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CSC6023 - Advanced Algorithms

Advanced Data Structures

Patrick Neff

Advanced Data Structures

This is a very fast paced course, After this four week topic and we are at the half the course. So, let’s see the greedy algorithms today.

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Agenda Presentation

Tree Review and AVL Trees

- Types of Trees
- AVL Trees
- Code for AVL Trees
- Worksheet Assignment for AVL Trees

Graphs

- Types of Graphs
- Common Graph Applications
- Common Problems with Graphs
- Dijkstra’s Algorithm
- Dijkstra’s Algorithm Code
- Dijkstra’s Algorithm Example
- Programming Assignment for Dijkstra’s Algorithm

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Tree Review:

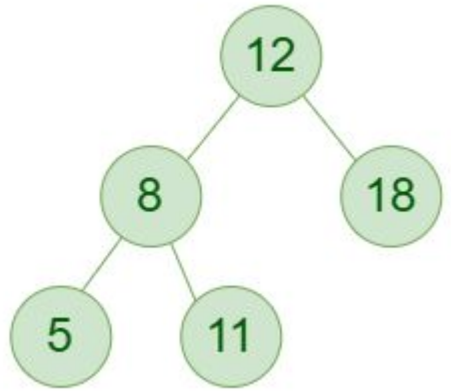
- **Full Binary Tree**: Every node has either 0 or 2 child nodes, i.e., left and right or no children
- **Complete Binary Tree**: All levels, except possibly the last, are filled, and all nodes are as left as possible
- **Perfect Binary Tree**: All nodes have exactly two children, and all leaf nodes are at the same level
- **Balanced Binary Tree**: The heights of any node’s left and right subtrees differ by at most one
- **Binary Search Tree (BST)**: A binary tree in which: (1)The left subtree of a node contains only nodes with keys lesser than the node’s key. (2) The right subtree of a node contains only nodes with keys greater than the node’s key. (3) The left and right subtree each must also be a binary search tree.

Source for definitions and images of trees on slides 26-30:

(https://www.geeksforgeeks.org/binary-tree-data-structure/?ref=lbp)

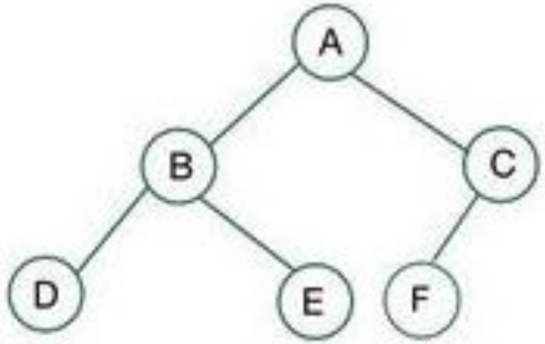
Tree Review:

- **Full Binary Tree**: Every node has either 0 or 2 child nodes, i.e., left and right or no children



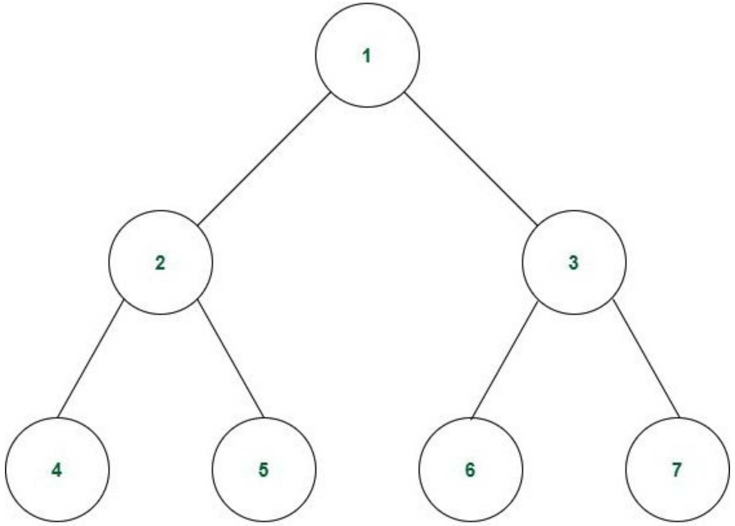
Tree Review:

- **Complete Binary Tree**: All levels, except possibly the last, are filled, and all nodes are as left as possible



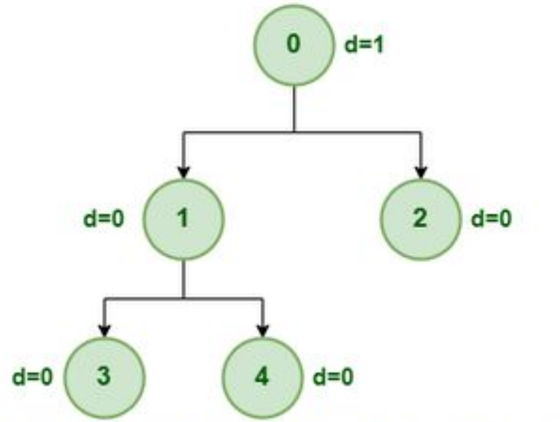
Tree Review:

- **Perfect Binary Tree**: All nodes (except leaf nodes) have exactly two children, and all leaf nodes are at the same level

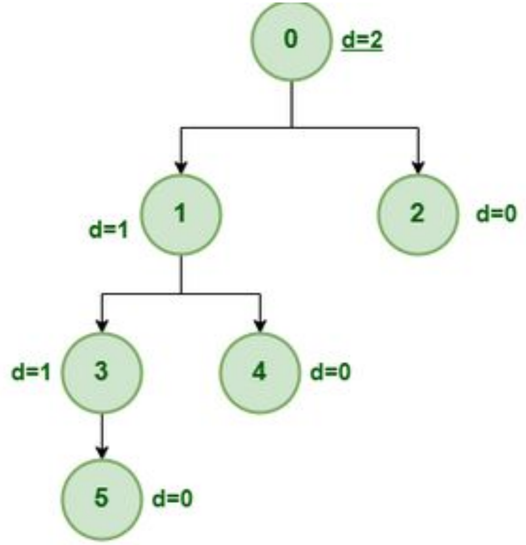


Tree Review:

- **Balanced Binary Tree**: The heights of any node's left and right subtrees differ by at most one



Balanced Binary Tree with depth at each level indicated
Depth of a node = |height of left child - height of right child|

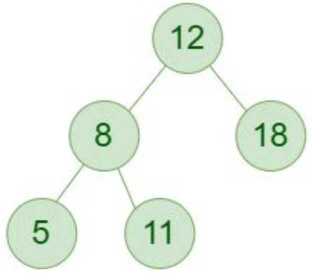


Unbalanced Binary Tree with depth at each level indicated
Depth of a node = |height of left child - height of right child|

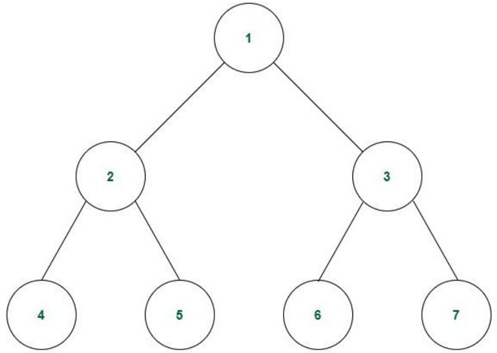
Tree Review:

- **Binary Search Tree (BST)**: A binary tree in which: (1)The left subtree of a node contains only nodes with keys lesser than the node's key. (2) The right subtree of a node contains only nodes with keys greater than the node's key. (3) The left and right subtree each must also be a binary search tree.

Binary Search Tree:



Not a Binary Search Tree:



Transform-and-Conquer Algorithms

Third Example

AVL Trees

- Named after the authors: Adelson, Velsky, and Landis (1962)
- Each insertion (or deletion) may require balancing - called **rotation**
    - Simple rotation
        - Left rotation or Right rotation
            - With or without children
    - Double rotation
        - Right-left rotation or Left-right rotation
            - With or without children
- A formal definition:
    - https://en.wikipedia.org/wiki/AVL_tree

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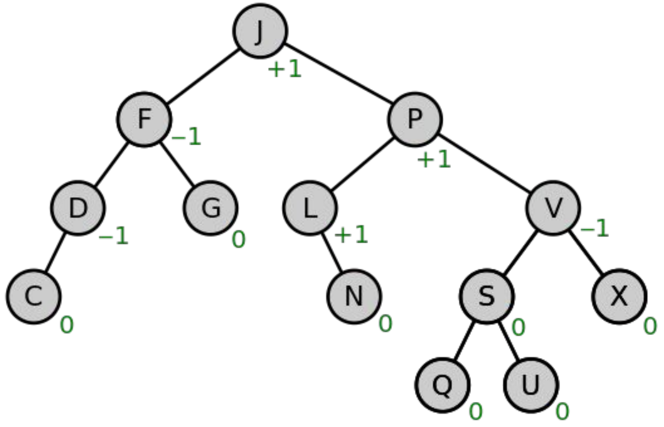
Transform-and-Conquer Algorithms

Third Example



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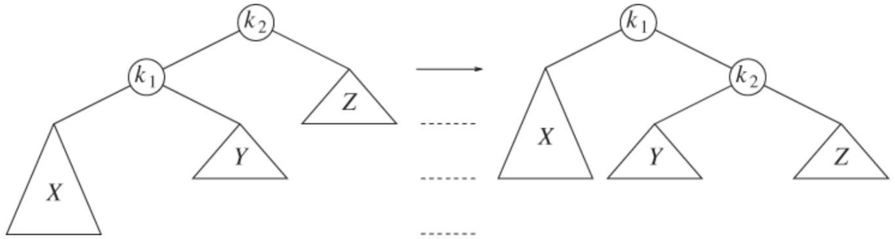
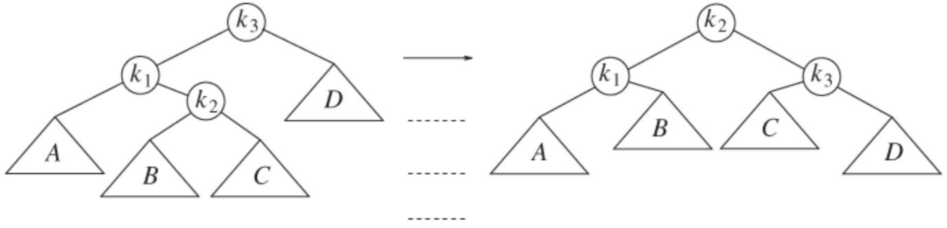
AVL Trees

- Single Rotation



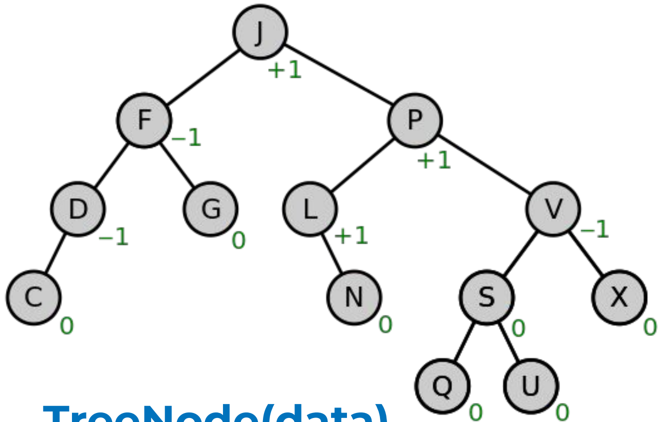
- Double Rotation



White board demonstration; race between AVL tree and linked list

Transform-and-Conquer Algorithms

Third Example



TreeNode(data)
AVLTree()

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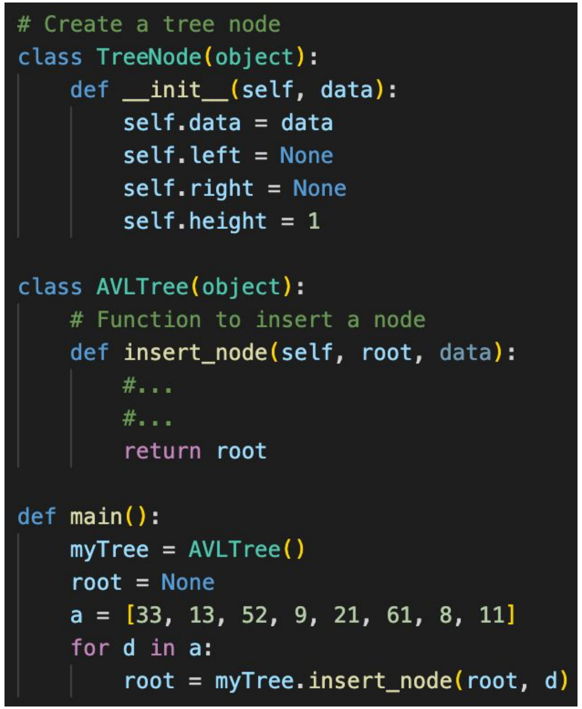
AVL Trees



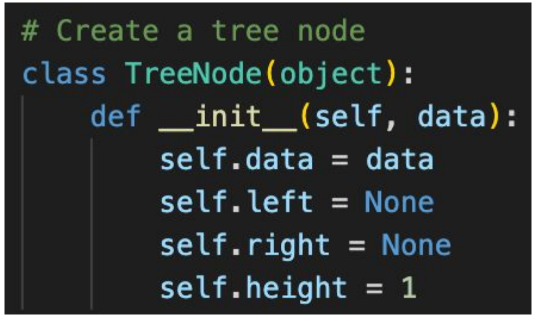
```
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1

class AVLTree(object):
    # Function to insert a node
    def insert_node(self, root, data):
        #...
        #...
        return root

def main():
    myTree = AVLTree()
    root = None
    a = [33, 13, 52, 9, 21, 61, 8, 11]
    for d in a:
        root = myTree.insert_node(root, d)
```

- A tree is defined by a root node (and its children)
- No need to have a tree constructor
- Just inserting a node in a root TreeNode that may be None (empty) or not

- Implementation details: https://www.programiz.com/dsa/avl-tree

Transform-and-Conquer Algorithms

Third Example



```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

insert_node(root, data)

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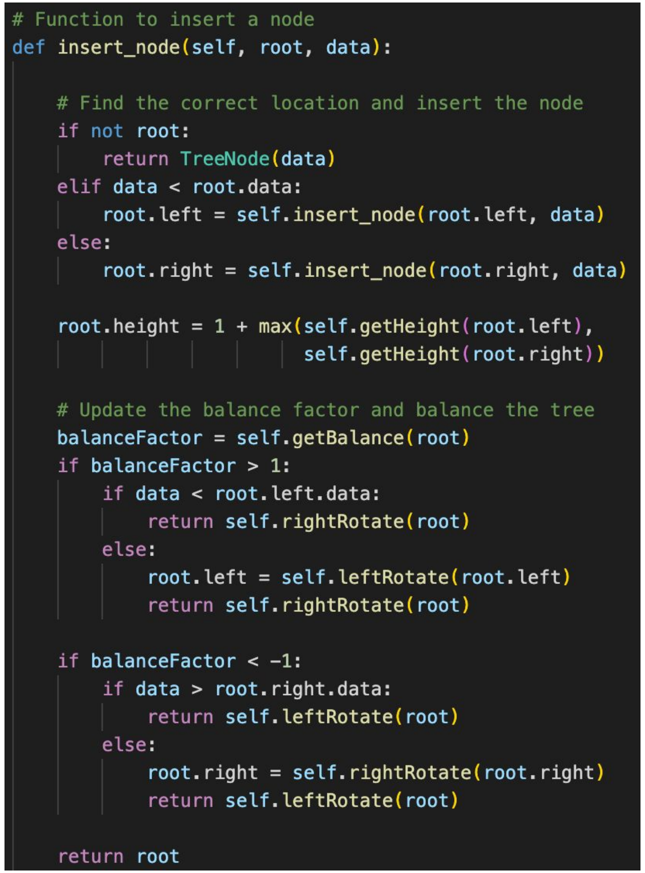
AVL Trees

- If the root is empty, insert in the root, otherwise, insert left or right
- Update the height
- If unbalanced (tallest right - balanceFactor > 1), rotate right
- If unbalanced (tallest left - balanceFactor < -1), rotate left

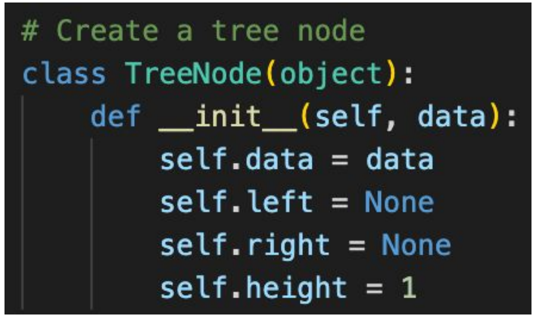


```python
# Function to insert a node
def insert_node(self, root, data):

    # Find the correct location and insert the node
    if not root:
        return TreeNode(data)
    elif data < root.data:
        root.left = self.insert_node(root.left, data)
    else:
        root.right = self.insert_node(root.right, data)

    root.height = 1 + max(self.getHeight(root.left),
                          self.getHeight(root.right))

    # Update the balance factor and balance the tree
    balanceFactor = self.getBalance(root)
    if balanceFactor > 1:
        if data < root.left.data:
            return self.rightRotate(root)
        else:
            root.left = self.leftRotate(root.left)
            return self.rightRotate(root)

    if balanceFactor < -1:
        if data > root.right.data:
            return self.leftRotate(root)
        else:
            root.right = self.rightRotate(root.right)
            return self.leftRotate(root)

    return root
```

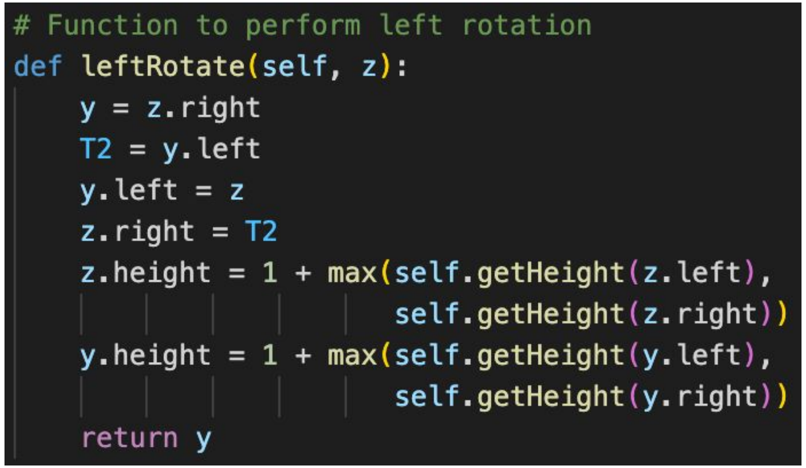
Transform-and-Conquer Algorithms

Third Example

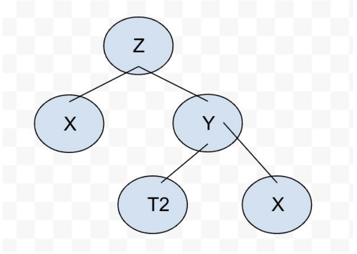
AVL Trees

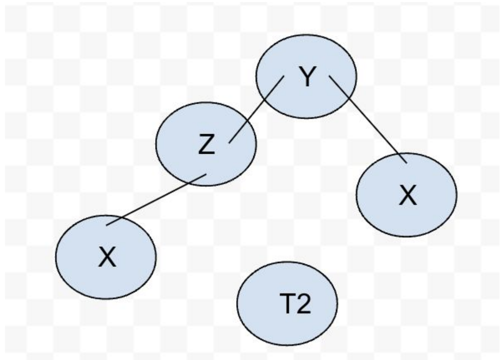


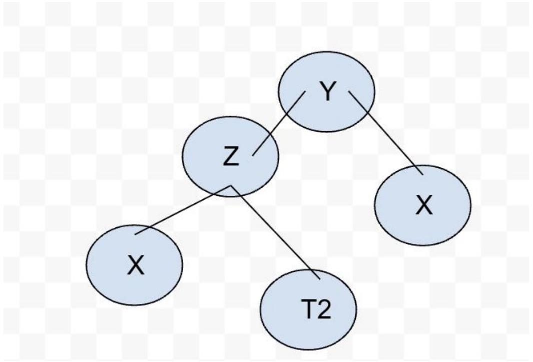
```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```



```python
# Function to perform left rotation
def leftRotate(self, z):
    y = z.right
    T2 = y.left
    y.left = z
    z.right = T2
    z.height = 1 + max(self.getHeight(z.left),
                       self.getHeight(z.right))
    y.height = 1 + max(self.getHeight(y.left),
                       self.getHeight(y.right))
    return y
```







leftRotate(z)

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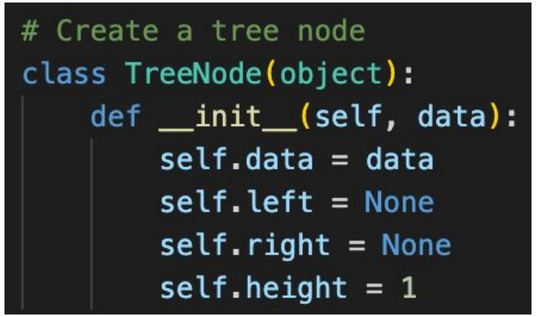
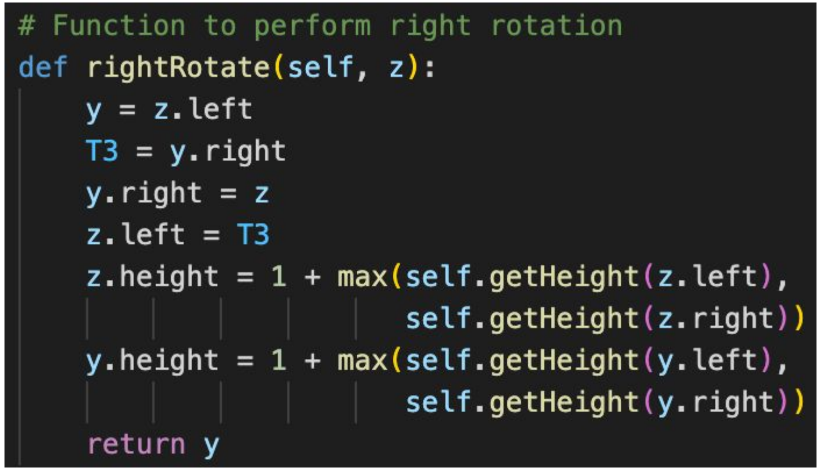
Transform-and-Conquer Algorithms

AVL Trees

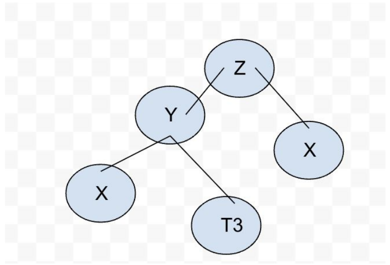
Third Example

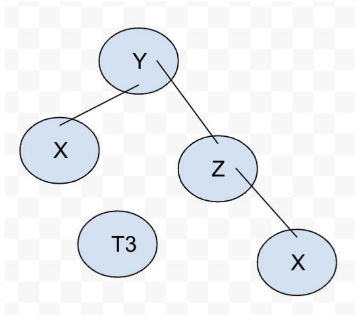


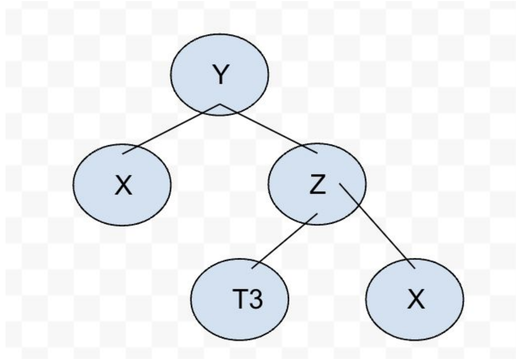
```
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```



```
# Function to perform right rotation
def rightRotate(self, z):
    y = z.left
    T3 = y.right
    y.right = z
    z.left = T3
    z.height = 1 + max(self.getHeight(z.left),
                        self.getHeight(z.right))
    y.height = 1 + max(self.getHeight(y.left),
                        self.getHeight(y.right))
    return y
```







rightRotate(z)

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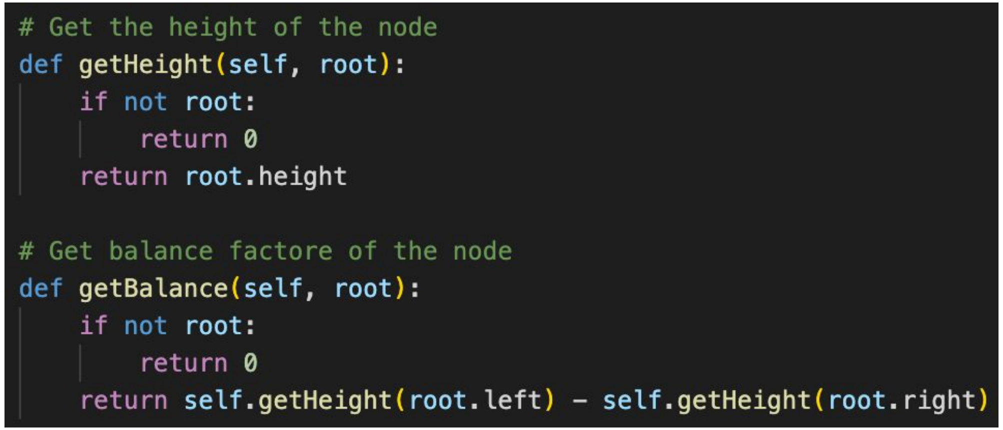
Transform-and-Conquer Algorithms

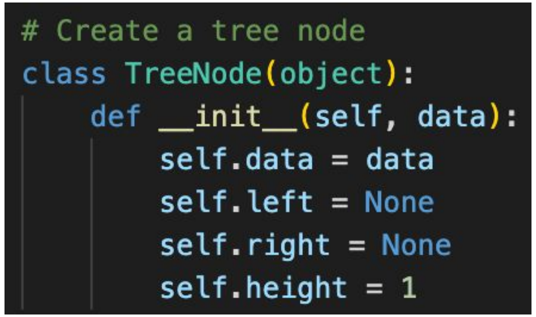
Third Example

AVL Trees



```python
# Get the height of the node
def getHeight(self, root):
    if not root:
        return 0
    return root.height

# Get balance factore of the node
def getBalance(self, root):
    if not root:
        return 0
    return self.getHeight(root.left) - self.getHeight(root.right)
```



```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

getHeight(root)
getBalance(root)

- Recursive implementations used in the insertion and deletion functions
  - insert_node
  - delete_node

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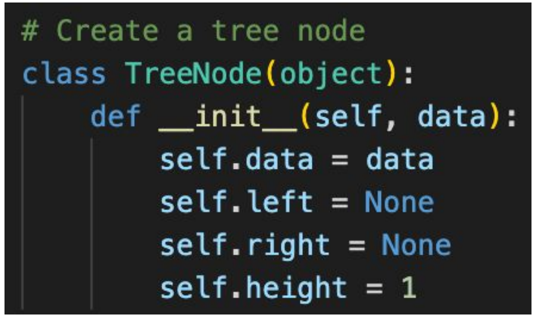
Let’s run some code. demo2

Transform-and-Conquer Algorithms

Third Example



```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

delete_node(root, data)

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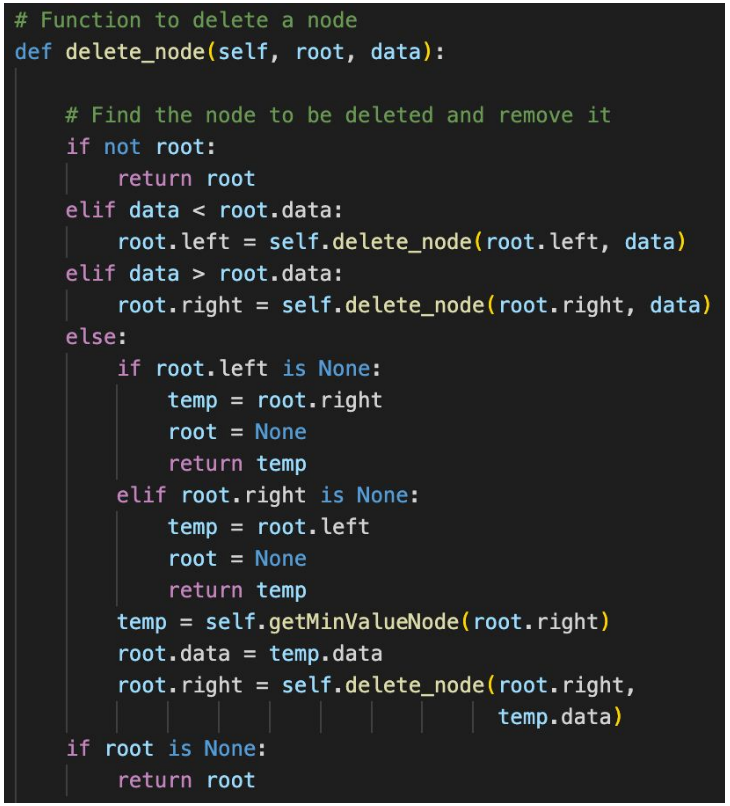
AVL Trees

- If empty, do nothing
- If smaller search left
- If greater, search right
- If the node is the one to delete, pull right if left is empty, or pull left if right is empty, otherwise, pull the minimum data at right



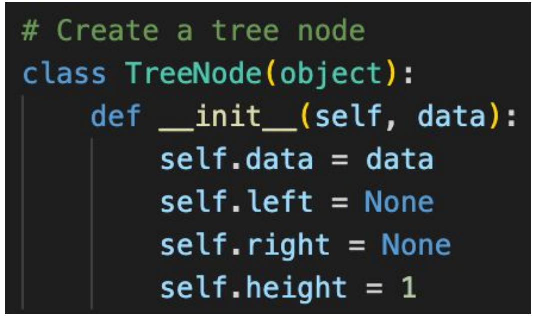
```python
# Function to delete a node
def delete_node(self, root, data):

    # Find the node to be deleted and remove it
    if not root:
        return root
    elif data < root.data:
        root.left = self.delete_node(root.left, data)
    elif data > root.data:
        root.right = self.delete_node(root.right, data)
    else:
        if root.left is None:
            temp = root.right
            root = None
            return temp
        elif root.right is None:
            temp = root.left
            root = None
            return temp
        temp = self.getMinValueNode(root.right)
        root.data = temp.data
        root.right = self.delete_node(root.right,
                                      temp.data)
    if root is None:
        return root
```

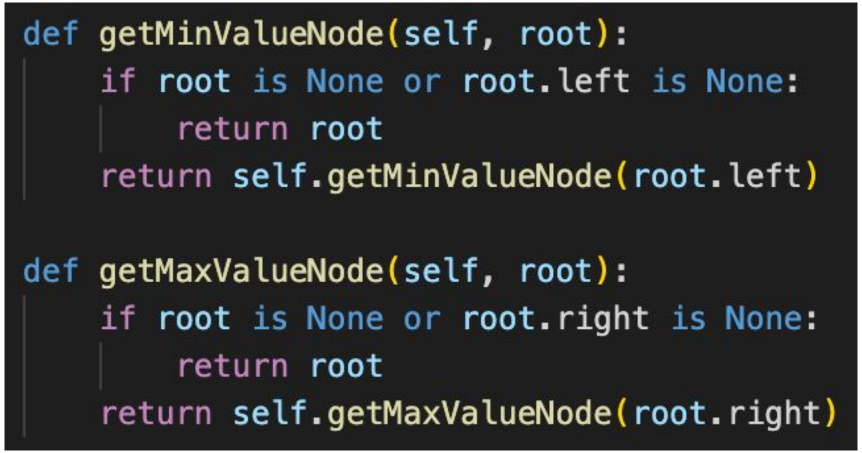
Transform-and-Conquer Algorithms

Third Example

AVL Trees


```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

getMinValueNode(root)
getMaxValueNode(root)


```python
def getMinValueNode(self, root):
    if root is None or root.left is None:
        return root
    return self.getMinValueNode(root.left)

def getMaxValueNode(self, root):
    if root is None or root.right is None:
        return root
    return self.getMaxValueNode(root.right)
```

- Recursive implementations used in the deletion function (and also available to search)
    - delete_node

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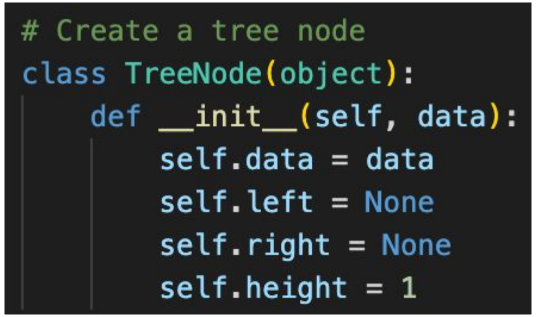
Transform-and-Conquer Algorithms

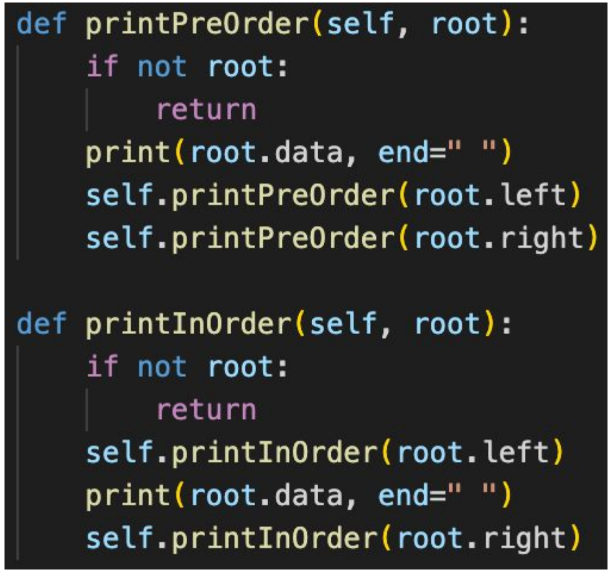
Third Example



```
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

printPreOrder(root)
printInOrder(root)

AVL Trees



```
def printPreOrder(self, root):
    if not root:
        return
    print(root.data, end=" ")
    self.printPreOrder(root.left)
    self.printPreOrder(root.right)

def printInOrder(self, root):
    if not root:
        return
    self.printInOrder(root.left)
    print(root.data, end=" ")
    self.printInOrder(root.right)
```

- Printing using tree traversals
  - Preorder
  - Inorder
- Can you implement Postorder?
- See In-class exercises #3 in the next slide…

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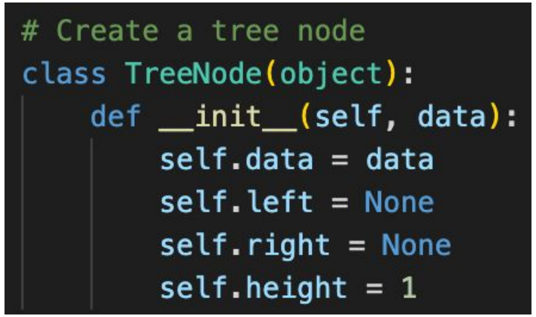
Transform-and-Conquer Algorithms

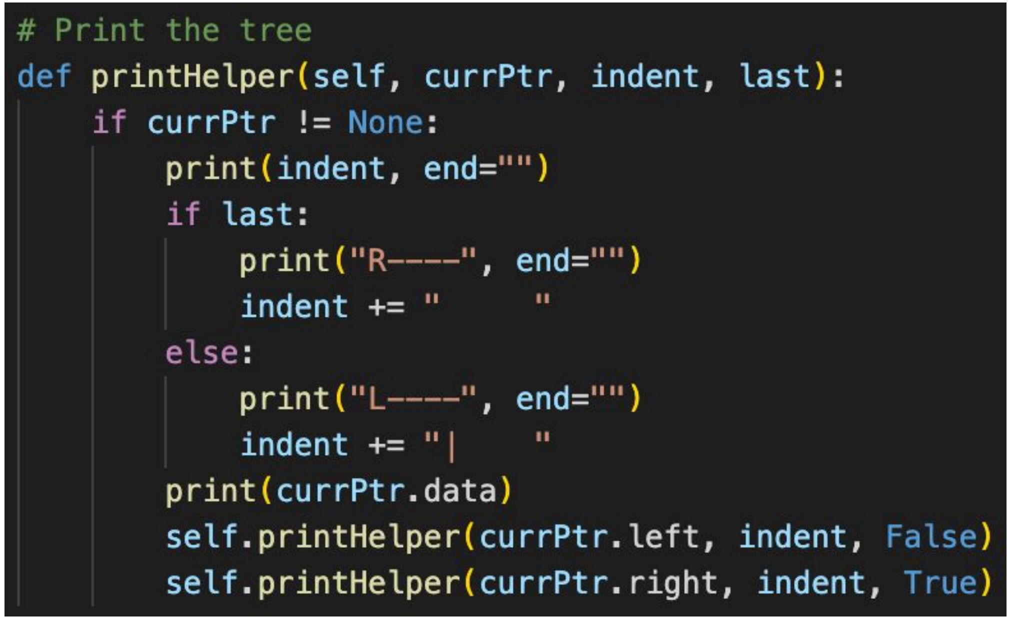
Third Example

AVL Trees



```python
# Create a tree node
class TreeNode(object):
    def __init__(self, data):
        self.data = data
        self.left = None
        self.right = None
        self.height = 1
```

printHelper(root)

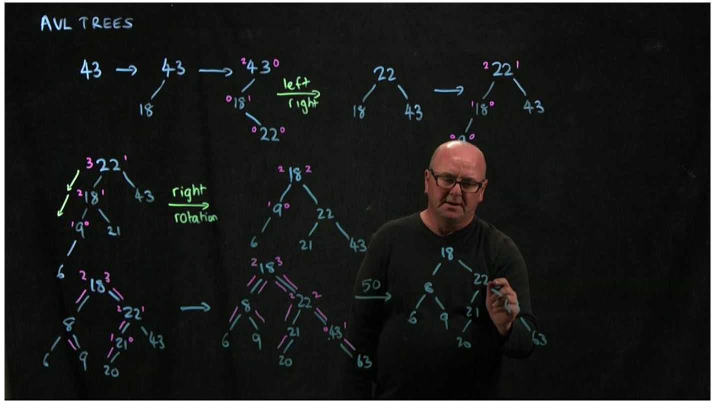


```python
# Print the tree
def printHelper(self, currPtr, indent, last):
    if currPtr != None:
        print(indent, end="")
        if last:
            print("R----", end="")
            indent += "     "
        else:
            print("L----", end="")
            indent += "|    "
        print(currPtr.data)
        self.printHelper(currPtr.left, indent, False)
        self.printHelper(currPtr.right, indent, True)
```

- Just a human readable print version to visualize the AVLTree (and yes, it is recursive)

Transform-and-Conquer Algorithms

Third Example



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AVL Trees

- Now try it out the usage example downloading the avl.py file
  - https://drive.google.com/file/d/1hSnE3w4fXmWfupeMFKhJTshU0f66Fes3/view?usp=sharing
  - Feel free to edit it inserting and deleting elements to see the example at work
- The complexity of the insertion and deletion operations is ***O***(log *n*)
- Formal definition:
  - https://en.wikipedia.org/wiki/AVL_tree
- Implementation details:
  - https://www.programiz.com/dsa/avl-tree

Advanced Data Structures

Worksheet

Task #1 for this week's Worksheet

- Create a program that asks the user repeatedly positive integer numbers and stores/deletes it in an AVL tree (use the code seen in the third example)
    - *If the user enters a number already in the tree, delete it*
    - If the user enters a number that is not in the tree, insert it
    - After each insertion/deletion, the program prints the current tree using the printHelper method
    - When the user enters a non positive integer, the program ends
    - The entire program must run in logarithmic time.
- Besides the implementation of your program, write a short report describing your experiences.

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Advanced Data Structures

Worksheet

Task #1 for this week's Worksheet

- This program must be your own, do not use someone else's code
- Any specific questions about it, please bring to the Office hours meeting this Monday or contact me by email
- This is a challenging program to make sure you are mastering your Python programming skills, as well as your asymptotic analysis understanding
- Don't be shy with your questions

Go to IDLE and try to program it
Save your program in a .py file and submit it in the appropriate delivery room

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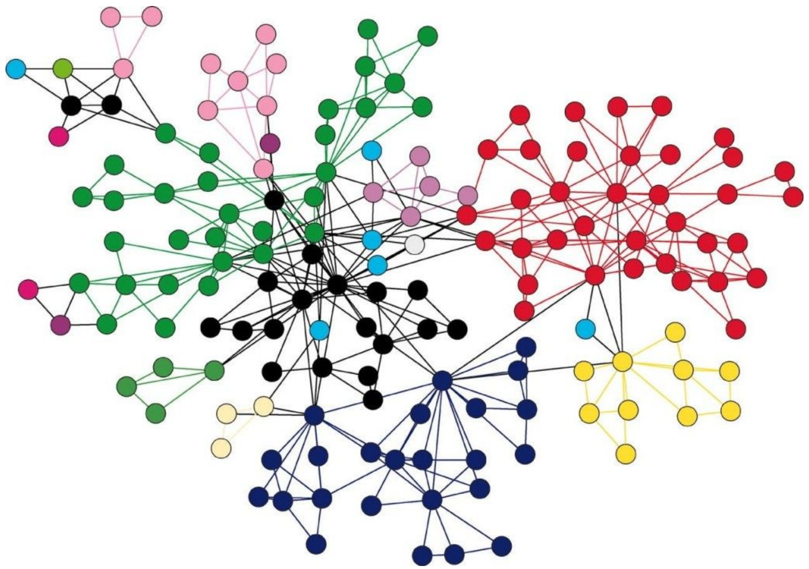
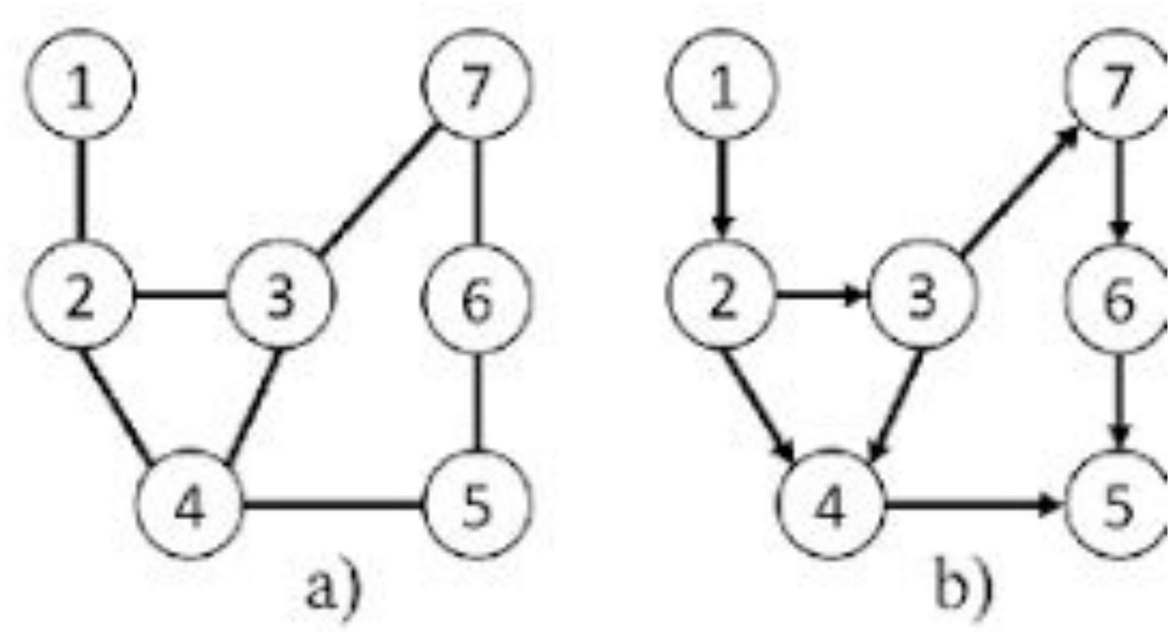
Graphs as Data Structures



- Data structure which:
  - Consists of a finite set of nodes (or vertices)
  - Together with a set of unordered pairs (or edges) of these vertices for an undirected graph or a set of ordered pairs for a directed graph
  - In a weighted graph, a numeric value is attributed to each edge - for example representing the distance between two points in space

Types of Graphs: undirected vs directed

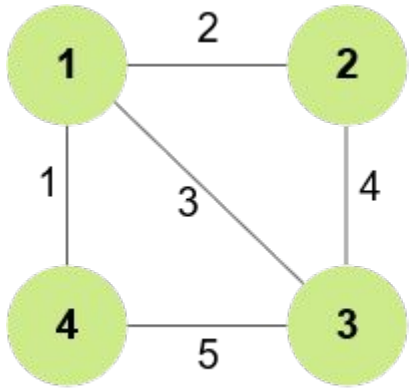


Undirected

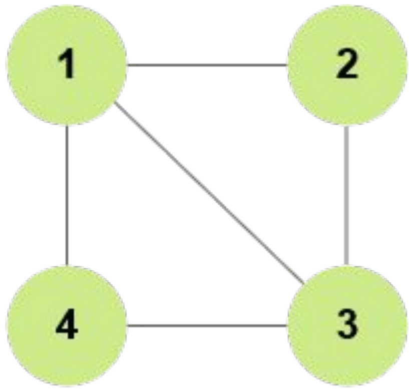
Directed

Source: researchgate.net

Types of Graphs: weighted vs unweighted



Weighted Graph



Unweighted Graph

Source: prepfortech.io

Common Applications for Graphs

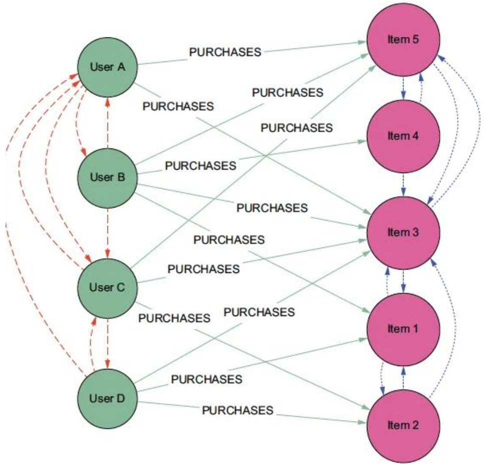
Social Networks: Representing relationships between users, where nodes are users and edges are connections.

Web Page Linking: Modeling the internet as a graph, where web pages are nodes and hyperlinks are directed edges.

Routing Algorithms: Using graphs to find the shortest path in networks, such as in GPS navigation systems.

Recommendation Systems: Leveraging user-item interactions as a graph to suggest products or services.

Cranberry Twp
Wexford
Indiana
Tyrone
Altoona
Hollidaysburg
2 hr 19 min
97.3 miles
Bethel Park
Greensburg
Latrobe
Johnstown
Washington
70
1 hr 48 min
107 miles
Somerset
Bedford



Sources:
facebook.com
Google maps
graphaware.com

Common Problems for Graphs

Graph Traversal: Exploring nodes and edges systematically (e.g., Depth-First Search, Breadth-First Search).

Minimum Spanning Tree: Connecting all nodes with the minimum total edge weight (e.g., Prim's algorithms).

Cycle Detection: Identifying cycles in a graph, which is crucial for certain applications like deadlock detection.

Shortest Path: Finding the shortest route between nodes (e.g., Dijkstra's algorithm).

Brute force approach to shortest path

Brute Force Approach:

1. **Generate all possible paths** from the source node to the destination.
2. **Calculate the total weight** of each path.
3. **Select the shortest path** among them

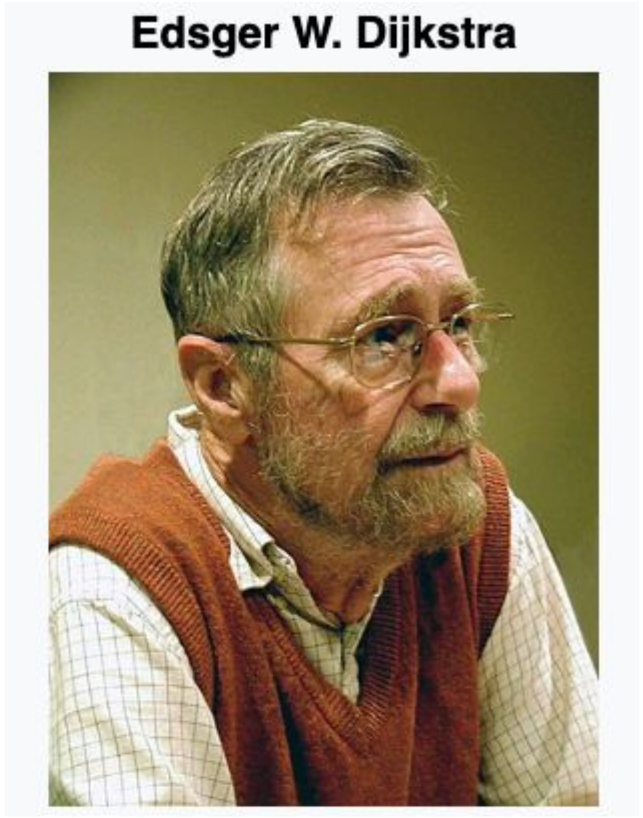
If there are **V** vertices, the number of ways to visit them is **V!**.

Checking each path requires summing the weights of at most **V−1** edges, which is **O(V)**.

The overall time complexity is **O(V! · V)** in the worst case.

Dijkstra's Algorithm Background

- Proposed by computer scientist Edsger Dijkstra in late 1950s.
- Finds the shortest path from a source node to every other node
- Many applications

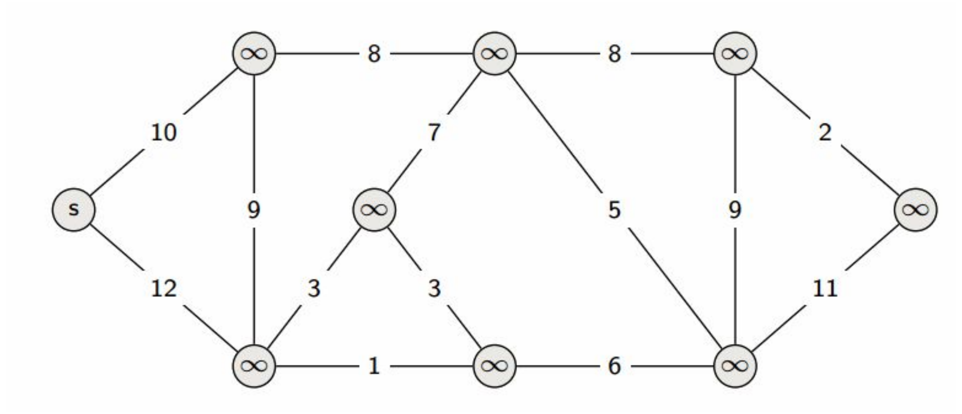


Dijkstra's Algorithm Basic Approach

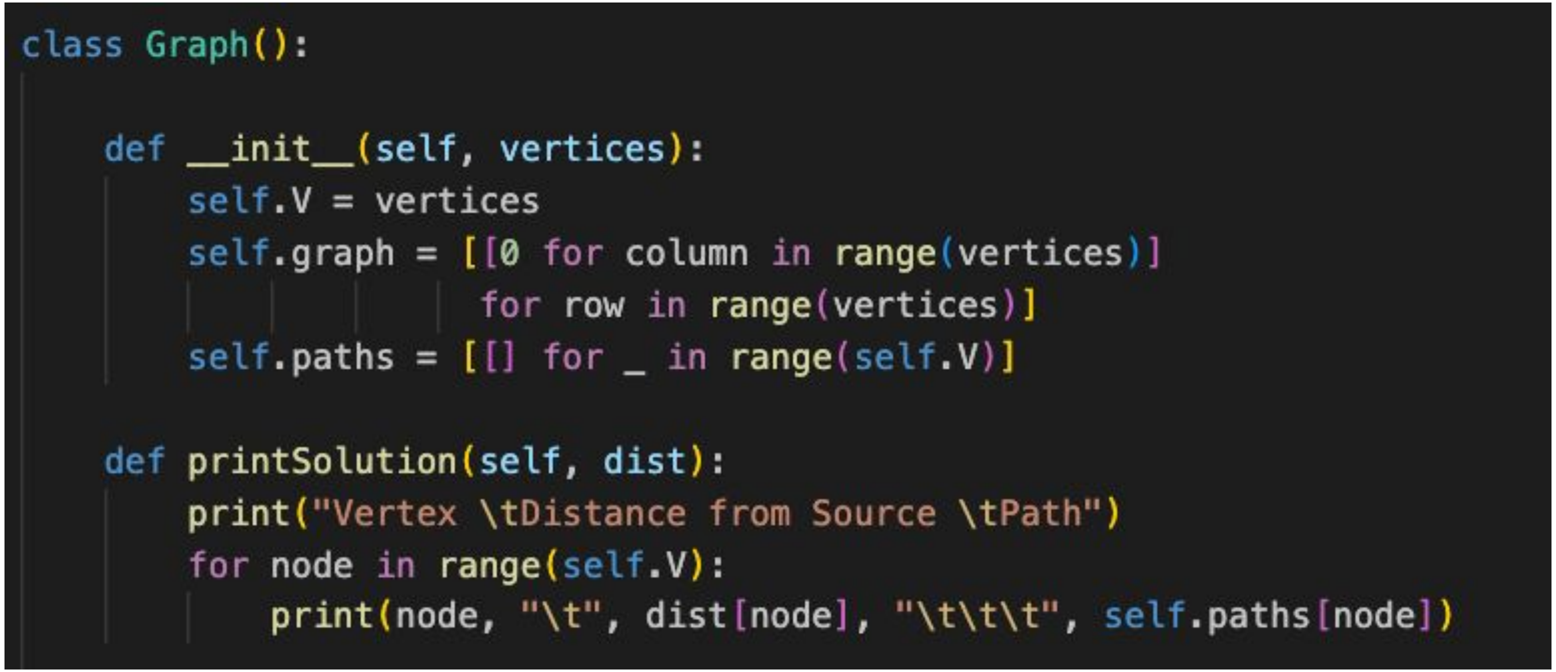
1. Create a set of unvisited nodes (sptSet in our code)
2. Assign every node a distance from the source node, with the default set to an infinite or very high value. During execution of the algorithm, the value will change to the value of the shortest discovered path.
3. From the univisted set, select the node with the smallest distance; this will start with the source node whose distance is 0.

   This is “minDistance” in our code.

4. Consider all neighbors of the next node:
   a. (This step is found in the y loop in our code)
   b. Mark the node as processed/visited in the sptSet
   c. Consider all unvisited neighbors
   d. If shorter paths than previously known are discovered, update.
5. Once the outer loop is complete and all reachable nodes have been visited, all distances have been updated with the shortest path distance and the paths have been saved.

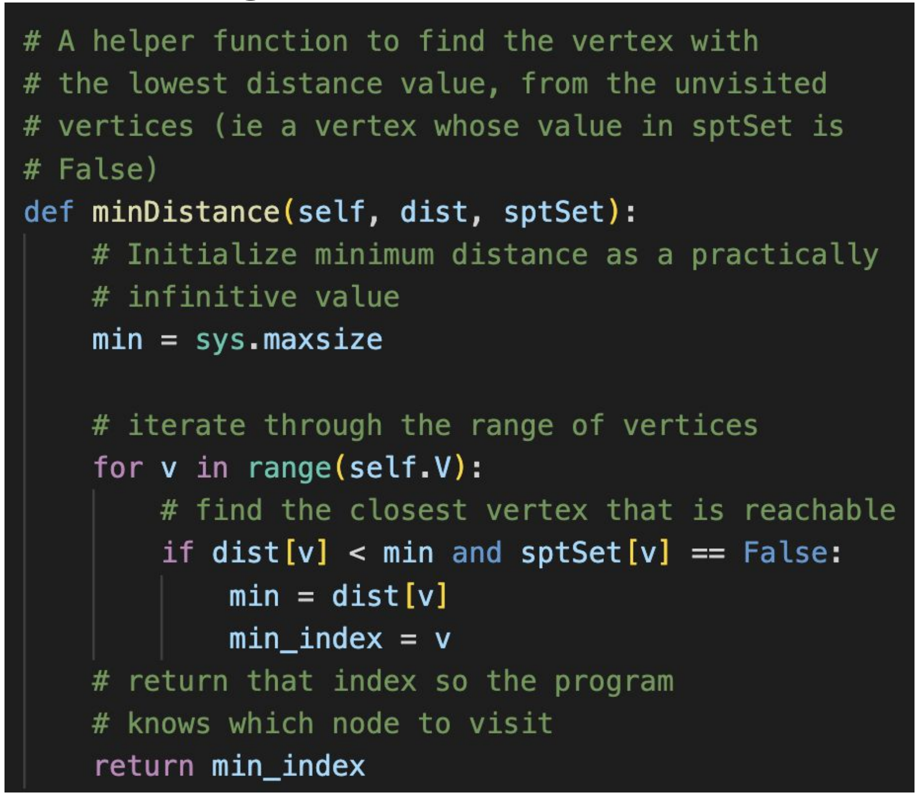


Dijkstra’s Algorithm Code



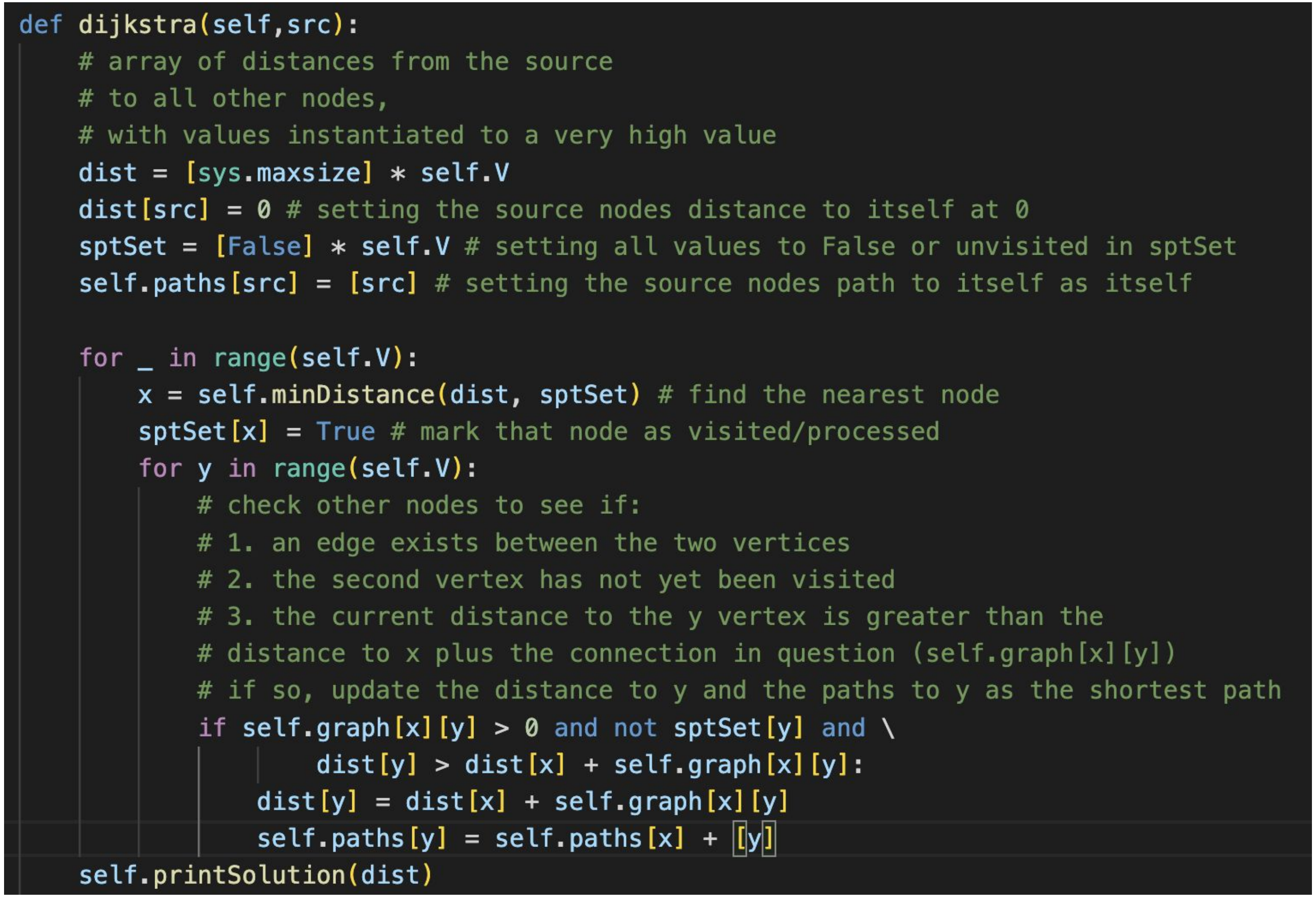
```python
class Graph():

    def __init__(self, vertices):
        self.V = vertices
        self.graph = [[0 for column in range(vertices)]
                      for row in range(vertices)]
        self.paths = [[] for _ in range(self.V)]

    def printSolution(self, dist):
        print("Vertex \tDistance from Source \tPath")
        for node in range(self.V):
            print(node, "\t", dist[node], "\t\t\t", self.paths[node])
```

Dijkstra’s Algorithm Code



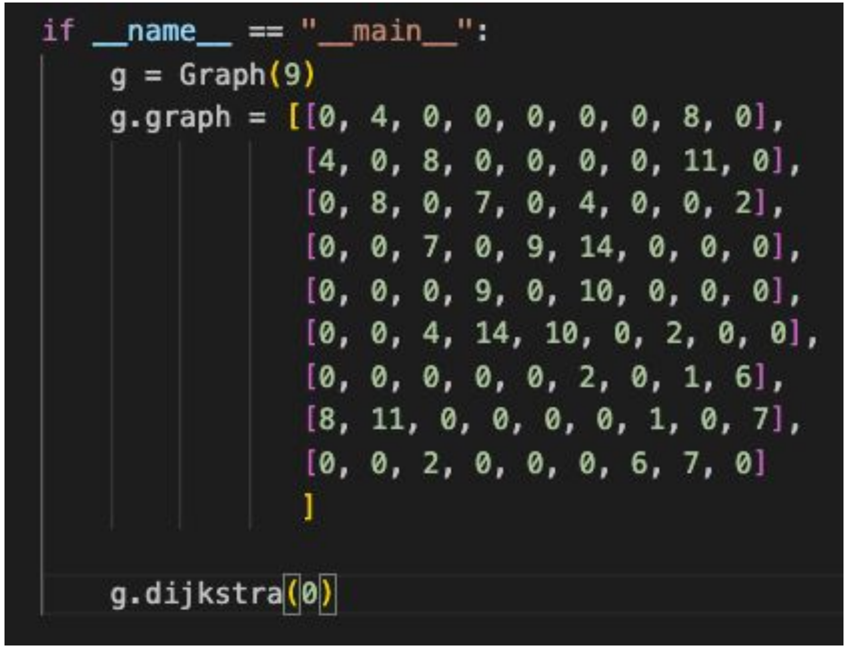
```python
# A helper function to find the vertex with
# the lowest distance value, from the unvisited
# vertices (ie a vertex whose value in sptSet is
# False)
def minDistance(self, dist, sptSet):
    # Initialize minimum distance as a practically
    # infinitive value
    min = sys.maxsize

    # iterate through the range of vertices
    for v in range(self.V):
        # find the closest vertex that is reachable
        if dist[v] < min and sptSet[v] == False:
            min = dist[v]
            min_index = v
    # return that index so the program
    # knows which node to visit
    return min_index
```

Dijkstra’s Algorithm Code

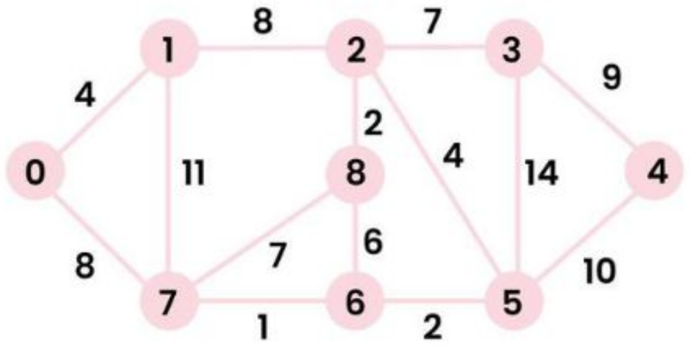


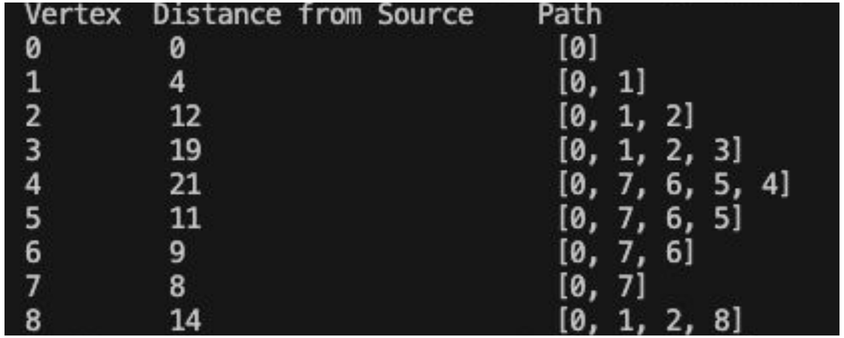
```python
def dijkstra(self,src):
    # array of distances from the source
    # to all other nodes,
    # with values instantiated to a very high value
    dist = [sys.maxsize] * self.V
    dist[src] = 0 # setting the source nodes distance to itself at 0
    sptSet = [False] * self.V # setting all values to False or unvisited in sptSet
    self.paths[src] = [src] # setting the source nodes path to itself as itself

    for _ in range(self.V):
        x = self.minDistance(dist, sptSet) # find the nearest node
        sptSet[x] = True # mark that node as visited/processed
        for y in range(self.V):
            # check other nodes to see if:
            # 1. an edge exists between the two vertices
            # 2. the second vertex has not yet been visited
            # 3. the current distance to the y vertex is greater than the
            # distance to x plus the connection in question (self.graph[x][y])
            # if so, update the distance to y and the paths to y as the shortest path
            if self.graph[x][y] > 0 and not sptSet[y] and \
                    dist[y] > dist[x] + self.graph[x][y]:
                dist[y] = dist[x] + self.graph[x][y]
                self.paths[y] = self.paths[x] + [y]
    self.printSolution(dist)
```

Dijkstra’s Algorithm Example



```
if __name__ == "__main__":
    g = Graph(9)
    g.graph = [[0, 4, 0, 0, 0, 0, 0, 8, 0],
               [4, 0, 8, 0, 0, 0, 0, 11, 0],
               [0, 8, 0, 7, 0, 4, 0, 0, 2],
               [0, 0, 7, 0, 9, 14, 0, 0, 0],
               [0, 0, 0, 9, 0, 10, 0, 0, 0],
               [0, 0, 4, 14, 10, 0, 2, 0, 0],
               [0, 0, 0, 0, 0, 2, 0, 1, 6],
               [8, 11, 0, 0, 0, 0, 1, 0, 7],
               [0, 0, 2, 0, 0, 0, 6, 7, 0]
               ]

    g.dijkstra(0)
```





```
Vertex  Distance from Source    Path
0        0                       [0]
1        4                       [0, 1]
2        12                      [0, 1, 2]
3        19                      [0, 1, 2, 3]
4        21                      [0, 7, 6, 5, 4]
5        11                      [0, 7, 6, 5]
6        9                       [0, 7, 6]
7        8                       [0, 7]
8        14                      [0, 1, 2, 8]
```

Complexity of Dijkstra’s Algorithm

- Answer depends on a number of factors like what type of graph implementation is being used.
- In our implementation, where we are using adjacency matrix representations of the graphs, the time complexity is $O(V^2)$ where V is the number of vertices.

Programming Assignment

Project #7 - this week's Assignment

- Create a program that implements the Dijkstra's algorithm program with the Lord of the Rings example case
- You will need to represent the map of Middle Earth as an adjacency matrix and then feed it into the program
- Your program should print the shortest path from the Shire to various points in Middle Earth including the paths and the distances along the way.

Dijkstra’s Algorithm Assignment: Lord of the Rings

Help Frodo plan a trip from the Shire to the crags of Mt. Doom so that he can destroy the One Ring and save Middle Earth.

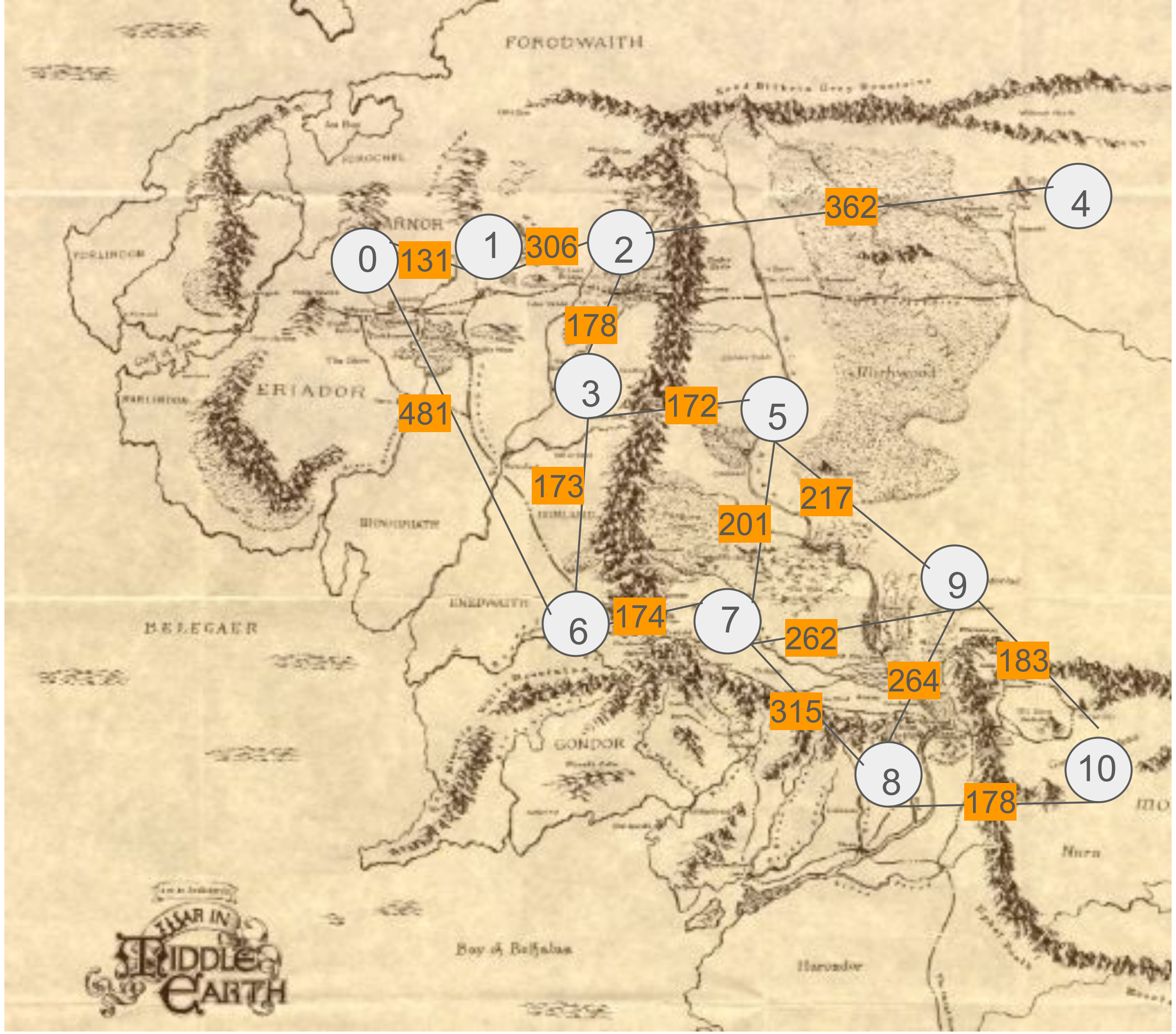
NORTHERN WASTE
FORODWAITH
ARNOR
ERIADOR
BELEGAER
GONDOR
MORDOR
RHÛN
Mirkwood
Nurn
Bay of Belfalas
Harondor
KHAND
HARADWAITH
The War in Middle Earth

Dijkstra’s Algorithm Assignment: Lord of the Rings

- Write a program that uses the Dijkstra’s Algorithm program to determine the shortest paths from the Shire to other places Middle Earth.
- Specifically the program should determine the minimum distance Frodo would have to travel from the Shire to Rivendell, and then from Rivendell to Mt. Doom, so your program will have to run the algorithm twice.
- Your program should print out the results and the total amount Frodo needs to travel.

0. The Shire
1. Bree
2. Rivendell
3. Moria
4. Dale
5. Lorien
6. Isengard
7. Edoras
8. Minas Tirith
9. Emyn Muil
10. Mt. Doom

Note: path suggested by the algorithm may differ from actual path.



The road goes ever on and on …

Programming Assignment

Project #2 - this week's Assignment

- This program must be your own, do not use someone else's code
- Any specific questions about it, please bring to the Office hours meeting this Monday or contact me by email
- This is a challenging program to make sure you are mastering your Python programming skills, as well as your asymptotic analysis understanding
- Don't be shy with your questions

Go to IDLE and try to program it
Save your program in a .py file and submit it in the appropriate delivery room

That's all for today folks!

This week's tasks

- Discussion: initial post by this Friday/replies by next Tuesday
- Tasks #1, #2, #3 and #4 for the In-class exercises
  - Deadline: Friday 11:59 PM EST
- Quiz #4 to be available this Friday
  - Deadline: Next Monday 11:59 PM EST
- Project #4 assignment
  - Deadline: Next Monday 11:59 PM EST

- Try all exercises seen in class and consult the reference sources, as the more you practice, the easier it gets

MERRIMACK COLLEGE

Have a Great Week!