

Output:

# 1

$$T(n) = 2T(n - 1) + 1$$

$$T(0) = 1$$

$$T(n) = 2T(n - 1) + 1$$

$$T(n - 1) = 2T((n - 1) - 1) + 1$$

$$T(n - 1) = 2T(n - 2) + 1$$

$$T(n) = 2(2T(n - 2) + 1) + 1$$

$$T(n) = 4T(n - 2) + 2 + 1$$

$$T(n - 2) = 2T((n - 2) - 1) + 1$$

$$T(n - 2) = 2T(n - 3) + 1$$

$$T(n) = 4(2T(n - 3) + 1) + 2 + 1$$

$$T(n) = 8T(n - 3) + 4 + 2 + 1$$

$$T(n - 3) = 2T((n - 3) - 1) + 1$$

$$T(n - 3) = 2T(n - 4) + 1$$

$$T(n) = 8(2T(n - 4) + 1) + 4 + 2 + 1$$

$$T(n) = 16T(n - 4) + 8 + 4 + 2 + 1$$

$$T(n) = 2^k * T(n - k) + (2^k) - 1$$

$$n - k = 0 \text{ so } k = n$$

$$T(n) = 2^n * T(0) + (2^n) - 1$$

$$T(n) = 2^n * 1 + 2^n - 1$$

$$T(n) = 2^{(n+1)} - 1$$

$O(2^n)$ , class exponential

#2

$$T(n) = T(n - 2) + n^2$$

$$T(0) = 1$$

$$T(n) = T(n - 2) + n^2$$

$$T(n - 2) = T((n - 2) - 2) + (n - 2)^2$$

$$T(n - 2) = T(n - 4) + (n - 2)^2$$

$$T(n) = [T(n - 4) + (n - 2)^2] + n^2$$

$$T(n) = T(n - 4) + (n - 2)^2 + n^2$$

$$T(n - 4) = T((n - 4) - 2) + (n - 4)^2$$

$$T(n - 4) = T(n - 6) + (n - 4)^2$$

$$T(n) = [T(n - 6) + (n - 4)^2] + (n - 2)^2 + n^2$$

$$T(n) = T(n - 6) + (n - 4)^2 + (n - 2)^2 + n^2$$

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$$T(n) = aT(n/b) + f(n)$$

$$T(n) = a^*T(n/b) + n^d$$

#4

$$T(n) = 2T(n/4) + 1$$

$$T(0) = 1$$

$$A = 2$$

$$B = 4$$

$$D = 0$$

$$n^{\log_b(a)} = n^{\log_4(2)} = n^{0.5}$$

$$T(n) = O(n^{\log_b(a)}) = O(n^{\log_4(2)}) = O(n^{0.5})$$

$O(n^{0.5})$  class polynomial

#5

$$T(n) = 2T(n/4) + n^{1/2}$$

$$T(0) = 1$$

$$A = 2$$

$$B = 4$$

$$D = 1/2$$

$$n^{\log_b(a)} = n^{\log_4(2)} = n^{0.5}$$

$$T(n) = O(n^d \log n) = O(n^{0.5} \log n)$$

$O(n^{0.5} \log n)$  class polynomial \* log

#6

$$T(n) = 2T(n/4) + n^2$$

$$T(0) = 1$$

$$A = 2$$

$$B = 4$$

$$D = 2$$

$$n^{\log_b(a)} = n^{\log_4(2)} = n^{0.5}$$

$$T(n) = O(n^d) = O(n^2)$$

$O(n^2)$  class polynomial

#7

$$T(n) = 10T(n/3) + n^2$$

$$T(0) = 1$$

$$A = 10$$

$$B = 3$$

$$D = 2$$

$$n^{\log_b(a)} = n^{\log_3(10)} = n^{2.10}$$

$$T(n) = O(n^{\log_b(a)}) = O(n^{\log_3(10)}) = O(n^{2.10})$$

$O(n^{2.10})$  class polynomial

#8

$$T(n) = 2T(2n/3) + 1$$

$$T(0) = 1$$

$$T(n) = 2T(n / (3/2)) + 1$$

$$A = 2$$

$$B = 3/2$$

$$D = 0$$

$$n^{\log b(a)} = n^{\log(3/2)(2)} = n^{1.71}$$

$$T(n) = O(n^{\log b(a)}) = O(n^{\log(3/2)(2)}) = O(n^{1.71})$$

$O(n^{1.71})$  class polynomial