ARIMA FORECASTING MODELS

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Theoretical Concepts for Time Series Analysis

- A time series is a sequence of n values
- The **backshift operator** is a function which acts on a time series sequence
- It is a commonly used shorthand for many types of time series models
- The backshift operator B acting on a time series y shifts the series back one term
  - $By_t = y_{t-1}$
  - $B^2y_t = y_{t-2}$
- The backshift operator provides a convenient expression for time series models
- **Stationarity** is an important property of time series data
  - A **stationary** time series has constant mean and variance over time, and the autocovariance function depends only on the time lag and not on the absolute time
  - Differencing can be used to make a non-stationary time series stationary
    - In terms of a backshift operator, differencing looks like D = I - B
  - Non-constant mean value → significant trend component

ARIMA Models

- ARIMA models are a more complex version of time series models
- AR = autoregressive (think autoregression)
- I = integrated (related to differencing)
- MA = moving average
- AR and MA can consider 1 or multiple lags
- Autoregressive terms can be inferred from pacf analysis

An AR(1) Model

- Any model will have some error
- This error will have the form of ε_t, which is assumed to be normally distributed with mean 0
- Random walk model:
  - $y_t = y_{t-1} + \varepsilon_t$
  - In backshift form: $y = By + \varepsilon_t$ or $(I – B)y = \varepsilon_t$
- Looking at the simplest ARIMA model: AR(1)
  - This means there is a regression between a series and the lag 1 of the series.
  - This can be viewed as a regression with the series own values
  - $y_t = cy_{t-1} + \varepsilon_t$
  - This can be written with the backshift operator $(I – cB)y = \varepsilon_t$
  - $c = 1 \rightarrow$ random walk

General Autoregressive Models

- General autoregressive models AR(n) use more terms of the series
  - $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$
  - $(I - \phi_1 B + \phi_2 B^2 + \cdots + \phi_p B^p)\ y = \varepsilon_t$
- Differencing? The I in ARIMA refers to this.
  - For this, $Dy = (I - B)y$ replaces y in the model
- Moving average: Looking at a MA(1) model → similar to simple exponential smoothing model
  - $y_t = \phi y_{t-1} + (1 - \phi)^2\ y_{t-2} + \cdots + \varepsilon_t$
  - Can be express with inverses of backshift operators

Multivariate Forecasting Models

- Multivariate forecasting model use one time series to predict another.
- Some examples of multivariate forecasting models
  - Lagged Regression models
  - ARIMAX (ARIMA models with X's included)
- Lagged regression models can be differenced or undifferenced
  - Model levels vs. model changes
- For ARIMAX models, the univariate models of ARIMA are combined with lagged regression models
  - Order matters in the modeling (model X's first and then the univariate or, the other way around?)
  - Statisticians prefer univariate first, but modeling X's first works better in my opinion
  - The lagged regressions can be with single or combinations of different lags of X's (these combinations are called ***transfer functions***)

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Multivariate Forecasting Models

- An important tool for Exploratory Data Analysis for multivariate time series is the cross-correlation function.
- The ***cross-correlation function*** gives the correlation between two different time series at different lags
- The cross-correlation function at lag 0 is equal to the standard correlation
- Lags are both positive and negative
- Format is ccf(x, y)
- Multivariate models can be differenced
  - Usually both x and y, but not always