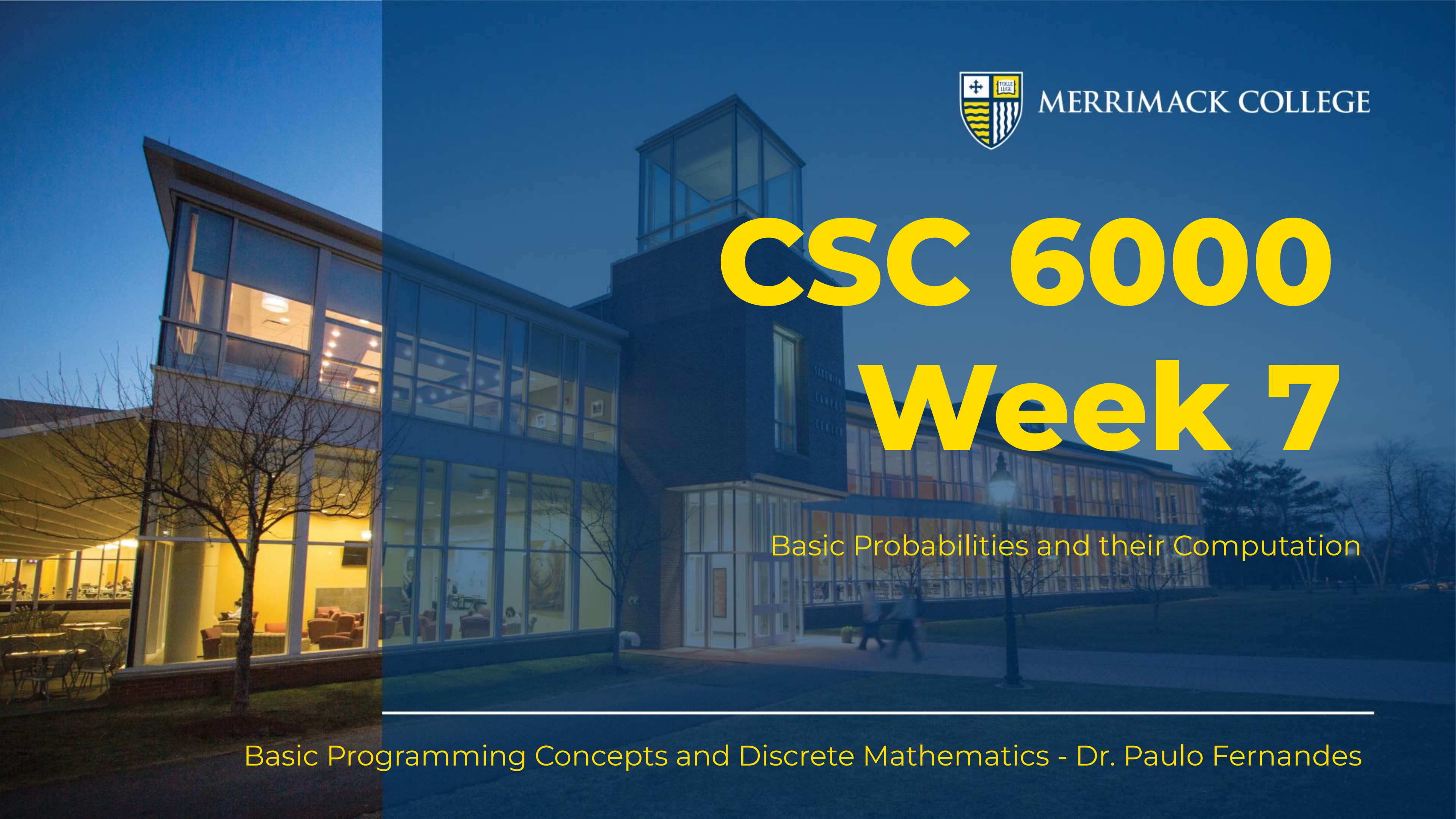


MERRIMACK COLLEGE

CSC 6000 Week 7

Basic Probabilities and their Computation

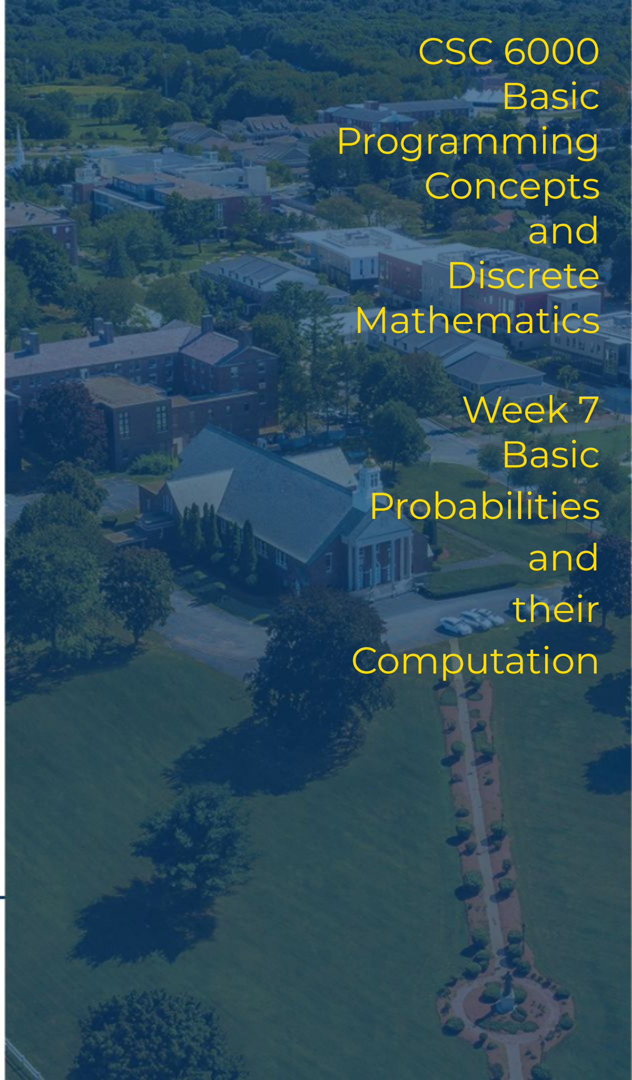
Basic Programming Concepts and Discrete Mathematics - Dr. Paulo Fernandes

Presentation Agenda

Week 7

- Probabilities
  a. Probabilities range
  b. Sum of all probabilities
  c. Probability of some events
- Basic Rules
  a. Addition rule
  b. Multiplication rule
  c. Conditional probability rule
  d. Complement rule
- This Week's tasks

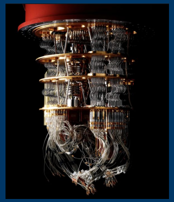
MERRIMACK COLLEGE



Probabilities

The most important questions of life are indeed, for the most part, really only problems of probability.

Pierre-Simon de Laplace



How can you be certain of something?

Usually, we just estimate the odds of situations, and we tend to assume that some things are certain, while others might not be.

Setting aside the situation where we are certain, is it possible to estimate with a given number how likely something can actually be?

This number is basically defined by:

- **Probabilities range**,
- Sum of all probabilities, and
- Probability of some events.

Probability

How likely is it that an event will occur?

According to Jeffrey, "*Before the middle of the seventeenth century, the term 'probable' (Latin* probabilis*) meant approvable, and was applied in that sense, univocally, to opinion and to action. A probable action or opinion was one such as sensible people would undertake or hold, in the circumstances.*"

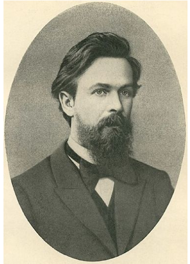
However, by the the seventeenth century, gambling became extremely popular, especially among the French nobility, so there was funding to study how likely one was to win or lose. Since then probability has led to, among other fields, Quantum Physics and Artificial Intelligence.

Richard C. **Jeffrey**
1926 - 1999 - 2002

Gerolamo **Cardano**
1501 - 1539 - 1576

Christiaan **Huygens**
Lord of Zeelhem
1629 - 1671 - 1695

Pierre-Simon
Marquis de **Laplace**
1749 - 1817 - 1827



Andrei Andreyevich
Markov
1856 - 1886 - 1922

MERRIMACK COLLEGE

Wikipedia: Probability.

Probability Range

How probable is it for something to happen?

- If I throw a basketball towards the basket I might score it, or I might not.
- Someone better than me would score more often...

How can we quantify the likelihood of something to happen?

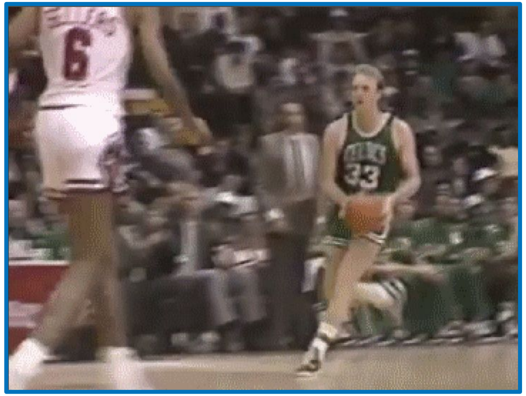
- If we know nothing about it, a good answer is there is a 50% chance, but...
- If I know more details I can say there is a ***x***% chance to happen:
  - where **x = 0** is "sure to not happen" and **x = 100** is "sure to happen"!

Remember
50% actually means 50 divided by 100
from Latin: per cent

MERRIMACK COLLEGE

Probabilities are Real numbers ranging from 0 to 1.

Probability and Randomness



How probable is it for something to happen?

The assignment of a probability is always a choice at the beginning.

- Seeing the image on the side, what is the probability of the player scoring his shot?
  - 110%
  - 100%
  - 85%
  - 50%
  - 25%
  - 0%

This answer may be correctly answered by five different guys:

- John answering 50% - John knows nothing about the image
- Jack answering 100% - Jack knows the player
- Joe answering 85% - Joe thinks: must be something worthy recording
- Jim answering 25% - Jim thinks: the player is running, so it is hard
- Jeff answering 0% - Jeff thinks: the previous slide had a miss

A probability is not right because it predicts the future.

Probability and Randomness

What is a probability?

It is an educated (but scientific!) guess about the outcome of a certain number of events of a specific kind.

- Now that you know the outcome of this particular shot, the probability becomes 100%, which is not random anymore;
- However, knowing only what you knew before, the five answers were correct.

This scene and other amazing shots from Larry Bird can be found everywhere, including here.

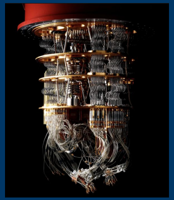
A probability is bound to be correct by **the law of large numbers**. This means, you can state a probability that will not predict the outcome of one try, but that will predict the **average outcome of a sufficiently large number of tries**.

Probabilities 0% and 100% are deterministic events.
All other probabilities in between are random events.

Probabilities

The most important questions of life are indeed, for the most part, really only problems of probability.

Pierre-Simon de Laplace



How can you be certain of something?

Usually, we just estimate the odds of situations, and we tend to assume that some things are certain, while others might not be.

Setting aside the situation where we are certain, is it possible to estimate with a given number how likely something can actually be?

This number is basically defined by:

- Probabilities range,
- **Sum of all probabilities**, and
- Probability of some events.

Sum of All Probabilities

What is a probability?

It is the prediction of the **average outcome of a sufficiently large number of tries**.

For example, assume you have two perfectly balanced coins.

What is the probability of tossing both coins and get two heads?

The probability of tossing one coin and landing heads is 50%, and the probability of landing tails is also 50% (if you toss many, many times).

The sum has to be 100% (1) because in each toss it will always be either heads or tails (no other outcome possible).

Wikipedia: Law of Large Numbers.

Sum of All Probabilities

When you toss two coins there are four possible outcomes:

- Heads on the first coin, Heads on the second coin;
- Heads on the first coin, Tails on the second coin;
- Tails on the first coin, Heads on the second coin;
- Tails on the first coin, Tails on the second coin;

Since:

- all cases are equally likely to happen;
- those are all the possible cases (their sum is 1);
- It is possible to say that each case has a probability of ¼.

What is the probability of tossing the two coins and get two heads?

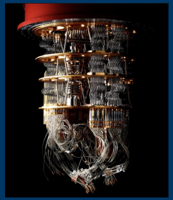
- one case, ¼ (**25%**).

Video: [All Probabilities sum to 1](#).

Probabilities

The most important questions of life are indeed, for the most part, really only problems of probability.

Pierre-Simon de Laplace



How can you be certain of something?

Usually, we just estimate the odds of situations, and we tend to assume that some things are certain, while others might not be.

Setting aside the situation where we are certain, is it possible to estimate with a given number how likely something can actually be?

This number is basically defined by:

- Probabilities range,
- Sum of all probabilities, and
- **Probability of some events**.

Probability of Some Events

Assuming a perfectly balanced die with six faces numbered from 1 to 6:

- All faces have the same probability to occur;

If you roll this die, what is the probability of getting a number 6?

- Since there are six possible outcomes and the sum of their probabilities is 1:
  - The probability is ⅙ (**16.6667%**).

What is the probability of rolling this die and getting a prime number as the outcome ?

- the possible outcomes are 1, 2, 3, 4, 5, and 6
  - 3 out of 6 are prime numbers, so the answer is 3/6 = **50%**.

Remember: 1 is not a prime number!

Probability of Some Events

What is the probability of rolling two of those perfectly balanced dice and getting two 6s?

- Each die has 6 possible outcomes, and there are two dice:
  - There are 6 times 6 = 36 possibilities.
- In only one possibility are there two 6s:
  - The answer is 1 out of 36, thus 1/36 (**2.7778%**).

If you roll 5 dice, what is the probability to get just one 6?

- Hint: You can get the 6 either on the first, second, third, fourth, or fifth die, and for each case all the other dice are not 6.

Don't go to the next slide before trying to solve it!

Probability of Some Events

If you roll 5 dice, what is the probability of getting just one 6?

- The total number of outcomes is ***6^5*** because each of the five dice can have six possible outcomes.
- Among those outcomes, the number of outcomes that has just one six and it is in the *first* die is ***5^4*** because only five outcomes (1, 2, 3, 4, and 5) can happen and there are four dice to hold them.
- That single number six can also be in the *second*, *third*, *fourth*, or *fifth* position, and in each case you have the same number of outcomes (***5^4***).

Thus: the probability is given by the number of cases with one 6 divided by the total number of cases:

$$\frac{5(5^4)}{6^5} = \frac{(5^5)}{6^5} = \left(\frac{5}{6}\right)^5 = \frac{3125}{7776} = 40.1878\%$$

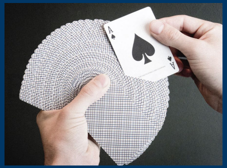
MERRIMACK COLLEGE

The probability of some events is given by the number of these events over the total number of events!

Basic Rules

When a coincidence seems amazing, that's because the human mind isn’t wired to naturally comprehend probability & statistics.

Neil deGrasse Tyson

To compute probabilities you can always go to enumerate cases with equal probability (equiprobable cases).

However, there are some shortcuts, called probability rules, that can give you a more general approach to combine the probabilities of events.

The rules we will see are:

- **Addition rule**,
- Multiplication rule,
- Conditional probability rule,
- Complement rule.

Addition Rule

The addition rule has been employed already in the earlier cases. For example, we know that in rolling a die you have the probability of ⅙ to get each of its faces. It is the Addition Rule that allows us to say that rolling a prime number is equal to 50%. Since the sum of:

- ⅙ of getting 2, plus
- ⅙ of getting 3, plus
- ⅙ of getting 5 is equal to 3/6, thus 1/2 , thus 0.5, thus 50%.

However, the Addition rule is more general than that, because it states:

> The probability of one event occurring among ***n*** events is the sum of the probability of each of these ***n*** events if, and only if, these events are *mutually exclusive*.

Example

If you have a **30%** probability to take the weekly quiz Saturday and a **10%** probability to take the weekly quiz Sunday,

You have a **40%** probability to take the weekly quiz on the weekend!

The addition rule justifies the fact that the sum of all possible outcomes is equal to 1.

Addition Rule

The "*mutually exclusive*" mentioned before is a complication to this rule, as the events you are adding up cannot have an intersection (happen at the same time).

If there is an intersection, you need to know the size of such an intersection to compute the probability of one of these events happening.

If we call the probability of event ***A*** happening ***P(A)***, and ***P(B)*** the probability of event ***B***, it is possible to say that the probability of ***A*** or ***B*** happening, denoted as ***P(A or B)***, is equal to the probability of ***P(A) + P(B)*** minus the probability of ***A*** and ***B***, denoted as ***P(A and B)***.

Example

If you have a **30%** probability to wake up before 8am and if you have a **50%** probability of waking up between 7am and 9am,

You have a probability less than or equal to **80%** to wake up before 9am!

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) \leq P(A) + P(B)$$

MERRIMACK COLLEGE

Video: [Addition Rule of Probability - Explained](#).

Basic Rules

When a coincidence seems amazing, that's because the human mind isn't wired to naturally comprehend probability & statistics.

Neil deGrasse Tyson

To compute probabilities you can always go to enumerate cases with equal probability (equiprobable cases).

However, there are some shortcuts, called probability rules, that can give you a more general approach to combine the probabilities of events.

The rules we will see are:

- Addition rule,
- **Multiplication rule**,
- Conditional probability rule,
- Complement rule.

Multiplication Rule

The multiplication rule is useful to compute the probability of *independent events* happening together.

If we use ***P(A)*** to name the probability of event ***A***, and ***P(B)*** the probability of event ***B***, it is possible to say that the probability of ***A*** and ***B*** happening, denoted as ***P(A and B)*** is equal to the probability of ***P(A) * P(B)***.

However, it is important to keep in mind that this is true only if your events are independent, because this means that one event doesn't affect the others' odds.

Knowing the addition and multiplication rules you can solve the large majority of probability problems.

Example

If you have a **30%** probability of eating bacon at breakfast and you have a **50%** probability of eating eggs at breakfast,

You have a **15%** probability of eating bacon and eggs at breakfast.

This is true assuming that having bacon does not increase or reduce your probability of having eggs and vice-versa.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Video: Multiplication Rule of Probability - Explained.

Multiplication Rule

Combining the Addition and Multiplication rules you can deal with events which are not mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

However, they need to be independent events so the multiplication rule works.

Usually, it is just a matter of clearly defining the events, taking into account their specific conditions.

Example

If you have a **30%** probability of eating bacon at breakfast and you have a **50%** probability of eating eggs at breakfast,

You have a probability of **65%** of eating bacon or eggs at the breakfast.

Answer: 0.3 + 0.5 - (0.3 * 0.5)

This is true assuming that having bacon does not increase or reduce your probability of having eggs and vice-versa.

Video: Independent and Dependent Events.

Basic Rules

When a coincidence seems amazing, that's because the human mind isn't wired to naturally comprehend probability & statistics.

Neil deGrasse Tyson

To compute probabilities you can always go to enumerate cases with equal probability (equiprobable cases).

However, there are some shortcuts, called probability rules, that can give you a more general approach to combine the probabilities of events.

The rules we will see are:

- Addition rule,
- Multiplication rule,
- **Conditional probability rule**,
- Complement rule.

Conditional Probability Rule

The probability of an event ***B*** to occur, given that another event ***A*** has already occurred, is called the conditional probability of ***B*** given ***A***, and denoted by ***P(B|A)***.

If events ***A*** and ***B*** are independent, the probability of ***P(B|A)*** is simply equal to the probability of ***B*** regardless of ***A***:

P(B|A) = P(B)

Otherwise, it is necessary to take into account the chance for ***A*** to occur and the chance for ***A*** and ***B*** to occur:

P(B|A) = P(A and B) / P(A)

Example

Considering two 6-faced dice, what is the probability that we roll at least one die with 3 (***B = 11/36***), given that we rolled a sum of dice that is less than 6 (***A = 10/36***)?

You have a probability of **40%** (**4/10**), since out of the 10 possible events adding up less than 6, in 4 of them there is a dice with 3 on it.

1 & 1 - 1 & 2 - **1 & 3** - 1 & 4
2 & 1 - 2 & 2 - **2 & 3**
3 & 1 - **3 & 2** - 4 & 1

P(A and B) = 4/36

MERRIMACK COLLEGE

Video: [Introduction to Conditional Probability].

Conditional Probability Rule

Conditional Probability is Tricky

- What is the probability of tossing a coin and getting tails, if you have already tossed the coin 5 times before and you had tails in all 5 previous tossings?
- What is the probability of drawing a King in the fourth card you take from a deck of 52 cards (4 suits, 13 cards in each suit), if you took three Kings in the first three cards?
- What is the probability of rolling a die for the second time and getting a number smaller than the one you had before, knowing that in the first time you rolled a number smaller than 4?

Don't go to the next slide before trying to solve it!

STOP

Conditional Probability Rule

Conditional Probability is Tricky

- What is the probability of tossing a coin and getting tails, if you have already tossed the coin 5 times before and you had tails in all 5 previous tossings?
  - The events are independent!
  - The probability of landing a tails in the sixth toss, knowing that you have already tossed 5 tails, is still **50%**
  - Note that the probability of tossing tails 6 times in a row is $\mathbf{0.5^6}$, which is **1.5625%**

Conditional Probability Rule

Conditional Probability is Tricky

- What is the probability to draw a King from a deck of 52 cards (4 suits, 13 cards in each suit) in the fourth card you take, if you took three Kings in the first three cards?
  - The events are not independent!
  - However, when you draw 3 Kings out of 52 cards, it remains just 1 King in 49 cards, so the answer is 1/49 which is **2.0408%**

Conditional Probability Rule

Conditional Probability is Tricky

- What is the probability of rolling a die for the second time and getting a number smaller than the one you had before, knowing that in the first time you rolled a number smaller than 4?
  - The events are dependent!
  - Probability of having a first number smaller than 4
    - **P(A) = 18/36 = ½**
  - Probability of having a second number smaller than the first
    - **P(B) = 15/36 = 5/12**
  - Probability of having **P(A and B) = 3/36 = 1/12** - *3 & 2 - 3 & 1 - 2 & 1*
  - **P(B|A) = (1/12) / (1/2) = ⅙ = 16.6667%**

Conditional Probability is Tricky!

Basic Rules

When a coincidence seems amazing, that's because the human mind isn't wired to naturally comprehend probability & statistics.

Neil deGrasse Tyson

To compute probabilities you can always go to enumerate cases with equal probability (equiprobable cases).

However, there are some shortcuts, called probability rules, that can give you a more general approach to combine the probabilities of events.

The rules we will see are:

- Addition rule,
- Multiplication rule,
- Conditional probability rule,
- **Complement rule**.

Complement Rule

If an event ***A*** has a probability equal to ***P(A)***, then the probability of this event not happening is equal to ***1 - P(A)***.

This is actually a consequence of the Sum of All Probabilities seen before.

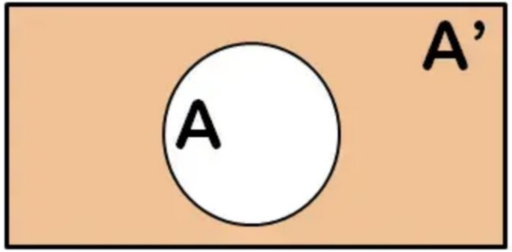
This simple rule is useful to facilitate the computation of probabilities, as sometimes it is simpler to express an event by its non-occurrence.

The complement probability of event ***A*** can be denoted as ***P(A')***.

Example

Considering two 6-sided dice, what is the probability that we roll them together and get different values in the dice?

We have a probability of **83.3333%** (**5/6**), since there are 6 outcomes with equal values in the dice out of the 36 possible outcomes.



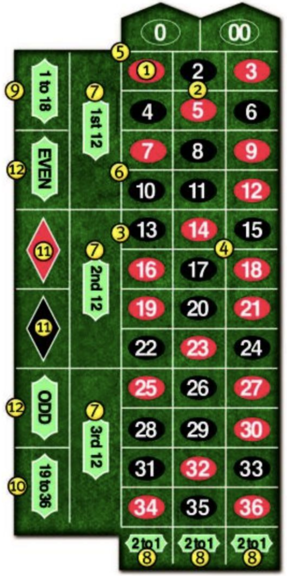
Video: Probability of Complementary Events.

Complement Rule

It is also possible to use the complement rule to solve problems such as finding the probability of a missing event, as for example:

There are in a bag candies, toys, and lumps of coal. The probability to randomly take a candy out of the bag is 20%, the probability of taking a lump of coal is 5%, so what is the probability of taking a toy?

However, more complex problems could also be solved by the complement rule. For example, let's say you are playing American roulette (numbers from 1 to 36, plus 0 and 00) and all 38 possible outcomes are equiprobable. Assuming that neither 0, 00, 1, 2, or 3 were drawn, but an odd number was drawn, what are the odds if you bet on the third column (a multiple of 3)?



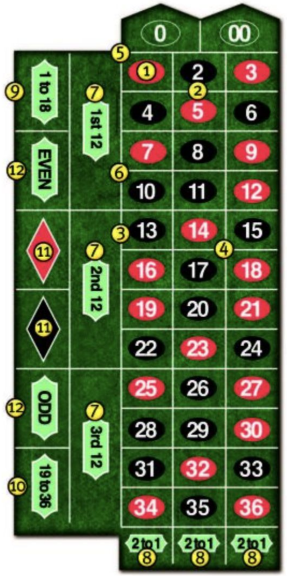
MERRIMACK COLLEGE

Don't go to the next slide before trying to solve it!

STOP

Complement Rule

Let's say you are playing American roulette (numbers from 1 to 36, plus 0 and 00) and all 38 possible outcomes are equiprobable. Assuming that neither 0, 00, 1, 2, or 3 were drawn, but an odd number was drawn, what are the odds if you bet on the third column (a multiple of 3)?



- **P(A)** - an odd number not in (0, 00, 1, 2, 3) was drawn - ***16/38***
- **P(B)** - you win with a number in the third column - ***12/38***
- Solution is **P(B'|A)**
  - **P(A and B) = *5/38***
  - **P(A and B') = *11/38*** because out 5 out of 16 (of **A**) are in **B**
  - **P(B') = *26/38***
  - **P(B'|A) = P(A and B') / P(A) = (*11/38*) / (*16/38*) = *11/16* = 68.75%** (loose)
  - **P(B|A) = 1 - P(B'|A) = *5/16* = 31.25%** (win)

MERRIMACK COLLEGE

Video: Complement in Conditional Probability.

This Week's tasks

- Post discussion D#7
- Coding Project P#7
- Quiz Q#7

Tasks

- Post in the discussion one practical example of probabilities.
- Coding Project #7, compute the probability to win a lottery game.
- Quiz #7 about this week topics.

Post Discussion - Week 7 - D#7

Post in the discussion one practical example of probabilities:

- Create an example of a problem that can be solved using probabilities.
- It can be one using coins, dice, cards, objects, etc.

Post your problem, but not the solution.

- Go read your colleagues' posts, try to find the solution and post your result.
- Once the deadline is over, or if one of your colleagues got the right answer, feel free to reply to your own post confirming or refuting the result.

Your task:

- Post your discussion in the message board by this Monday;
- Reply to your colleagues' posts in the message board by next Thursday.

This task counts towards the Discussions grade.

Seventh Coding Project - Week 7 - P#7

- Write a Python program that receives information about a lottery game based on the player guessing ***k*** numbers out of ***n*** possible numbers. Assume that:
  - the game consists of the players betting on ***k*** numbers;
  - the lottery agent randomly draws ***k*** numbers;
  - the player wins big if they hit all ***k*** drawn numbers;
  - the player wins a smaller prize if they hit ***k-1*** drawn numbers.
  - Your program should ask the users the values of ***n*** and ***k***;
    - Then your program should compute the probability of winning big (hitting all ***k*** drawn numbers) and the probability of winning smaller (hitting ***k-1*** drawn numbers).

Example in the next slide

Your task:

- Go to Canvas, and submit your Python file (.py) within the deadline:
  - The deadline for this assignment is next Thursday.

MERRIMACK COLLEGE

This assignment counts towards the Projects grade.

Seventh Coding Project - Week 7 - P#7

- For example, if the user enters ***n = 20*** and ***k = 5***, the program should compute:
    - the probability of the player hitting all 5 drawn numbers
        - which is ***1 / 15,504***;
    - the probability of the player hitting 4 drawn numbers
        - which is ***5*15 / 15,504***.

- For example, if the user enters ***n = 30*** and ***k = 20***, the program should compute:
    - the probability of the player hitting all 20 drawn numbers
        - which is ***1 / 30,045,015***;
    - the probability of the player hitting 19 drawn numbers
        - which is ***20*10 / 30,045,015***.

This is the last coding assignment of the course!

Seventh Quiz - Week 7 - Q#7

- The seventh quiz in this course covers the topics of Week 7;
- The quiz will be available this Saturday, and it is composed of 10 questions;
- The quiz should be taken on Canvas (Module 7), and it is not timed:
  - You can take as long as you want to answer it;
- The quiz is open book, open notes, and you can even use any language interpreter to answer it;
- Yet, the quiz is evaluated and you are allowed to submit it only once.

Your task:

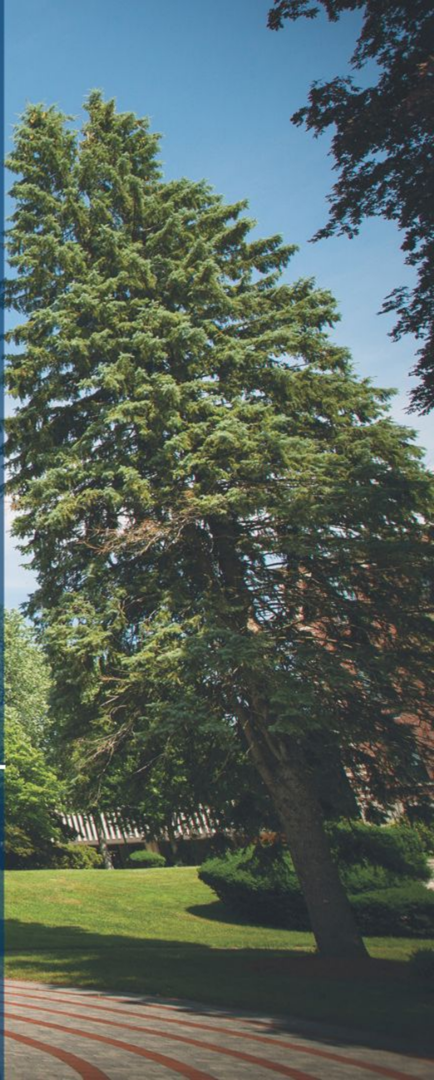
- Go to Canvas, answer the quiz and submit it within the deadline:
  - The deadline for the quiz is next Thursday.

This quiz counts towards the Quizzes grade.

We are in Week 7 of CSC 6000

- **Post discussion D#7 by Monday, reply to colleagues by next Thursday;**
- **Do quiz Q#7 (available Saturday) by next Thursday;**
- **Develop coding project P#7 by next Thursday.**

Next Week - Basic Statistics and their Computation
... plus the Final Exam!



MERRIMACK COLLEGE