

## Assignment - II

Ques. Explain Quick sort algorithm? Also sort the following elements using quick-sort algorithms  
[5, 7, 9, 4, 10, 2, 8, 1]

Ans: Quick sort is a divide and conquer sorting algorithm.

Steps:

- 1) Choose a pivot  $\rightarrow$  select an element from array (first, last, middle, or random element).
- 2) Partitioning  $\rightarrow$  Rearrange array such that:
  - All elements smaller than pivot are placed before it.
  - All elements greater than pivot are placed after it.
- 3) Recursively sort  $\rightarrow$  Recursively apply quick sort to the left and right partitions until the array is sorted.

Time Complexity:

- Best case :  $O(n \log n)$  (when pivot divides well)
- Worst case :  $O(n^2)$  (when pivot is always smallest / largest)
- Avg. case :  $O(n \log n)$

Example: Sort [5, 7, 9, 4, 10, 2, 8, 1]

we'll use last element as pivot

$\rightarrow$  Step-1 : Initial array

[5, 7, 9, 4, 10, 2, 8, 1]

Pivot = 1

• After partition  $\rightarrow$  [1, 7, 9, 4, 10, 2, 8, 5]

• Left part = [], Right part = [7, 9, 4, 10, 2, 8, 5]

→ Step-2 : Apply quick sort on right part

[7, 9, 4, 10, 2, 8, 5]

Pivot = 5

• After partition  $\rightarrow [4, 2, 5, 10, 9, 8, 7]$

• Left part = [4, 2], Right part = [10, 9, 8, 7]

So far [1, 2, 4, 5, 10, 9, 8, 7]

→ Step-3 : Sort left [4, 2]

Pivot = 2

• After partition  $\rightarrow [2, 4]$

So far  $\rightarrow [1, 2, 4, 5, 10, 9, 8, 7]$

→ Step-4 : Sort right [10, 9, 8, 7]

Pivot = 7

• After partition  $\rightarrow [7, 9, 8, 10]$

• Left part = [], Right part = [9, 8, 10]

→ Step-5 : Sort [9, 8, 10]

Pivot = 10

• After partition  $\rightarrow [9, 8, 10]$

• Left = [9, 8], Right = []

Now [9, 8]  $\rightarrow$  pivot 8  $\rightarrow [8, 9]$

Final sorted array : [1, 2, 4, 5, 7, 8, 9, 10]

**Ques:** Write Prim's algorithm for finding the minimum spanning tree. Execute the algorithm for the given below graph. Consider 'a' as starting vertex.

**Ans:** In Prim's algorithm, grow a tree from any start vertex. At each step, add the cheapest edge that connects a visited vertex to an unvisited vertex.

Pseudocode (adjacency list, weights w):

- 1) For all vertices  $v$ :  $\text{key}[v] = \infty$ ,  $\text{parent}[v] = \text{NIL}$ .
- 2) Pick a start  $s$ . Set  $\text{key}[s] = 0$ .
- 3) While some vertex remains unpicked:
  - choose the unpicked vertex  $u$  with minimum  $\text{key}[u]$
  - Mark  $u$  picked (add to MST)
  - For each neighbour  $v$  and  $u$  with edge weight  $w(u, v)$ 
    - If  $v$  unpicked and  $w(u, v) < \text{key}[v]$
    - $\text{key}[v] = w(u, v)$ ,  $\text{parent}[v] = u$
- 4) The MST edges are  $\{(v, \text{parent}[v]) | \text{parent}[v] \neq \text{NIL}\}$

Applying Prim's Ago. to given graph (start a)

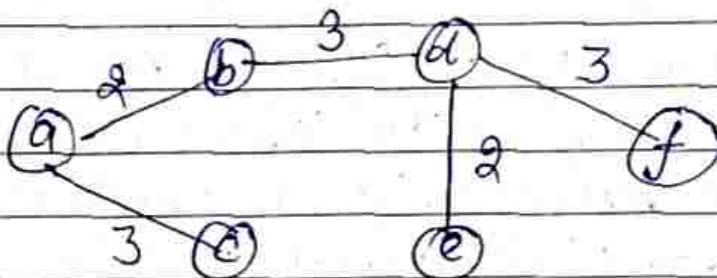
Edges (undirected from figure):

$$\begin{aligned} a-b &: 2, a-c : 3, b-c : 5, b-d : 3, b-e : 4, \\ c-e &: 4, d-e : 2, d-f : 3, e-f : 5 \end{aligned}$$

Initialize : key [a] = 0, other =  $\infty$

Iter Picked u Edge added

|   |   |         |
|---|---|---------|
| 1 | a | -       |
| 2 | b | a-b (2) |
| 3 | c | a-c (3) |
| 4 | d | b-d (3) |
| 5 | e | a-e (2) |
| 6 | f | a-f (3) |



$$\begin{aligned} \text{Total MST weight} &= 2 + 3 + 3 + 2 + 3 \\ &= 13 \end{aligned}$$

Ques 3. Explain greedy method with example.

Ans: The Greedy method is a problem-solving approach where decisions are made step-by-step, always choosing the option that seems (best) optimal at current step, with the hope that this leads to global optimum.

- It works well when the greedy choice property and optimal substructure property hold:

- Greedy choice property
- Optimal substructure

General steps in greedy algorithm

- Initialize soln set as empty.
- At each step, select best possible choice according to some criterion.
- Check feasibility  $\rightarrow$  if choice is valid, add it to the solution.
- Repeat until soln is complete.

Example: Fractional Knapsack Problem

| Item           | Value | Weight | Value/Weight |
|----------------|-------|--------|--------------|
| I <sub>1</sub> | 60    | 10     | 6            |

|                |     |    |   |
|----------------|-----|----|---|
| I <sub>2</sub> | 100 | 20 | 5 |
|----------------|-----|----|---|

|                |     |    |   |
|----------------|-----|----|---|
| I <sub>3</sub> | 120 | 30 | 4 |
|----------------|-----|----|---|

Greedy choice : Pick item with highest value / weight first.

Steps :

- Take all of  $I_1$  (20 weight, value = 60)
- Take all of  $I_2$  (20 weight, value = 100)
- left capacity = 20  $\rightarrow$  take  $20/30$  of  $I_3$  = value 80

Total value = 240