TAKING APART ARIMA MODELS

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An AR(1) Model

- Any model will have some error
- This error will have the form of ε_t, which is assumed to be normally distributed with mean 0
- Random walk model:
  - $y_t = y_{t-1} + \varepsilon_t$
  - In backshift form: $y = By + \varepsilon_t$ or $(I – B)y = \varepsilon_t$
- Looking at the simplest ARIMA model: AR(1)
  - This means there is a regression between a series and the lag 1 of the series.
  - This can be viewed as a regression with the series own values
  - $y_t = cy_{t-1} + \varepsilon_t$
  - This can be written with the backshift operator $(I – cB)y = \varepsilon_t$
  - $c = 1 \rightarrow$ random walk

An AR(1) Model

- The form of the AR(1) model is
  - $y_t = cy_{t-1} + \varepsilon_t$
- This sounds like it makes sense, but does it?
- If $c > 1$, you get geometric growth
- So $c < 1$, but what do we get in that case?
- What is really going on
  - There is an average value for the series that is constant (stationarity requirement)
  - Without knowledge of the previous point, the expected value is the mean
  - The error term is the deviation of the expected value, which is the mean, not some other conditional prediction

An AR(1) Model

- A better way to understand the steps in generating an AR(1) forecast
- Suppose the AR(1) parameter is β
  - At each step calculate the difference between actual time series value and mean
    - Difference between actual and predicted
  - **<u>Forecast for next step will be β * (difference) + the mean value</u>**
- The forecast is a regression fitted on how the previous points differ from the mean
- This is a 1-step-ahead forecast
- How to get multiple steps ahead?
  - Prediction for first step is treated as actual value for second step
  - And so on

Other AR models

- Higher-order AR models will work the same way
- Take an AR(2) model with 2 coefficients, β_1 and β_2
  - At each step calculate the difference between actual time series value and mean
    - Difference between actual and predicted
- Forecast = mean + β_1*(difference 1 step back) + β_2*(difference 2 steps back)
- Differencing
  - An AR model with differencing means you
    - Take the difference of the series (diff(ts))
    - Work with the AR model in the same way as above on the differenced series
    - Reintegrate the forecasted differences to get the forecasts
      - Reintegrate → if last point is x, the forecasted difference is d, the new forecast is x + d
- For AR(1) we said coefficients have to be < 1 in absolute value. What about AR(p)?
  - Form a polynomial of the backshift operator with the coefficients. The roots of this polynomial have to be within the unit circle in the complex plane ($Re^2 + Im^2 < 1$)
    - Criteria is called the **unit root test**

An MA(1) Model

- Creating a forecast for an MA forecast is a bit more complicated
- Recall that for a simple exponential smoothing model the form of the prediction is $f = \alpha x_n + (1 - \alpha)^2 x_{n-1} + (1 - \alpha)^3 x_{n-2} + \ldots$
- A MA(1) model combines this with the AR form
  - Create the sequence of differences between actual and mean
  - The prediction at each point puts those differences in the above exponential smoothing formula (infinite regression)
  - Because time series are finite, have to normalize for truncation (usually small for many terms after)
- You can do the same with an MA(p) in general, but you have combination of different values
  - For MA(2), the exponential smoothing expression includes every second term

Multivariate Forecasting Models

- Multivariate forecasting model use one time series to predict another.
- Some examples of multivariate forecasting models
  - Lagged Regression models
  - ARIMAX (ARIMA models with X's included)
- Lagged regression models can be differenced or undifferenced
  - Model levels vs. model changes
- For ARIMAX models, the univariate models of ARIMA are combined with lagged regression models
  - Order matters in the modeling (model X's first and then the univariate or, the other way around?)
  - Statisticians prefer univariate first, but modeling X's first works better in my opinion
  - The lagged regressions can be with single or combinations of different lags of X's (these combinations are called ***transfer functions***)

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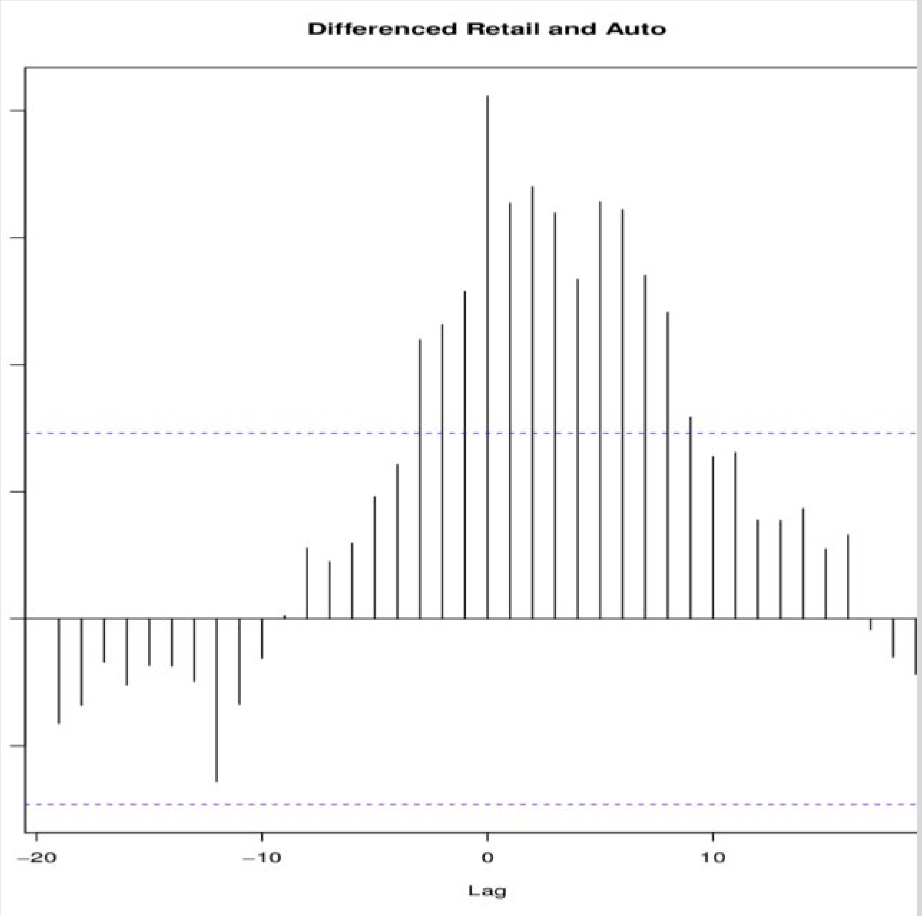
Multivariate Forecasting Models

- An important tool for Exploratory Data Analysis for multivariate time series is the cross-correlation function.
- The ***cross-correlation function*** gives the correlation between two different time series at different lags
- The cross-correlation function at lag 0 is equal to the standard correlation
- Lags are both positive and negative
- Format is ccf(x, y)
- Multivariate models can be differenced
  - Usually both x and y, but not always



Taking Apart an ARIMAX Model

- The way to deconstruct an ARIMAX model is not too complicated
  - Set up just like AR or MA models, with or without differencing
  - The linear regression for the X’s replace the mean
    - If model is differenced, X’s will have to be differenced for the regression
  - Keep on going