Earth-Moon Spaceflight Mission Planning

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I. Executive Summary

A series of impulsive maneuvers are described to transport a spacecraft from an initial launch vehicle orbit, to a parking orbit. Then a Hohmann transfer was used to reach the lunar SOI. Another transfer then reached to a lunar target orbit of ($a = 56648$[km]$, e = 0, i = 90° \Omega = 90°, \omega = 0°, \theta = 180°$).

The total delta v for the mission is: $\Delta v_{TOT} = 10.8913[km/s]$

II. Methodology

A. Task 1

The initial state of the spacecraft places the spacecraft's orbit's periapse below the surface of the Earth. As this is problematic for a lunar mission, the first maneuver is collision avoidance. We select a LEO periapse of 2000km above Earth's surface, yielding $r_p = 8371$ as the target parameter. We define v_t and v_i as the target orbital velocity and initial orbital velocities respectively, given by the vis-viva equation:

$$v = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}} \tag{1}$$

We then need to solve for the semi-major axis of the target orbit, a_t, given by the following trigonometric relation:

$$a = \frac{r_p}{1+e} \tag{2}$$

Then the first delta-v or magnitude of the orbit insertion is found with:

$$\Delta v = v_{\text{after}} - v_{\text{before}} \tag{3}$$
$$= v_{\text{transfer orbit}} - v_{\text{launch orbit}} \tag{4}$$

Now to achieve reach a circular parking orbit, we perform a circularisation burn. This makes the planning of further burns easier, as every point about this orbit is it's periapsis. However, from the initial burn, we wait until the spacecraft reaches $\theta = 180°$. The velocity of a circular orbit is given by:

$$v = \sqrt{\frac{\mu}{a}} \tag{5}$$

From which we can change the eccentricity of our orbit to $e = 0$. The final maneuver is an inclination change, placing the satellite in-plane with the lunar orbit. If performed at periapsis or, in our case, apoapsis, this is given by:

$$\Delta v = 2v \sin \frac{\Delta i}{2} \tag{6}$$

And the total delta v for these maneuvers can be found with:

$$\Delta v = \sum_i |v_i| \tag{7}$$

The initial orbital elements were:

$$a = 5137[\text{km}], e = 0.3, i = 39°$$
$$\Omega = 90°, \omega = 270°, \theta = 160°$$

After our impulsive maneuvers, the orbital elements of the parking orbit are:

$$a = 8371[\text{km}], e = 0, i = 28.58°$$
$$\Omega = 90°, \omega = 0°, \theta = 180°$$

B. Task 2

A Hohmann transfer from our parking orbit of 8371km will create an elliptical transfer orbit

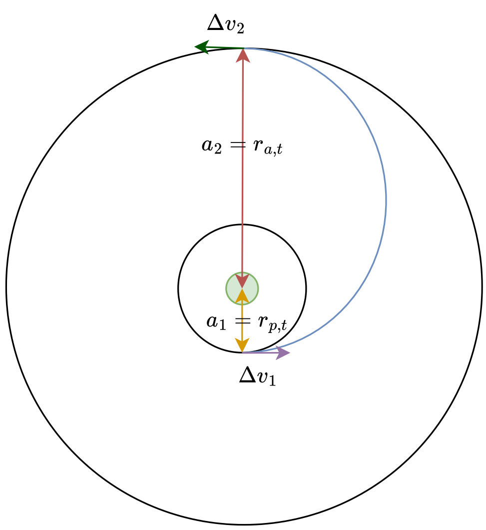

Fig. 1: Earth-Moon Hohmann transfer.

Parameters displayed in Figure 1 use distance in [km] and velocity in [km/s]. These are defined by:

$$a_2 = r_{a,t} = D - r_{SOI} \tag{8}$$
$$r_{p,t} = a_{\text{parking}} \tag{9}$$
$$a_t = \frac{1}{2}(r_{p,t} + r_{a,t}) \tag{10}$$

From this, we can define the velocity required for the transfer burn, given by:

$$v_{p,t} = \sqrt{\frac{2\mu_E}{r_{p,t}} - \frac{\mu_E}{a_t}} \tag{11}$$
$$\Delta v_1 = v_{p,t} - v_{\text{parking}} \tag{12}$$

C. Task 3

Since we approximate the Moon's orbit around the Earth as circular, the perigree is not well defined, so the argument of latitude is used. This is given by:

$$u = \theta_1 + \omega \tag{13}$$

Where ω is the argument of periapsis, from the spacecraft's departure. For circular orbits, however, from the definition:

$$\omega = \arccos\left(\frac{n \cdot e}{|n| \cdot |e|}\right) \tag{14}$$

We would have division by zero, due to $e = 0$, so by definition $\omega = 0$. As such, $u = \theta_1$ in this case.

For a Hohmann transfer, the transfer orbit is an ellipse, where the points of peri- and apoapsis meet the inner and outer circular orbits. As such, by geometry, the true anomaly swept by the transfer orbit must be $180°$.

Another critical factor for the Moon's placement is the time of flight. This is given by:

$$\mathbb{TOF} = \pi\sqrt{\frac{a_t^3}{\mu_E}} \tag{15}$$

D. Task 4

In the previous section we found the true anomaly of the spacecraft at it's lunar orbit insertion point. Since we have also set the radius of the lunar orbit to the Moon's sphere of influence, we have definitions for three orbital elements already. Eccentricity is zero for a circular orbit at the SOI. We are given the inclination of the lunar orbit as $i = 28.58°$. Also provided is $\Omega = 90°$. Finally, the argument of periapsis is zero, as outlined in II-C.

E. Task 5

1) Orbit period: The first constraint of this task requires an orbital period of less than 14 days. As the initial selenocentric orbit is at the interface of the Moon's SOI, the semi-major axis of this arrival orbit is large, which means a long orbital period. This is given by:

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{16}$$
$$14 \times 86400 > 2\pi\sqrt{\frac{a^3}{\mu}} \tag{17}$$

Rearranging yields a constraint on the semi-major axis:

$$\sqrt[3]{\mu_{\text{selene}}\left(\frac{14 \times 86400}{2\pi}\right)^2} > a \tag{18}$$
$$\Rightarrow a < 56648.4[\text{km}] \tag{19}$$

2) Crossing all latitudes: For crossing all latitudes, this requires adjusting the inclination of the orbit. Looking at the ground track of a satellite as a wave-like pattern, the inclination sets the amplitude. The closer to the extremes of the interval $i \in [-90, 90°]$, the further the orbit ground track moves towards the poles. To cross all latitudes, a polar orbit is needed. We therefore choose $i = 90°$.

3) Moon visualisation: We also would like to visualise the ground track of the Moon orbit and show the ground track for 5 orbital periods. The open source *moon_geology_atlas_of_space* repository by Eleanor Lutz [8] provides an example of geo-referencing a high resolution *.png* of the Moon. This has been used as a starting point in the *ground_track.ipynb* file in the project repository for this assignment [7]. Briefly, the *.png* was loaded then geo-referenced to the Robinson projection using Cartopy and a spline36 interpolation method. To confirm that this worked, four markers at coordinates: $[(0, 0), (90, 0), (-90, 0), (0, -180), (0, 180)]$ were placed to verify that these locations are plotted where expected.

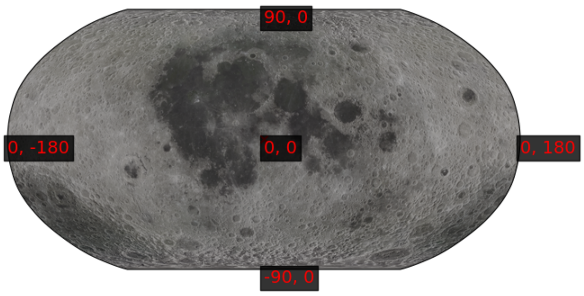

Fig. 2: Validation of geo-referencing method.

4) Calculating : To plot a ground track on the moon, latitudes and longitudes of the spacecraft need to be calculated. We can do so by transforming the position of the spacecraft, given by $[x, y, z]$ cartesian coordinates. This is relative to the centre of the primary, in this case, the Moon. Treating the Moon as a sphere simplifies the geometry. Dr. T.S. Kelso illustrated how to find the latitude and longitude for a spherical Earth and compared this value to a true oblate Earth [5]. For our purpose, we will ignore any lunar oblateness, as well as the correction factor of the Greenwich Mean Sidereal Time on the longitude calculation.

From this we can define the latitude with:

$$\phi = \tan^{-1}\left(\frac{z}{\sqrt{x^2+y^2}}\right) \tag{20}$$

And the longitude with:

$$\lambda = \tan^{-1}\left(\frac{y}{x}\right) \tag{21}$$

The location of the satellite along its orbit is required to plot the ground track. We therefore need to solve Kepler's equation for true anomaly:

$$M = E - e\sin E \tag{22}$$

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right) \tag{23}$$

We can then model the moon as a sphere and use the true anomaly, inclination and semi-major axis as θ, ϕ, ρ respectively, as defined in Figure 5

From this we can find our Cartesian coordinates:

$$x = \rho\sin\phi\cos\theta \tag{24}$$

$$y = \rho\sin\phi\sin\theta \tag{25}$$

$$z = \rho\cos\phi \tag{26}$$

Additionally, to proceed the ground track, the rotation of the Moon must be used. This is given by:

$$\Delta\psi_1 = -2\pi\frac{\mathbb{P}}{\tau_{\text{selene}}} \tag{27}$$

Where τ_{selene} is the sidereal period. This will be used to increment the polar orbit after a full orbital period.

Finally, the surface observation capability of the satellite is driven by its view angle, which is primarily a function of the altitude of the satellite. This is shown by (Soler et. al. 1994 [9]) to be:

$$\cos\gamma = \cos\psi' \cos(\lambda_s - \lambda) \tag{28}$$

In reference to a spherical primary as illustrated in Figure 3.

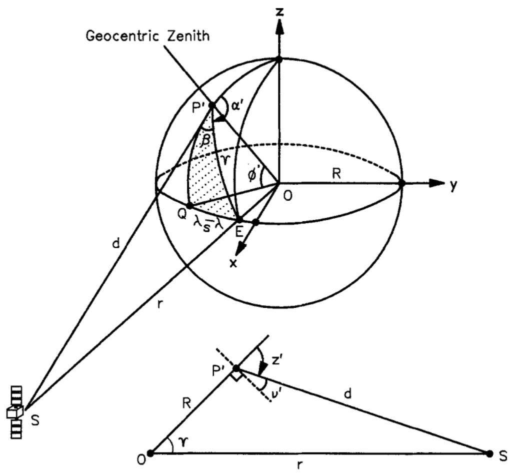

Fig. 3: Look angles assuming spherical Earth by (Soler et. al. 1994 [9]).

F. Task 6

Constructing a sequence of impulsive maneuvers for the target Lunar orbit, does not require new methods to be elaborated on here. The semi-major axis will need to be reduced, then followed by a circularisation burn. The lunar orbit will require a polar orbital inclination, so an inclination change is needed. This is the inverse of the process outlined in Task 1.

G. Task 7

A simple state machine approach will be used. To propagate the location of the spacecraft along orbits, the eccentric anomaly (22) needs to be numerically solved to find the true anomaly (23).

As such, we will use the time of flight of the respective maneuver as an interval, alongside the keplerian elements of the orbit. The output will be the true anomaly, indicating the position of the spacecraft along that specific orbital segment. We arbitrarily choose ten time steps for each respective orbital segment.

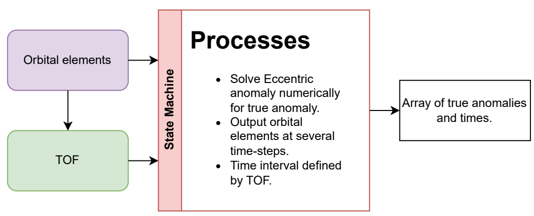

Fig. 4: State machine design.

We use TOF, to input a range of Mean anomalies, defined by:

$$M = n(t - tp) \tag{29}$$

$$n = \frac{2\pi}{\mathbb{P}} = \frac{4\pi}{\mathbb{TOF}} \tag{30}$$

To solve (22), we use the *kepler* Python package [3], which has a wrapper for Newton's method. For visualisation, we use Python's *matplotlib* library [4].

A result of the state machine approach is discrete orbital segments. However, since impulsive maneuvers are assumed to be instantaneous, this actually yields a full simulation of the entire trajectory, as required.

III. Results and Discussion

See V-A for sourcing of some values.

A. Task 1

For the first burn, we raise periapsis, to a target of $r_p = 8371km$, which is $r_{Earth} + 2000km$. This gives us a semi-major target axis of $a = 6439.23[km]$ using (2). The delta-v of the first maneuver is given by (1). Inserting values into (4) yields:

$$v_{\text{raised}} = \sqrt{\frac{2 \times 398600}{8371} - \frac{398600}{6439.23}} = 5.773[km/s]$$
$$v_{\text{launch}} = \sqrt{\frac{2 \times 398600}{6678.1} - \frac{398600}{5137}} = 6.464[km/s]$$
$$\Delta v_1 = -0.691[km/s]$$

We then circularise at $\theta = 180°$. The target circular velocity is given by (5). Similarly to before this yields:

$$v_{\text{circ}} = \sqrt{\frac{398600}{8371}} = 6.9[km/s]$$
$$v_{\text{raised}} = \sqrt{\frac{2 \times 398600}{6678.1} - \frac{398600}{5137}} = 5.773[km/s]$$
$$\Delta v_2 = 1.13[km/s]$$

Then the inclination change is given by (6). Which for moving from 28.58° to 39° and using v_{circ} yields $\Delta v_3 = 1.2531[km/s]$. This gives a total of $\Delta v_{\text{TOT}} = 3.0741[km/s]$.

B. Task 2

We construct a Hohmann transfer orbit from the Earth to the Moon. The parking orbit radius is $a_1 = r_{p,t} = 8371[km]$. The outer circular orbit's distance can be found with $D - r_{SOI,selene} = r_{a,t} = 318200[km]$. Then the semi-major axis of the transfer orbit is $a_t = \frac{1}{2}(r_{p,t} + r_{a,t}) = 163285.5[km]$.

The transfer orbit is an ellipse, which gives the velocity from (1) as:

$$v_{p,t} = \sqrt{\frac{2 \times 398600}{8371} - \frac{398600}{163285.5}}$$
$$= 9.6328[km/s]$$

C. Task 3

From the Method section II-C, the argument of latitude, u needs to be 180° away from the starting true anomaly point of the spacecraft. The in-plane and inclination change maneuvers can be assumed to occur so quickly as to be instantaneous. See Section V-B for motivation.

The time taken for the Earth-Moon Hohmann transfer is given by (15):

$$\mathbb{TOF} = \pi\sqrt{\frac{163285.5^3}{398600}} = 328324[s]$$
$$= 3.8[\text{days}]$$

The Moon moves East at a rate of 13.176° per day, so the starting position of the moon should be $13.176° \times 3.8 = 50.069°$, or at a longitude different from the true anomaly of the spacecraft by 230°. For our parking orbit, departing at $\theta = 180°$, this requires $u = 50.069°$.

D. Task 4

The orbital elements of the spacecraft on entering the lunar SOI is effectively a transition to a circular orbit at the SOI. By definition of a circular orbit and given $\Omega = 90°$, we also know that $\theta_{selene} = 180°$ by geometry. This is because the starting true anomaly is $\theta_{selene} = 180°$ and the transfer arc sweeps an additional 180°. However, from the Moon's reference frame, the spacecraft is at the location of Δv_1 in Figure 1, hence at $\theta_{selene} = 180°$. This gives:

$$a = 66200[\text{km}], e = 0, i = 28.58°$$
$$\Omega = 90°, \omega = 0°, \theta = 180°$$

E. Task 5

From the requirements outlined in Section II-E, we maximise a, under constraint (19) to $56648[km]$. This gives a 400m overhead for unforeseen perturbations. We require a polar orbit, so select $i = 90°$ and select a circular orbit so the ground track has a wider pattern, for $e = 0$. Ω, ω are the same as at the SOI. This yields:

$$a = 56648[\text{km}], e = 0, i = 90°$$
$$\Omega = 90°, \omega = 0°, \theta = 180°$$

F. Task 6

We first perform the inclination change from 28.58 to 90 degrees at the SOI circular orbit. This is because inclination changes require less delta v at greater radii. Using (6):

$$v_{SOI} = \sqrt{\frac{4905}{66200}} = 0.2722[km/s]$$
$$\Delta v_1 = 2 \times 0.2722 \times \sin\left(\frac{61.42}{2}\right)$$
$$= 0.278[km/s]$$

We then perform a Hohmann transfer, in reverse order of our Task 2, as we are going from a higher radius, transferring to a lower one. For this, we have:

$$\begin{aligned}\Delta v_1 &= v_{a,t} - v_2 \\ &= \sqrt{\frac{2 \times 4905}{66200} - \frac{4905}{61424}} - \sqrt{\frac{4905}{66200}} \\ &= 0.2614 - 0.2722 = -0.0108[km/s] \\ \Delta v_2 &= \sqrt{\frac{4905}{56648}} - \sqrt{\frac{2 \times 4905}{56648} - \frac{4905}{61424}} \\ &= 0.2943 - 0.305 = 0.0118[km/s]\end{aligned}$$

Then combining, we get a total delta v of 0.0226 [km/s]. The time of flight is:

$$\mathbb{TOF} = \pi\sqrt{\frac{61424^3}{4905}} = 682869 = 7.9[days]$$

G. Task 7

The state machine architecture outlined in Section II-G was successfully implemented into *state_machine.py*, available in the GitHub repository and added to the Appendix V-D. A key change needed in the *calc_mean_anomaly* function, was returning values as a remainder of two pi:

```
n * (time_array[i + 1] - t_0) % (2 * np.pi)
```

The initial delta-v maneuvers from the launch orbit to the parking orbit are instantaneous, shown in Figure 9. The Hohmann transfers are visualised as Figure 1 first as shown for the Earth-Moon transfer, then should be viewed as swapped $\Delta v_1, \Delta v_2$ for the radius lowering Moon transfer. We fully simulated the Earth to Moon Hohmann transfer in Figure 6, followed by the lunar lowering transfer in Figure 7.

We also then simulated five orbital periods around the chosen lunar orbit in Figure 8.

H. Extension

Improvements can be made to the visualisations chosen for the mission, from the state machine's output. The set of impulses used for Task 1 should be investigated and a Non-Hohmann transfer with a common apse line should be attempted using a flight path angle and applied radial and tangential velocity components in stead. The ground track visualisation was not completed, though the method was thoroughly outlined and partially completed in the */moon* directory of the GitHub repository for this project [7]. The state machine was to be used for generating the needed theta values. The orbital period of the lunar operational polar orbit may need to be reduced if the sweep angle of the ground track is too slow for the use case of the satellite.

IV. Conclusion

A complete set of impulses needed to maneuver a spacecraft from a launch orbit to a custom orbit around the Moon has been constructed. Hohmann transfers were used to conserve delta v throughout the mission. These consisted of a transfer from the Earth to the Moon's SOI. Then from the Moon's SOI to a custom polar orbit. The entire mission was simulated using a Newton's method solver of the Kepler equation, illustrating the movement of the satellite throughout its different transfer orbits. A novel state machine approach was utilised to account for the assumption of instantaneous impulses.

References

[1] W. contributors, "Orbit of the moon," 10 2024. [Online]. Available: https://en.wikipedia.org/wiki/Orbit_of_the_Moon

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[3] D. Foreman-Mackey, "Kepler: Fast and stable solver for kepler's equation extracted from exoplanet." https://github.com/dfm/kepler.py/tree/main, 2022.

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[5] T. Kelso, "CelesTrak: "Orbital Coordinate Systems, Part III"." [Online]. Available: https://celestrak.org/columns/v02n03/

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[7] S. Lowis, "orbit-mech: Simulating an orbit transfer from earth to the moon." https://github.com/shaunlowis/orbit-mech/tree/main, 2024.

[8] E. Lutz, "The Geology of the Moon: This repository explains how to make a geologic map of the moon using open-source data from the usgs, iau, and nasa." https://github.com/eleanorlutz/moon_geology_atlas_of_space, 2019.

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V. Appendix

A. Units, Constants

Values were supplied for assignment or from course notes unless cited here. v_M from [2]. τ_{selene} from [1].

$$\begin{aligned}\mu_E &= 398600, \mu_{selene} = 4905[km^3/s^2] \\ a_{selene} &= 384400, r_{SOI,selene} = 66200[km] \\ D &= \text{Mean Earth-Moon distance} \\ &= 384400[km] \\ r_{Earth} &= 6371[km] \\ v_{\text{inf},Earth} &= 2.946[km/s] \\ v_{\text{M}} &= v_{avg}\text{of Moon, w.r.t Earth barycenter} \\ &= 1.022[km/s] \\ \tau_{\text{selene}} &= 2015020.8[s]\text{seconds in a day} \qquad = 86400[s]\end{aligned}$$

B. Impulsive durations

A crude distance change can be used for a time estimate for the maneuver increasing periapsis:

$$\begin{aligned}\Delta a &= a_{final} - a_{initial} = 6439.23 - 5137 \\ &= 2487.69 \\ v_{\text{raised}} &= \frac{\Delta a}{\Delta t} \\ \therefore \Delta t &= \frac{\Delta r_p}{v_{\text{raised}}} = \frac{1302.23}{5.773} = 225.57[s]\end{aligned}$$

In this very simplified case we still show that the maneuver would take in the order of seconds, so can be treated as instantaneous, relative to the Moon transfer.

The time taken for the inclination change can be found as a fraction of its orbital period. This can be approximated as a section of a circular orbit.

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{8371^3}{398600}} = 7622.145[s]$$

$$t_{inc} = \frac{\Delta i}{360} \times \mathbb{P} = \frac{10.42}{360} \times 7622.145 = 220.618[s]$$

Again, this shows the starting time is not dependent on the initial impulsive maneuvers done.

C. Figures

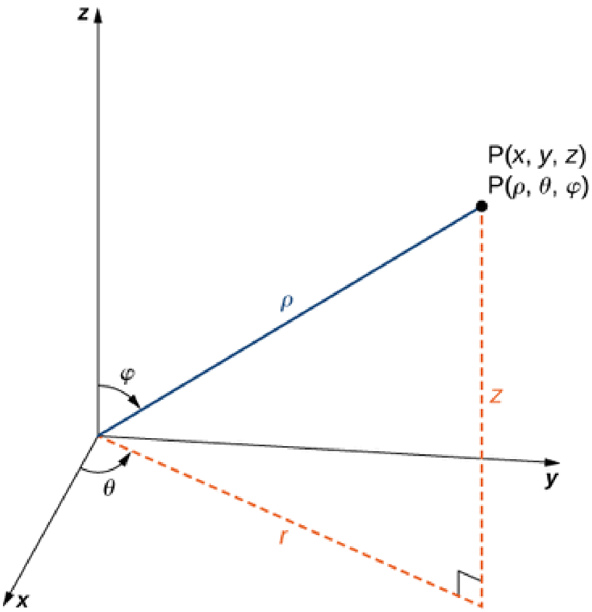

Fig. 5: Spherical coordinate frame (Math libretext [6]).

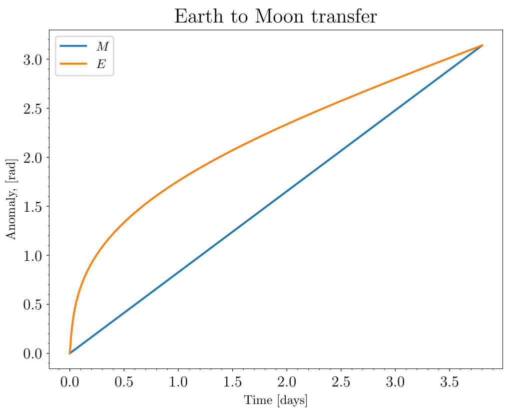

Fig. 6: Earth to Moon transfer).

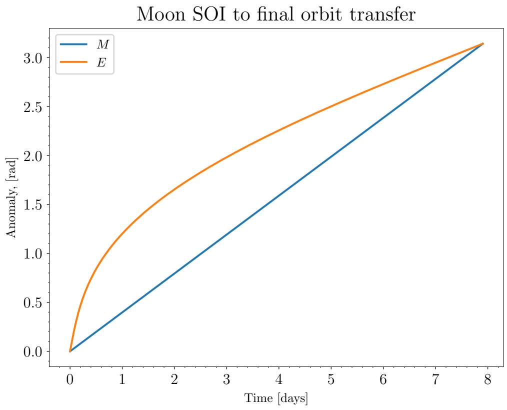

Fig. 7: Moon lowering transfer).

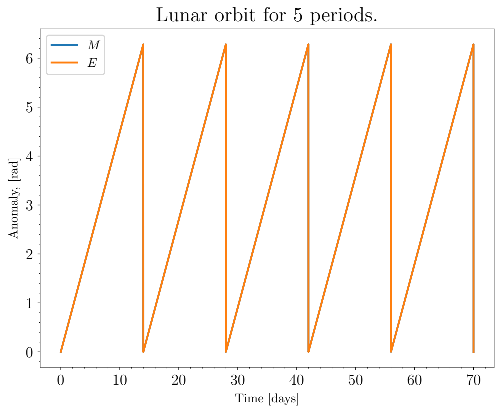

Fig. 8: Moon lowering transfer).

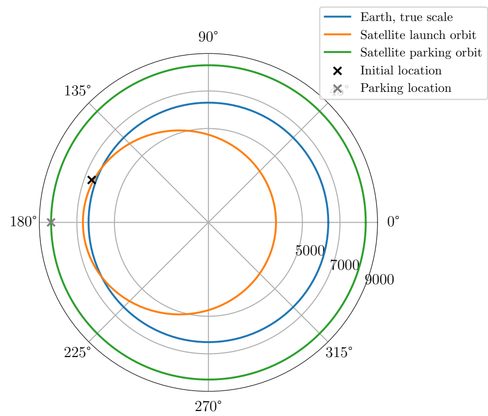

Fig. 9: Launch to parking orbit firing locations).

D. Code

```
import kepler
import numpy as np

import matplotlib.pyplot as plt
import matplotlib_rc


def state_machine(e: float, mean_anomalies:
↪ np.ndarray) -> None:
```

```python
    """
    __init__ Solve Kepler's equation
    for the given mean anomaly
    range and update the inputted
    Keplerian elements.

    Parameters
    ----------
    oe_t0 : eccentricity

    mean_anomalies : np.ndarray
        The range of mean anomalies
        to use as inputs to the solver.
    """
    eccentricity = e

    true_anomalies = []

    for mean_anomaly in mean_anomalies:

        ↪ true_anomalies.append(kepler.solve(mean_anomaly,
        ↪ eccentricity))

    return np.array(true_anomalies)


def calc_TOF(a, gravitational_parameter:
↪ float):
    # This is for a Hohmann transfer.
    return np.pi * np.sqrt((a**3) /
    ↪ gravitational_parameter)


def calc_mean_anomaly(
    t_0: float, tof: float, a,
    ↪ gravitational_parameter,
    ↪ finer_grain=False
):
    # NOTE: t_0 is time since periapsis.
    # Construct 11 steps for the inputted
    ↪ times.
    if finer_grain:
        time_array = np.linspace(t_0, t_0 +
        ↪ tof, 10000)
    else:
        time_array = np.arange(t_0, t_0 +
        ↪ tof, 11)

    # n is constant
    n = np.sqrt(gravitational_parameter /
    ↪ (a**3))

    # Return n-1 steps of Mean anomaly
    mean_anomalies = []

    for i, _ in enumerate(time_array):
        if i < len(time_array) - 1:
            # M = n(t-tp)
            mean_anomalies.append(n *
            ↪ (time_array[i + 1] - t_0) %
            ↪ (2 * np.pi))

    return np.array(mean_anomalies),
    ↪ time_array / 86400


def plot_anom_vs_time(time, anomaly):
    fig, ax = plt.subplots()
    ax.plot(time, anomaly)

    plt.show()


def calc_e(ra, rp):
    if ra != rp:
        return (ra - rp) / (ra + rp)
    else:
        return 0


def calc_orbital_period(a,
↪ gravitational_parameter):
    return 2 * np.pi * np.sqrt((a**3) /
    ↪ gravitational_parameter)


def plot_anomalies(x, mean, true, title,
↪ savepath):
    fig, ax = plt.subplots()
    ax.plot(x[0:-1], mean, label="$M$")
    ax.plot(x[0:-1], true, label="$E$")

    ax.legend()

    ax.set_xlabel("Time [days]")
    ax.set_ylabel("Anomaly, [rad]")
    ax.set_title(title)

    plt.savefig(savepath)


def earth_moon_transfer(mu_earth):
    a_transfer_arc = 163285.5

    tof_transfer_arc =
    ↪ calc_TOF(a_transfer_arc, mu_earth)
    print(f"Time of flight:
    ↪ {tof_transfer_arc}")
    mean_anomalies, times =
    ↪ calc_mean_anomaly(
        0, tof_transfer_arc, a_transfer_arc,
        ↪ mu_earth
    )

    true_anomalies = state_machine(
        e=calc_e(ra=318200, rp=8371),
        ↪ mean_anomalies=mean_anomalies
    )

    plot_anomalies(
        times,
        mean_anomalies,
        true_anomalies,
        "Earth to Moon transfer",
        "plots/earth_moon_transfer.pdf",
    )


def moon_lowering_transfer(mu_moon):
    a_transfer_arc = 61424

    # Moon SOI -> Moon final orbit transfer:
    tof_transfer_arc =
    ↪ calc_TOF(a_transfer_arc, mu_moon)
    print(f"Time of flight:
    ↪ {tof_transfer_arc}")
    mean_anomalies, times =
    ↪ calc_mean_anomaly(
        0, tof_transfer_arc, a_transfer_arc,
        ↪ mu_moon
    )

    true_anomalies = state_machine(
        e=calc_e(ra=66200, rp=4905),
        ↪ mean_anomalies=mean_anomalies
    )

    plot_anomalies(
        times,
        mean_anomalies,
        true_anomalies,
        "Moon SOI to final orbit transfer",
        "plots/moon_lowering_transfer.pdf",
    )
```

```python
def lunar_orbit(mu_moon):
    a_lunar_orbit = 56648

    # 5 orbital periods:
    tof_orbit = 5 *
    ↪  calc_orbital_period(a_lunar_orbit,
    ↪  mu_moon)
    print(f"Time of 5 orbital periods:
    ↪  {tof_orbit}")
    mean_anomalies, times =
    ↪  calc_mean_anomaly(
        0, tof_orbit, a_lunar_orbit, mu_moon,
         ↪  finer_grain=True
    )

    true_anomalies = state_machine(e=0,
    ↪  mean_anomalies=mean_anomalies)

    plot_anomalies(
        times,
        mean_anomalies,
        true_anomalies,
        "Lunar orbit for 5 periods.",
        "plots/moon_orbit.pdf",
    )


def main():
    mu_earth = 398600
    mu_moon = 4905

    # Maneuvers from launch to parking orbit
    ↪  are instantaneous
    earth_moon_transfer(mu_earth)

    # Inclination change set to instantaneous
    moon_lowering_transfer(mu_moon)

    # Spacecraft now around moon.
    lunar_orbit(mu_moon)


if __name__ == main():
    main()
```