## Linear Optimization Math 354 Section 03 Homework 1 Linear Algebra Review

## Guidelines.

- Please staple work.
- Late work not accepted.
- Responses should be direct and clear for the sake of the grader.

**Problem 1.** Show that jth column of AB is given by  $AB_j$ , where  $B_j$  is the jth column of B.

**Problem 2.** Show that if Ax = b has more than one solution, then it has infinitely many solutions.

**Problem 3.** Let 
$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$
.

(A) Show the column vectors of A furnish a basis for  $\mathbb{R}^3$ .

- (B) Express the vector  $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  as a linear combination of the columns of A.
- (C) State your answer from B using the equation Ax = b, explaining how the coefficients in part (B) correspond to the vector  $x \in \mathbb{R}^3$ .

**Problem 4.** Prove that any n+1 vectors in  $\mathbb{R}^n$  must be linearly dependent.

**Problem 5.** Let A be an  $n \times n$  matrix. Show that A is nonsingular if and only if it is the product of elementary matrices.

Suggested Problems (Do not turn in.): Section 0.5: 1, 5, 18, 19