CS-182
Homework 3

1. The greedy algorithm provides us with a solution to make changes for n cents using the fewest coin denominations of quarters, dimes and pennies.

a. For 87 cents, we first take 3 quarters, which leaves us with 62 cents. Then we select a dime leaving us with 4 cents. Finally, we select 2 pennies.
Therefore, 87 cents = **3 quarters, 1 dime and 2 pennies**

b. For 49 cents, we first take 1 quarter, which leaves us with 24 cents. Then we select a dime leaving us with 4 cents. Finally, we select 4 pennies.
Therefore, 49 cents = **1 quarter, 2 dimes and 4 pennies**

c. For 99 cents, we first take 3 quarters, which leaves us with 25 cents. Then we select a dime leaving us with 5 cents. Finally, we select 4 pennies
99 cents = **3 quarters, 2 dimes and 4 pennies**

d. For 33 cents, we first take 1 quarter, which leaves us with 8 cents. Then we select 8 pennies as no nickel is allowed. Finally, we select 2 pennies
33 cents = **1 quarter and 8 pennies** as **no nickel** can be selected.

2. To determine the relative size of these functions, we take the logarithm of all of them which gives us the result as: $19\log n$, $\sqrt{n}$, $n^2 \log n$, $n^2/10^5$, $14n^n$, 3.2^n, 2.3^n, $3n!$

3. a. {(a,b)| a and b are of the same age}

REFLEXIVE

Since, a is of the same age as a. This implies that a is related to a for all values of a.
Therefore, relation R is Reflexive.

SYMMETRIC

If a is of the same age as b. Then, this implies that b is of the same age as a.
Therefore, relation R is Symmetric.

TRANSITIVE

If a is of the same age as b and b is of the same age as c. Then, this implies that a and c are also of the same age.
Therefore, relation R is Transitive.

Since, R is reflexive, symmetric and transitive. Therefore, it is an Equivalence Relation.

b. {(a,b)| a and b have the same parents.}

REFLEXIVE

Since, a has the same parents as a. This implies that a is related to a for all values of a. Therefore, relation R is Reflexive.

SYMMETRIC

If a and b have the same parents. Then, this implies that b and a also have the same parents.
Therefore, relation R is Symmetric.

TRANSITIVE

If a and b have the same parents and b and c have the same parents. Then, a and c also have the same parents. Therefore, relation R is Transitive.

Since, R is reflexive, symmetric and transitive. Therefore, it is an Equivalence Relation.

c. {(a,b)| a and b share a common parent}

REFLEXIVE

Since, a shares a common parent with a. This implies that a is related to a for all values of a. Therefore, relation R is Reflexive.

SYMMETRIC

If a and b share common parents then this implies that b and a share common parents. Therefore, relation R is Symmetric.

TRANSITIVE

If a and b share common parents and b and c share common parents, then this **does not** imply that a and c necessarily have to share the common parents. Therefore, relation R is **not** Transitive.

Since, R is reflexive, symmetric but **not** transitive. Therefore, it is **not** an Equivalence Relation.

d. {(a,b)| a and b have met}

REFLEXIVE

a always met a for all a. Therefore, the Relation R is Reflexive.

SYMMETRIC

If a met b. Then, this implies that b met a. Therefore, relation R is Symmetric.

TRANSITIVE

If a met b and b met c. Then, it need **not** imply that a met c.
Therefore, relation R is **not** Transitive.

Since, R is reflexive, symmetric but **not** transitive. Therefore, it is **not** an Equivalence Relation.

e. {(a,b)| a and b speak a common language}

REFLEXIVE

Since, a speaks the same language as a. This implies that a is related to a for all values of a. Therefore, relation R is Reflexive.

SYMMETRIC

If a speaks a common language as b. Then, this implies that b also speaks the same language as a.
Therefore, relation R is Symmetric.

TRANSITIVE

If a and b speak a common and b and c also speak a common language. Then, this does **not** imply that a and c speak a common language.
Therefore, relation R is **not** Transitive.

Since, R is reflexive, symmetric but **not** transitive. Therefore, it is **not** Equivalence Relation.