.*



(a) Indicate the pre- and post- numbers of the nodes.

(b) What are the sources and sinks of the graph?

(c) Write down one topological ordering found by the algorithm.

(d) How many topological orderings does this graph have?

Question 2 (10 points). *A tree is a connected and acyclic graph. Show that a tree with n vertices has exactly $n-1$ edges. (Hint: use induction.)*

Question 3 (15 points). *Give an efficient algorithm that takes as input a directed acyclic graph $G = (V; E)$, and two vertices $s; t \in V$, and outputs the number of different directed paths from s to t in G.*

Question 4 (15 points). *For each vertex u in an undirected graph, let twodegree[u] be the sum of the degrees of u's neighbors. Given an undirected graph $G = (V, E)$ in adjacency-list format, show how to compute the entire array of twodegree[·] values in time linear in $|V|$ and $|E|$.*

Question 5 (20 points). *A multigraph is a directed graph which is permitted to have more than one edges with the same source and target nodes. Given an adjacency list representation of a multigraph $G = (V, E)$, describe an efficient algorithm to compute the adjacency list representation of the equivalent undirected graph $G' = (V, E')$, where E' consists of the edges in E with all multiple edges between two vertices replaced by a single edge, and with all self-loops removed.*

Question 6 (30 points). *A bipartite graph is a graph $G = (V; E)$ whose vertices can be partitioned into two sets ($V = V1 \cup V2$ and $V1 \bigcap V2 = \Phi$) such that there are no edges between vertices in the same set (for instance, if $u; v \in V1$, then there is no edge between u and v).*

(a) Give a linear-time algorithm to determine whether an undirected graph is bipartite.

(b) There are many other ways to formulate this property. For instance, an undirected graph is bipartite if and only if it can be colored with just two colors.
Prove the following formulation: an undirected graph is bipartite if and only if it contains no cycles of odd length.

(c) At most how many colors are needed to color in an undirected graph with exactly one oddlength cycle?