CS182
Homework 2

1.Let us assume that p= Logic is difficult, q= many students like logic and r= mathematics is easy. Therefore,

1) p ∨ ~q

2) r(~p

a. q(~r
b. ~r(~q
c. ~r ∨ p
d. ~p ∨ ~r
e. ~q((~r ∨ ~p)

Thus, a, b, c and d are valid conclusions of the given assumptions.

2.a. Let n be any odd number. Thus, n=2k+1 for some integer k:

$3n+2=3(2k+1)+2$

$=6k+3+2$

$=2(3k+2)+1$

Therefore, 3n+2 is twice an integer n plus one. So, it is odd. We proved the statement by proving the contrapositive.

b. If the given statement is false, then it leads to a contradiction and hence it is false even when (3n+2) is even and n is odd. Hence, we'll assume that 3n+2 is even and n is odd.

This implies,

$n= 2k+1$
$3n+2=3(2k+1)+2$
$=6k+3+2$
$=6k+5$
$=6k+4+1$
$2(3k+2)+1$

So, 3n+2 is odd but, we assumed it was even. Thus, the contradiction completes the proof.

3. We will begin the question by dividing it into cases and studying the question on a case by case basis which are based on the fact whether a is greater than b or b is greater than c. Next, we will prove the statement for all cases.

Cases:

1. a>= b>= c
2. a>= c>= b
3. b>= c>= a
4. b>= a>= c
5. c>= b>= a
6. c>= a>= b

Case 1:

min(a, min(b, c))= min(min(a, b), c)

min(a, c)= min(b, c)

c=c

Case 2:

min(a, min(b, c))= min(min(a, b), c)

min(a, b)= min(b, c)

b=b

Case 3:

min(a, min(b, c))= min(min(a, b), c)

min(a, c)= min(a, c)

a=a

Case 4:

min(a, min(b, c))= min(min(a, b), c)

min(a, c)= min(a, c)

c=c

Case 5:

min(a, min(b, c))= min(min(a, b), c)

min(a, b)= min(a, c)

a=a

Case 6:

min(a, min(b, c))= min(min(a, b), c)

min(a, b)= min(b, c)

b=b

Since, all possible cases are true, the statement is true.

4. A-B is “that part of A which is <u>not</u> in B”
A ∩ B is “that part of a which is in B”

Definition of A-B is A ∩ B’ where, B’ is the complement of B

$$(A-B)\ (A \cap B) = (A \cap B')\ (A \cap B)$$

$$= A \cap (B'\ B) \quad \text{[Distributive property]}$$
$$= A \cap U \quad \text{[U is the universe]}$$
$$= A$$

5.a.

$$\sum^{2}\sum^{3}(i+j)=\sum_{i=1}^{2}((i+1)+(i+2)+(i+3))$$

$$=\sum_{i=1}^{2}(3i+6)$$

$$=(3(1)+6)+(3(2)+6)=21$$

b. $\sum^{8}(3j-2j) = \sum_{j=0}^{8} 3^j - \sum_{j=0}^{8} 2^j$

$$=((3^9-1/3-1)-(2^9-1/2-1))$$

$$=9330$$

6.a. R= {(x,y)|xy>= 1}

Reflexive: No, since (0,0) does not belong to R (as $0^2= 0< 1$)

Symmetric: Yes, by definition xy= yx

Anti symmetric: No, because (2,4) and (4,2) are both in R

Transitive: Yes, as (if xy>= 1 and yz>= 1, then xz>= 1)

Therefore, R is neither an equivalence relation nor a partial order relation.

b.R= {(x,y)x ≡ y mod 9}

Reflexive: Yes, since x ≡ x mod 9

Symmetric: Yes, as x ≡ y mod 9 y ≡ x mod 9

Anti symmetric: No, because (1,10) and (10,1) are both in R
Transitive: Yes, as (if (x ≡ y mod 9) and (y ≡ z mod 9), then (x ≡ z mod 9))

Therefore, R is an equivalence relation and the classes are as follows:

[0]= {…,-18,-9,0,9,18,…}
[1]= {…,-17,-8,1,10,19,…}
[2]= {…,-16,-7,2,11,20,…}
[3]= {…,-15,-6,3,12,21,…}
[4]= {…,-14,-5,4,13,22,…}
[5]= {…,-13,-4,5,14,23,…}
[6]= {…,-12,-3,6,15,24,…}
[7]= {…,-11,-2,7,16,25,…}
[8]= {…,-10,-1,8,17,26,…}