

Asymptotic Analysis - Example 2: Elements Uniqueness

Function being analyzed:

```
def uniqueElements(A):  
    for i in range(0, len(A) - 1):  
        for j in range(i + 1, len(A)):  
            if A[i] == A[j]:  
                return False  
    return True
```

This analysis explains the cost $T(n)$ of the 'uniqueElements' function, which checks whether all elements in an array A of size n are unique.

The function uses two nested loops and in the worst-case scenario (no duplicates), all comparisons are made.

$$1. T(n) = (n - 1) * c_1 + ((n - 1)n)/2 * c_2 + ((n - 1)n)/2 * c_3 + 0 * c_4 + c_5$$

- Line 5 runs $(n - 1)$ times
- Lines 6 and 7 (inner loop and comparison) run $(n(n - 1))/2$ times
- Line 8 not run in worst case; Line 9 runs once

$$2. T(n) \leq n * c_1 + n^2 * (c_2 + c_3) + c_5$$

- Approximated $(n - 1)$ as n , and dropped constant $1/2$ for Big-O

$$3. T(n) \leq n * c_1 + n^2 * c_6 + c_5$$

- $c_6 = c_2 + c_3$ to simplify expression

$$4. T(n) \leq n^2 * c_1 + n^2 * c_6 + n^2 * c_5$$

- Linear term replaced with quadratic upper bound

$$5. T(n) \leq n^2 * (c_1 + c_6 + c_5)$$

- Factored out n^2

$$6. T(n) \leq n^2 * c_7$$

- All constants grouped into a single constant c_7

Conclusion:

The total time complexity $T(n)$ of the function is $O(n^2)$, which means if the input size doubles, the effort increases by a factor of 4.

That is: $T(2n) = 4 * T(n)$.