



ONLINE TRAINING MODULES

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AVERAGES

Averages is one of the most important topic for most of the placements exams but a very simple topic once you understand. Therefore this topic helps you score well.

Average is generally defined as:

$$\text{Average} = (\text{Sum of items} / \text{Number of items})$$

- 1. Example:** Ten teachers have 80 children to teach. Find the average number of students for a teacher?

A. 6 B. 8 C. 9 D. 12

Solution:

$$\text{Average} = (\text{Sum of students} / \text{Number of teachers}) = (80/10) = 8$$

Therefore, we have 8 students for each teacher.

- 2. Example:** The average of the 5 members is 24 years. What is the sum of the ages of all the members of the family?

A. 90 B. 100 C. 120 D. 150

Solution:

$$\text{Avg} = (\text{S}/\text{N}) \rightarrow \text{S} = \text{Avg} \times \text{N} = 24 \times 5 = 120.$$

Therefore, the sum of the ages of the family members is 120kgs.

- 3. Example:** If \$300 is distributed among a group of people, the average money that they receive is \$25. How many people are there in the group?

A. 9 B. 12 C. 15 D. 18

Solution:

$$\text{S} = \$300 \quad \text{N} = ? \quad \text{A} = \$25$$

$$\text{N} = 300/25 = 12.$$

Therefore, there are 12 people in the group.

TRUE MEANING OF AVERAGES:

It is the balancing point or the point at which both the sides are balanced.



If you take the deviation on either sides of the average then the positive deviation and negative deviation will be equal.



For Example: What is the average of 15, 75, 100, 110?

Let us find the average of the above numbers:

$$= (15+75+100+110)/4 = 75.$$

Now let us see the deviation of each number with respect to the average.

-60	0	+25	+35
15	75	100	110

75

When I say deviation I mean the difference of the number from the average.

- 15 – how far is 15 from 75 = -60
- 75 – how far is 75 from 75 = 0
- 100 – how far is 100 from 75 = +25
- 110 – how far is 110 from 75 = +35

The positive deviation = $25+35 = +60$.

The negative deviation = -60.

Therefore, Positive deviation = Negative Deviation.

+ve Deviation = -ve Deviation

Note:
Average is always the middle of the magnitude.
It is a balance between weights.

We are using this concept to solve a lot of our problems.

Example:

1. Find the average of 100, 120 and 170.

Solution:

$$(100+120+170)/3 = 130.$$

Let us find the deviations:

- 100 – how far is 100 from 130 = -30
- 120 – how far is 120 from 130 = -10
- 170 – how far is 170 from 130 = +40

Hence, $+40 = -40$. Proved that concept works for all the averages.

The Concept's application:

1. Average of 81, 93, 99 and x is 87, find x.

Solution:



87

Now the deviations are:

- 81 – how far is 81 from 87 = -6
- 93 – how far is 93 from 87 = +6
- 99 – how far is 99 from 87 = +12
- X – has to be 12 far from 87 to make +ve deviation = -ve deviation

Therefore, $x = 87 - 12 = 75$.

2. There are eleven numbers. The average of first five numbers is 72 and the average of last five numbers is 81. If the average of all the eleven numbers is 7. Find the middle number.

Solution:

$$\begin{array}{ccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ & & & & & \underbrace{72} & & & & & 81 \\ & & & & & & & & & & & \end{array}$$

78

Now the average of first 5 is 72, the average of last 5 is 81 and the average of all 11 is 78.

The deviation of the averages:

- 72 from 78 = -6
Since 72 is the average of 5 the deviation will be $-6 \times 5 = -30$
- 81 from 78 = +3
Since 81 is the average of 5 numbers the deviation = $+3 \times 5 = +15$

We have -30 and +15 to make -ve deviation equal to +ve we need +15 more.

So the middle number which is the only number for which we have not found the deviation is $78 + 15 = 93$.

Answer: The middle number is 93.

Other Applications of the concept:

Case 1: When all the numbers in the list are same.

e.g. D = (5, 5, 5, 5)

$$\rightarrow (5+5+5+5)/4 = 20/4 = 5$$

Case 2: Numbers in arithmetic progression (AP)

e.g. The arithmetic progression 5, 10, 15, 20

Even number of numbers in the series: use $(\text{first number} + \text{last number})/2 = (a+l)/2$

$$(5+20)/2 = 12.5$$

Let us check:

- 5- is how far from 12.5 = -7.5
- 10 is how far from 12.5 = -2.5
- 15 is how far from 12.5 = +2.5
- 20 is how far from 12.5 = +7.5

Therefore the +ve is equal to -ve deviation.

e.g. 10, 20, 30, 40, 50

Odd Number of numbers in the series: The average is the middle number.

Here the average is 30.

Let us check:

- 10 is how far from 30 = -20
- 20 is how far from 30 = -10
- 30 is how far from 30 = 0
- 40 is how far from 30 = +10
- 50 is how far from 30 = +20

Therefore, the +ve and -ve deviation is equal.

Let us solve a tricky question:

1. There are seven consecutive integers in ascending order. The average of first five integers is n , what is the average of all the seven integers.
 A. $n+2$
 B. $n+1$
 C. $5n/7$
 D. $n+k$

Solution: Let us assume seven consecutive integers are: 1 2 3 4 5 6 7.

Since they are consecutive it is in AP with a difference of 1.

Now the average of 5 consecutive numbers will be the middle number. 1 2 3 4 5.

3rd number is the middle number and is the average which is n .

When we take all seven consecutive numbers. 1 2 3 4 5 6 7.

4th number is the middle number is the average. If 3rd number is n then 4th will be $n+1$.

Therefore, the option (B) is the answer.

There are three main types of average:

- **Mean** - The mean is what most people mean when they say 'average'. It is found by adding up all of the numbers you have to find the mean of, and dividing by the number of numbers. So the mean of 3, 5, 7, 3 and 5 is $23/5 = 4.6$
- **Median** - The median of a group of numbers is the number in the middle, when the numbers are in order of magnitude. For example, if the set of numbers is 4, 1, 6, 2, 6, 7, 8, the median is 6.
- **Mode** - The mode is the number in a set of numbers which occurs the most. So the modal value of 5, 6, 3, 4, 5, 2, 5 and 3 is 5, because there are more 5s than any other number.

1. **Example:** Find the mean, median, mode, and range for the following list of values:

13, 18, 13, 14, 13, 16, 14, 21, 13

Solution:

The **mean** is the usual average, so:

$$(13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13) \div 9 = 15$$

Note that the mean isn't a value from the original list. This is a common result. You should not assume that your mean will be one of your original numbers.

The **median** is the middle value, so I'll have to rewrite the list in order:

13, 13, 13, 13, 14, 14, 16, 18, 21

There are nine numbers in the list, so the middle one will be the $(9 + 1) \div 2 = 10 \div 2 = 5$ th number:

13, 13, 13, 13, 14, 14, 16, 18, 21

So the median is 14.

The **mode** is the number that is repeated more often than any other, so 13 is the mode.

The largest value in the list is 21, and the smallest is 13, so the range is $21 - 13 = 8$.

- mean: 15
- median: 14
- mode: 13

Note:

The formula for the place to find the median is " $([\text{the number of data points}] + 1) \div 2$ ", but you don't have to use this formula. You can just count in from both ends of the list until you meet in the middle, if you prefer. Either way will work.



QUESTIONS

1. The average monthly salary of 12 workers and 3 managers in a factory was Rs. 600. When one of the manager whose salary was Rs. 720, was replaced with a new manager, then the average salary of the team went down to 580. What is the salary of the new manager?
 - A. 570
 - B. 420
 - C. 690
 - D. 640
2. The average age of 30 students of a class is 12 years. The average age of a group of 5 of the students is 10 years and that of another group of 5 of them is 14 years. What is the average age of the remaining students?
 - A. 8 years
 - B. 12 years
 - C. 10 years
 - D. 14 years
3. The average of first five multiples of 3 is:
 - A. 3
 - B. 6
 - C. 8
 - D. 9
4. Average of all prime numbers between 30 and 50?
 - A. 37
 - B. 37.8
 - C. 39
 - D. 39.8
5. Scores in a classroom are broken into 5 different ranges, 51-60, 61-70, 71-80, 81-90 and 91-100. The number of students who have scored in each range is given below.
 51 to 60 - 3 students, 61 to 70 - 8 students, 71 to 80- 7 students, 81 to 90- 4 students, 91 to 100 - 3 students.
 Furthermore, we know that at least as many students scored 76 or more as those who scored below 75. What is the minimum possible average overall of this class?
6. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man F, whose weight is 3 kg more than that of D replaces A then the average weight of B, C, D and E becomes 79 kg. The weight of A is:
 - A. 70
 - B. 72
 - C. 75
 - D. 80
7. Average of 10 numbers is zero. At the most how many numbers may be greater than zero
 - A. 0
 - B. 1
 - C. 5
 - D. 9
8. The average of eight numbers is 14. The average of six of these numbers is 16. The average of the remaining two numbers is:
 - A. 4
 - B. 16
 - C. 8
 - D. Data inadequate
9. Average age of boys in a class is 16 years and average age of girls is 15 years, what is the average age of all
 - A. 15.5
 - B. 15

- C. 16
D. Can't be computed
10. The average temperature on Wednesday, Thursday and Friday was 25 degrees. The average temperature on Thursday, Friday and Saturday was 24 degrees. If the temperature on Saturday was 27 degrees, what was the temperature on Wednesday?
 A. 24 degrees
 B. 21 degrees
 C. 27 degrees
 D. 30 degrees
11. When a student weighing 45 kg left a class, the average weight of the remaining 59 students increased by 200g. What is the average weight of the remaining 59 students?
 A. 57
 B. 56.8
 C. 58.2
 D. 52.2
12. Three math classes: X, Y, and Z, take an algebra test. The average score in class X is 83. The average score in class Y is 76. The average score in class Z is 85. The average score of all students in classes X and Y together is 79. The average score of all students in classes Y and Z together is 81. What is the average for all the three classes?
 A. 81
 B. 81.5
 C. 82
 D. 84.5
13. The average wages of a worker during a fortnight comprising 15 consecutive working days was Rs.90 per day. During the first 7 days, his average wage was Rs.87 per day and the average wage during the last 7 days was Rs.92 per day. What was his wage on the 8th day?
 A. 83
 B. 92
 C. 90
 D. 97
14. The average age of a family of 5 members is 20 years. If the age of the youngest member be 10 years then what was the average age of the family just a day before the birth of the youngest member?
 A. 13.5 years
 B. 14 years
 C. 15 years
 D. 12.5 years
15. The average salary per month of 30 employees in a company is Rs 4000. If the manager's salary is added, the average salary increases to Rs 4300, what is the salary of the manager?
 A. Rs 10000
 B. Rs 12000
 C. Rs 13000
 D. Rs 13300
16. The average weight of a class of 24 students is 36 years. When the weight of the teacher is also included, the average weight increases by 1kg. What is the weight of the teacher?
 A. 60 kgs
 B. 61 kgs
 C. 37 kgs
 D. None of these
17. Average weight of 25 boys in a class is 48 kgs. The average weight of the class of 40 students is 45 kgs. What is the average weight of the 15 girls in the class?
 A. 44 kgs
 B. 42 kgs
 C. 40 kgs
 D. 39 kgs
18. A student finds the average of 10 positive integers. Each integer contains two digits. By mistake, the boy interchanges the digits of one

- number say ba for ab. Due to this, the average becomes 1.8 less than the previous one. What was the difference of the two digits a and b?
- A. 8
 - B. 6
 - C. 2
 - D. 4
19. Average cost of 5 apples and 4 mangoes is Rs. 36. The average cost of 7 apples and 8 mangoes is Rs. 48. Find the total cost of 24 apples and 24 mangoes?
- A. 1044
 - B. 2088
 - C. 720
 - D. 324
20. A library has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors in a month of 30 days starting with Sunday is
- A. 280
 - B. 285
 - C. 290
 - D. 295

RATIOS AND PROPORTIONS

The concept of ratio and proportion is an important one for the aptitude examinations. Beside, this concept is very important in the area of Data interpretation, where ratio changes and ratio comparisons are very popular question types.

Ratio: Gives the relation between two quantities of the same kind.



Ratio of oranges that Rina has to that of Tina is 3:4 or $\frac{3}{4}$

Proportion: Equality of two ratios. Two ratios $a:b$ and $c:d$ are said to be in proportion if $a:b = c:d$ and is represented as $a:b::c:d$.
 If $a:b::c:d$ then $ad = bc$.



The ratio of oranges to all fruits that Rina has is proportional to the ratio of oranges to all fruits that Tina has
 $3:6 :: 4:8$, We have $3 \times 8 = 6 \times 4$.

Ratio

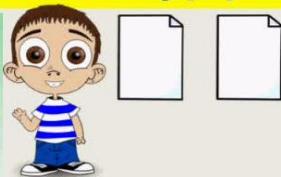
When comparing any two numbers, sometimes, it is necessary to find out how many times one number is greater (or less) than the other, in other words, we often need to express one number as a fraction of the other.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y .

The numbers that form the ratio are called the terms of the ratio. The numerator of the ratio is called "antecedent" and the denominator is called the "consequent" of the ratio. The ratio may be taken for homogenous quantities or for heterogeneous quantities. In the first case, the ratio has no unit(or unit less), while in the second case, the unit of the ratio is based on the units of the numerator and that of denominator.

In Mr. Tendilla's Math Class, he gives 2 pieces of bond paper for every pupil.

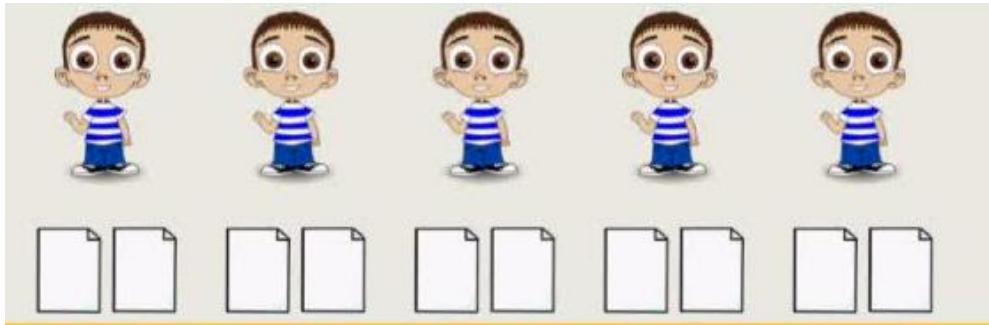
We say that the ratio of pupils to bond paper is "1 to 2".



So the ratio is of Pupil : Bond paper = 1:2 and the ratio of Bond paper to pupil = 2:1.
 Now what if it is the below case?

If there are 5 pupils in Mr. Tendilla's class, how many sheets of bond paper should he distribute?

In this case there are 5 pupil and the number of bond paper needed is 10. So the ratio becomes



Now the ratio of Pupil : Bond paper is = 5:10.

We can write $\frac{5}{10}$ in simplest form

$$\frac{5}{10} \div \frac{5}{5} = \frac{1}{2}$$

So from this we know that 1:2 is same as 5:10.

Hence, now if you get a fraction it means it might have had a common multiple that was removed. Let us say 4:5 is the ratio of blue paint to be mixed with yellow paint to get green paint of a particular shade, it might not be the original number of paint buckets used. They might have used $4 \times 2 : 5 \times 2 = 8:10$. i.e 8 buckets of blue paint and 10 buckets of yellow paint or $4 \times 3 : 5 \times 3 = 12:15$ or $4 \times 4 : 5 \times 4 = 16:20$ and so on.

Ratios can be expressed as percentages. To express the value of ratio as a percentage, we multiply the ratio by 100.

Thus $4/5 = 0.8 = 80\%$.

The calculation of a ratio:
Percentage and decimal values

- NOTE:**
1. If we multiply the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. That is, $a/b = ma(mb)$
 2. If we divide the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. Thus $a/b = (a/d)/(b/d)$.

The calculation of ratio is principally on the same line as the calculation of a percentage value. Hence, you should see it as:

The ratio 2/4 has a percentage value of 50% and it has a decimal value of 0.5.

It should be pretty obvious to you that in order to find out the decimal value of any ratio, calculate the percentage value using the percentage rule method illustrated in the chapter of percentage and then shift the decimal point 2 places to the left.

Thus a ratio which has a percentage value 62.47% will have a decimal value of 0.6247.

Some Important Properties of Ratios

1. If we multiply the numerator and the denominator of a ratio by the same number, the ratio remains unchanged.

That is, $a/b = ma(mb)$

2. If we divide the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. Thus
 $a/b = (a/d)/(b/d)$.

3. Denominator equation method:

The magnitudes of two ratios can be compared by equating the denominators of the two ratios and then checking for the value of the numerator. Thus if we have check for

$8/3$ vs. $11/4$

We compare $(8*1.33)/(3*1.33)$ vs. $11/4$

i.e. $10.66/4 < 11/4$

In fact, the value of a ratio has a direct relationship with the value of numerator of the ratio. At the same time, it has an inverse relationship with the denominator of the ratio. Since the denominator has an inverse relationship with the ratio's value, it involves an unnecessary inversion in the minds of reader. Hence, in my opinion, we should look at maintaining constancy in the denominator and work all the requisite calculations on the numerator's basis.

The reader should recall here the product Constancy table (or the denominator change to the ratio change table)

Product XY is Constant	X increases (%)	Y Decreases (%)
$A \rightarrow B \rightarrow A$	$A \rightarrow B \%$ increase	$B \rightarrow A \%$ decrease
X is inversely proportional to Y	X increases (%)	Y decreases (%)
Ratio change effect of Denominator change	Denominator increases (%)	(Ratio) decreases (%)
Denominator change effect of Ratio change	Ratio increases (%)	As Denominator decreases (%)
Standard Value 1	9.09	8.33
Standard Value 2	10	9.09
Standard Value 3	11.11	10
Standard Value 4	12.5	11.11
Standard Value 5	14.28	12.5
Standard Value 6	16.66	14.28
Standard Value 7	20	16.66
Standard Value 8	25	20
Standard Value 9	33.33	25
Standard Value 10	50	33.33
Standard Value 11	60	37.5
Standard Value 12	66.66	40
Standard Value 13	75	42.85
Standard Value 14	100	50

The value shows,

- 1) If we have two numbers X and Y. The value of XY will remain unchanged if X value is increased by 50% and Y value is decreased by 33.33% (Standard Value 10 in the above table)
 E.g. Let the no. X and Y be 100 and 100 which gives the product to be 10000. The value will remain same if X value is increased by 50 i.e. to 150 and Y value is reduced by 33.333 i.e. 66.666
 - 2) The ratio obtain the same initial value only if there is an increment of 50 and decrement by 33.33.
 E.g. Let the no. be 100 if there is an increment by 50 percent which gives new no. tends to be 150. We obtain the same value when we reduce 150 by 33.33 percent.
4. The ratio of two fractions can be expressed as a ratio of two integers. Thus the ratio:
 $a/b:c/d = (a/b)/(c/d) = ad/bc$
5. If either or both the terms of a ratio are a surd quantity, then the ratio will never evolve into integral numbers unless the surd quantities are equal. Use this principle to spot options in questions having surds.

Example: $\sqrt{3}/\sqrt{2}$ can never be represented by integers.

This principle can also be understood in other words as follows:

Suppose while solving a question, you come across a situation where $\sqrt{3}$ appears as a part of the process. In such case, it would be safe to assume that $\sqrt{3}$ will also

be part of the answer. Since the only way the $\sqrt{3}$ can be removed from the answer is by multiplying or dividing the expression by $\sqrt{3}$. Thus for instance, the formula for the area of an equilateral triangle is $(\sqrt{3}/4)a^2$.

Hence, you can safely assume that the area of any equilateral triangle will have $\sqrt{3}$ in its answer. The only case when this gets negated would be when the value of the side has a component which is the fourth root of three.

6. The multiplication of the ratios a/b and c/d yields.

$$a/b * c/d = ac/bd$$

7. When, the ratio a/b is compounded with itself, the resulting ratio is a^2/b^2 and is called the duplicate ratio. Similarly, a^3/b^3 is the triplicate ratio and $a^{0.5}/b^{0.5}$ is the sub-duplicate ratio of a/b .

8. If $a/b=c/d=e/f=g/h=k$ then $K=(a+c+e+g)/(b+d+f+h)$

9. If $a_1/b_1, a_2/b_2, a_3/b_3, \dots, a_n/b_n$ are unequal fractions. Then the ratio:

$(a_1+a_2+a_3+\dots+a_n)/(b_1+b_2+b_3+\dots+b_n)$ lies between the lowest and the highest of these fractions.

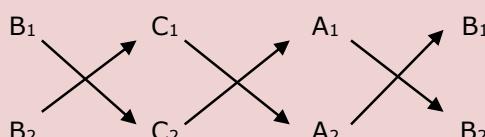
10. If we have two equations containing three unknowns as

$$a_1x+b_1y+c_1z=0 \quad (1)$$

$$a_2x+b_2y+c_2z=0 \quad (2)$$

Then, the value of x , y and z cannot be resolved without having a third equation. However, in the absence of a third equation, we can find the proportion $x:y:z$. This will be given by $b_1c_2-b_2c_1:c_1a_1-c_2a_1:a_1b_2-a_2b_1$.

This can be remembered by writing as follows:



Multiply the coefficients across the indicated always taking a multiplication as positive if the arrow points downwards and taking it as negative if the arrow point upwards. Thus x corresponds to $b_1c_2-b_2c_1$ and so on.

11. If the ratio $a/b > 1$ (called a ratio of greater inequality) and if k is a positive number:

$$(a+k)/(b+k) > a/b \text{ and } (a-k)/(b-k) > a/b$$

12. Maintenance of equality when numbers are added in both the numerator and the denominators. This is best illustrated through an example:

$$20/30 = (20+2)/(30+3)$$

i.e. $a/b = (a+c)/(b+d)$ if and only if $c/d = a/b$.

In other words, the ratio of the additions should be equal to original ratio maintain equality of ratios when two different numbers are added in the numerator and denominator.

Consequently, if $c/d > a/b$ then $(a+c)/(b+d) > a/b$ and

if $c/d < a/b$ then $(a+c)/(b+d) < a/b$

The practical applications of (11) and (12) is of immense importance for all aptitude exams.

PROPORTION

When two ratios are equal, the four quantities composing them are said to be proportional. Thus if $a/b=c/d$, then a, b, c, d are proportional. This is expressed by saying that a is to b as c is to d , and the proportion is written as

$$a:b::c:d \quad \text{or} \quad a:b=c:d$$

The terms a and d are called the extremes while the term b and c are called the means.

If four quantities are in proportions, the product of the extremes is equal to the product of the means.

Let a, b, c, d be the proportional.

Then by definitions $a/b=c/d$

$$ad=bc$$



Yes, these two ratios DO form a proportion, because the same relationship exists in both the numerators and denominators.

Hence if any three terms of proportion are given, the fourth may be found. Thus if a, c, d are given then $b=ad/c$.

$$\frac{35}{40} = \frac{7}{\boxed{}}$$

$\div 5$

$\div 5$

If three quantities a, b and c are in continued proportion, then $a:b=b:c$
 $ac=b^2$

In this case, b is said to be a mean proportional between a and c ; and c is said to be a third proportional to a and b .

If three quantities are proportional the first to the second, that is for $a:b::b:c$

$$a:c = a^2:b^2$$

If four quantities a, b, c and d form a proportion, many other proportions may be deduced by the properties of fractions. The result of these operations is very useful. These operations are:

1. **Invertendo:** If $a/b = c/d$ then $b/a = d/c$
2. **Alternendo:** If $a/b = c/d$ then $a/c = b/d$
3. **Componendo:** If $a/b = c/d$, then $(a+b)/b = (c+d)/d$
4. **Dividendo:** If $a/b = c/d$, then $(a-b)/b = (c-d)/d$
5. **Componendo and Dividendo:** If $a/b = c/d$, then $(a+b)/(a-b) = (c+d)/(c-d)$

QUESTIONS:

1. Jo's collection contains US, Indian and British stamps. If the ratio of US to Indian stamps is 5 to 2 and the ratio of Indian to British stamps is 5 to 1, what is the ratio of US to British stamps?
 - A. 10:5
 - B. 15:2
 - C. 20:2
 - D. 25:2
2. A bag contains 50 paise, 25 paise and 10 paise coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type respectively.
 - A. 360,160,200
 - B. 160,360,200
 - C. 200,360,160
 - D. 200,160,300
3. Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40:57. What is Sumit's salary?
 - A. 38000
 - B. 46800
 - C. 36700
 - D. 50000
4. A cubical block of metal weighs 6 pounds. How much will another cube of the same metal weigh if its sides are twice as long?
 - A. 48
 - B. 12
 - C. 36
 - D. 6
5. A mixture contains alcohol and water in the ratio 4 : 3. If 5 liters of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture.
 - A. 10
 - B. 12
 - C. 15
 - D. 18
6. Seats for Mathematics, Physics and Biology in a school are in the ratio 5:7:8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?
 - A. 1:2:3
 - B. 2:3:4
 - C. 3:4:5
 - D. 4:5:6
7. A dog takes 3 leaps for every 5 leaps of a hare. If one leap of the dog is equal to 3 leaps of the hare, the ratio of the speed of the dog to that of the hare is :
 - A. 9:5
 - B. 2:3
 - C. 4:7
 - D. 5:6
8. In a zoo, there are rabbits and pigeons. If heads are counted, there are 340 heads and if legs are counted there are 1060 legs. How many pigeons are there?
 - A. 120
 - B. 150
 - C. 170
 - D. 180
9. If 20 men or 24 women or 40 boys can do a job in 12 days working for 8 hours a day, how many men working with 6 women and 2 boys take to do a job four times as big working for 5 hours a day for 12 days?
 - A. 8 men
 - B. 12 men
 - C. 2 men
 - D. 24 men
10. If 76 is divided into four parts proportional to 7, 5, 3, 4, then the smallest part is:
 - A. 12
 - B. 15
 - C. 16

- D. 19
11. The ratio of water and milk in a 30 liter mixture is 7:3. Find the quantity of water to be added to the mixture in order to make this ratio 6:1.
- 30
 - 2
 - 33
 - 35
12. If 10 persons can clean 10 floors by 10 mops in 10 days, in how many days 8 persons can clean 8 floors by 8 mops?
- 12.5 days
 - 8 days
 - 10 days
 - 8.33 days
13. Rs.432 is divided amongst three workers A, B and C such that 8 times A's share is equal to 12 times B's share which is equal to 6 times C's share. How much did A get?
- Rs.192
 - Rs.133
 - Rs.144
 - Rs.128
14. The students in three batches at Mindworkzz are in the ratio 2:3:5. If 20 students are increased in each batch, the ratio changes to 4:5:7. The total numbers of students in the three batches before the increase were.
- 10
 - 100
 - 90
 - 150
15. If work done by $(x-1)$ men in $(x+1)$ days is to the work done by $(x+2)$ men in $(x-1)$ days is in the ratio 9:10, then the value of x is
- 10
 - 12
 - 8
 - 15
16. In the famous Bhojpur Island, there are four men for every three women and five children for every three men. How many children there in the island if it has 531 women?
- 454
 - 1180
 - 1070
 - 389
17. Rs 3000 is distributed among A, B and C such that A gets $\frac{2}{3}$ rd of what B and C together get and C gets $\frac{1}{2}$ of what A and B together get. Find C's share.
- 750
 - 1000
 - 800
 - 1200
- Three drunkards agree to pool their vodka and decided to share it with a fourth drunkard (who had no vodka) at a price equal to 5 roubles a liter. The first drunkard contributed a liter more than the second and the second contributed a little more than the third. Then all the four divided the vodka equally and drank it. The fourth paid money, which was divided in the ratio of each drunkard's contribution towards his portion. It was found that the first drunkard should get twice as much money as the second. Based on this information answer the questions 18-20. (Assume that all the shares are integral).
18. How much money did the second drunkard get (in roubles)?
- 8
 - 10
 - 5
 - Data insufficient.
19. How many liters of vodka were consumed in all by the four of them?
- 12
 - 16
 - 10
 - None of these.

20. What proportion of the fourth drunkard's drink did the second drunkard contribute?

- A. $\frac{1}{3}$
- B. $\frac{2}{3}$
- C. $\frac{1}{2}$
- D. None of these



PROBABILITY

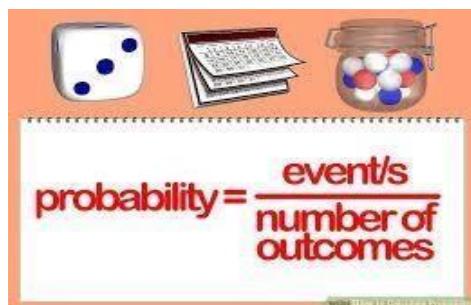
Probability is the measure of the likeliness of the occurrence of an event.

It is the ratio of favourable outcomes to the total number of outcomes.

Probability is denoted by $P(E)$. $P(E) = n(E)/n(S)$

where $n(E)$ is the no. of favourable outcomes and $n(S)$ is the total number of outcomes.

Probabilities all the events add to 1.



Examples:

Rolling a Dice:

PROBABILITY



When the dice is rolled there are six possible outcomes because a dice has numbers from 1 to 6.

Each possibility has one outcome so each has a probability of $1/6$. For example when we roll a dice if 3 occurs, then the probability or occurrence of 3 is $1/6$.

Tossing a coin

When a coin is tossed there are only two possible outcomes. Either heads will occur or tails will occur. The probability that head will occur is $1/2$. Similarly, the probability that tails will occur is $1/2$.

If we add the two probabilities the sum is 1.



Deck of Cards



A standard deck of cards has 52 cards. 2 to 9 and A, J, K, Q cards

The four kinds of cards are Hearts, Diamonds, Spade and Clubs. Each type has 13 cards in it.

There are three kinds of cards called face cards which are King, Queen and Jack making it 12 face cards.

When a card is removed from a deck, the probability that the card is queen of hearts is $4/13$ because there are 4 queens and thirteen heart cards.

Similarly the probability that the card is 6 of spades is $13/52$ or $1/4$ because there are 13 spade cards and 52 total cards in the deck.

Two or more events

If there are two or more events happening we need to consider if they are happening together at once or one after the other.

When they are happening together at once, then the probabilities need to be multiplied. It is generally implied with 'and'.

When the events are happening one after the other, the probabilities need to be added. It is generally implied with 'or'.

Eg: If a dice is rolled **and** a coin is tossed, then the probability that a 4 and a head occurs is,

$$1/2(\text{probability that a heads occurs}) * 1/6(\text{probability that a 4 occurs}) = 1/12$$

Eg: If a dice is rolled **or** a coin is tossed, then the probability that a tails and a 5 occurs is,

$$1/2(\text{probability that a tails occurs}) + 1/6(\text{probability that a 5 occurs}) = 1/2 + 1/6 = 4/6 = 2/3.$$

Random experiment:



An experiment where outcome is not certain is called a random experiment.

Eg: * Tossing a coin once or several times

*Throwing a dice once or several times

Sample Space

The set consisting of all the possible outcomes is called Sample Space.

Eg: When throwing two dice the sample space is $6*6 = 36$. (6 possible outcomes * 6 possible outcomes)

When throwing one dice the sample space is 6. (6 possible outcomes)

Sample point

Any one of the possible outcomes is called a simple event.

Eg : 6 of hearts is a sample point



Jack of diamonds is a sample point

Event

Event is a set of all possible outcomes or a collection of all simple events.

Eg: Rolling a dice and getting a 4 after rolling.

Getting a tail when tossing a coin.

Eg:

Experiment: Tossing a coin 4 times



Sample Space = {HHHH, HHHT, HHTT, HTHT, TTTT, TTTH, THTH, TTHH, THHH, HTTT, HTTH, TTHT, HHTH, THTT, THHT, HTHH}.

Examples of events:

- Atleast 3 heads

$$E = (\text{HHHH}, \text{HHHT}, \text{THHH}, \text{HHTH}, \text{HTHH})$$

- * Exactly one tail

$$E = (\text{HHHT}, \text{THHH}, \text{HTHH})$$

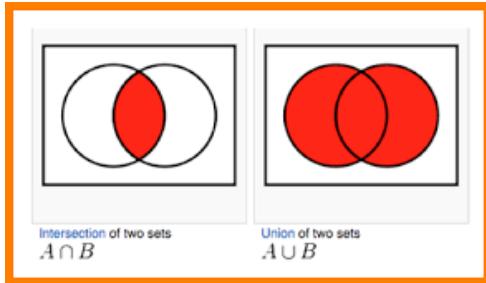
Some Concepts from Set Theory:

*The union of two events is the set of events occurring both in the first and second or either in first event or in second event.

It is denoted by $A \cup B$ where A is the set of events in A and B is the set of events in B.

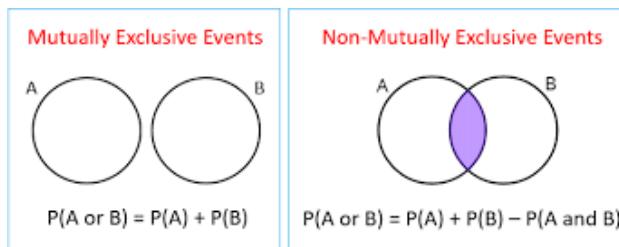
*The intersection of two events is the set of events occurring in both A and B where A is the set of events in A and B is the set of events in B.

It is denoted by $A \cap B$.



*The Complement of a set X is the set of all events not in set X but in the Universal set. It is denoted by X^c or X' .

*When the outcome of two events do not match they are called disjoint or mutually exclusive events.



Eg: Let A = {1,3,5,6,7,8}, B={2,4,6,8,7}, C = {0,9}

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A \cap B = \{6, 7, 8\}$$

$$A \cup C = \{0, 1, 3, 5, 6, 7, 8, 9\}.$$

$A \cap C = \emptyset$ or null set thus A and C are disjoint sets

$$C' = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A' = \{2, 4, 9, 0\}$$

$$B' = \{0, 1, 3, 5, 9\}$$

Laws of Set Theory:

* Commutative Law

$$X \cap Y = Y \cap X \text{ or } X \cup Y = Y \cup X$$

*Associative Laws



$$(X \cup Y) \cup Z = X \cup (Y \cup Z)$$

$$(X \cap Y) \cap Z = X \cap (Y \cap Z)$$

* Distributive Laws

$$(X \cup Y) \cap Z = (X \cap Z) \cup (Y \cap Z)$$

$$(X \cap Y) \cup Z = (X \cup Z) \cap (Y \cup Z)$$

Rules of Probability

* For an event E, $0 \leq P(E) \leq 1$

* $P(S) = 1$ where $P(S)$ is called probability of sample space

*For any event E, $P(E)^c = 1 - P(E)$

*For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

*For any three events A and B and C, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

*P(0) = 0. It is called Null set. If an outcome cannot occur its probability is 0.

*If X_1, X_2, X_3 -----is a collection of mutually exclusive events then $P(X_1 \cup X_2 \cup X_3) = P(X_1) + P(X_2) + P(X_3)$ -----

Independent events

The probability of Independent events A, B is given by $P(A, B) = P(A)P(B)$

Two events are independent when the occurrence of one event does not affect the occurrence of another event.

Conditional Probability

$$P(A|B) = P(A \cap B)/P(B)$$

It is the probability of an event such that another event has already occurred.

Examples:

1) There are 10 tickets marked with numbers 1 to 10. What's the probability of selecting a ticket having the following property?

a) even number

Solution:

$$n(E) = \{2, 4, 6, 8, 10\} = 5$$

$$n(S) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = 10$$

$$P(E) = n(E)/n(S) = 5/10 = 1/2$$

2) Determine the probability of following results when throwing 2 playing dice (a green one and a yellow one)?



a) sum equals to 7

Solution:

$$n(E) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} = 6$$

$n(S) = 36$ outcomes

$$P(E) = 6/36 = 1/6.$$

3) A bag contains 3 red, 4 green and 3 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is green?



Solution:

There are 6 non-green balls and 10 total balls.

Let S be the sample space

$$\text{Then } n(S) = \text{Number of ways of drawing 2 balls out of 10} = {}^{10}C_2 = 10*9/2 = 45$$

Let E = Event of drawing 2 balls none of which is green.

$$n(E) = \text{Number of ways of drawing 2 balls out of 6 balls} = {}^6C_2$$

$$= 6*5/2 = 15$$

$$\text{Thus } P(E) = n(E)/n(S) = 15/45 = 1/3.$$

4. Two unbiased coins are tossed. What is the probability of getting at most one head?



Solution:

$$n(S) = \{(H,H), (H,T), (T,H), (T,T)\} = 4$$

$$n(E) = \{(H,T), (T,H), (T,T)\} = 3$$

$$P(E) = n(E)/n(S) = 3/4$$

Getting atmost a head means either getting no heads or one head maximum.

5. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being queens?



Solution:

There are 4 queen cards and a total of 52 cards.

Let S be the sample space

$$n(S) = {}^{52}C_2 = 52*51/2 = 1326$$

Let E = event of getting 2 queens out of 4

$$n(E) = {}^4C_2 = 4*3/2 = 6$$

$$\text{Thus } P(E) = n(E)/n(S) = 6/1326 = 1/221.$$

Tricky question:

In a town there are 3 crossroads with traffic lights. Each traffic light opens or closes the traffic with the same probability of 0.3. Determine the probability of:

- a) a car crossing the first crossroad without stopping.
- b) a car crossing first two crossroads without stopping

Solution:

b) $P(A \cap B) = P(A)P(B)$

a) $P(A) = 0.3 = 30\%$

$$P(A \cap B) = 0.3 * 0.3 = 0.09 = 9\%$$

Practice Questions- Set I

1) Which of these numbers cannot be a probability?

- A. 0.5
- B. 1.01
- C. 1
- D. 0

2) A coin is tossed thrice. What is the probability of getting three consecutive tails?

- A) $1/4$
- B) $1/6$
- C) $1/2$
- D) $1/8$

3) What is the probability of drawing a Jack and an 'A' card consecutively from a deck of 52 cards, without replacement?

- A) $4/663$
- B) $5/773$
- C) $4/557$
- D) $6/567$

4) In a class, 30% of the students study English and Hindi. 70% of the students study English. What is the probability of a student studying Hindi given he/she is already studying English?

- A. $2/5$
- B. $4/7$
- C. $5/9$
- D. $3/7$

5) When two dice are rolled, find the probability of getting a smaller number on the first die than the one on the second, given that the sum should equal 9?

A. $1/3$

B. $1/2$

C. $1/4$

D. $1/7$

6) A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is $(1/6)$ and the probability of woman's selection is $(1/4)$. What is the probability that only one of them is selected?

- A) $1/4$
- B) $2/3$
- C) $3/7$
- D) $1/3$

7) A basket contains 20 mangoes and 40 grapes out of which 5 mangoes and 8 grapes are defective. If we choose two fruits at random, what is the probability that either both are grapes or both are non-defective?

- A. $281/365$
- B. $285/371$
- C. $273/354$
- D. $289/378$

8) I forgot the last digit of a 6-digit telephone number. If I randomly dial the final 4 digits after correctly dialling the 2, then what is the chance of dialling the correct number?

- A. $1/1000$
- B. $2/10949$
- C. $3/10344$
- D. $1/10000$.

9) The probability of occurrence of two events P and Q are $1/5$ and $1/4$ respectively. The probability of their

Simultaneous occurrence is 9/50. Find the probability that neither P nor Q occurs?

A. 71/100

B. 73/100

C. 62/100

D. 83/100

10) What is the probability of getting at least one four in a single throw of three unbiased dice?

A. 91/235

B. 17/345

C. 91/256

D. None of these

Practice Questions - Set II

1) If a is chosen at random from the set {3, 5, 6, 7} and b is to be chosen at random from the set {2, 4, 1}, what is the probability that $a \times b$ will be even?

A. 4/5

B. 6/7

C. 3/5

D. 3/4

2) Out of 10 applicants 5 boys and 5 girls. Two persons are to be selected for the job. Find the probability that at least one of the selected persons will be a girl?

A. 7/9

B. 6/7

C. 4/5

D. 3/7

3) There are 8 Letters and 8 correspondingly 8 different Address. If the letters are put into envelope

randomly, then find the Probability that Exactly 7 letters will be at the correct address?

A. 0.23

B. 0.34

C. 0

D. 0.5

4) The odds favoring the event of a person hitting a target are 4 to 6. The odds against the event of another person hitting the target are 4 to 3. If each of them fire once at the target, find the probability that both of them hit the target?

A. 5/37

B. 6/35

C. 7/34

D. 8/41

5) If $P(X)=3/9$, then the odd against the event X is?

A. 3/7

B. 4/9

C. 2/5

D. 2/1

6) The first 7 alphabets are written down at random. What is the probability that the letters c, d, e always come together?

A. 2/7

B. 1/7

C. 3/8

D. 4/9

7) A six-sided die with faces numbered 1 through 6 is rolled two times. What is the probability that the face with the

number 5 on it will not face upward on all the two rolls?

A. 24/37

B. 34/47

C. 22/45

D. 35/36.

8) A fair six-sided die is rolled twice. What is the probability of getting 5 on the first roll and not getting 4 on the second roll ?

A. 5/36

B. 4/37

C. 3/37

D. 7/31

9) A bag contains 10 apples and 8 orange. Four fruits are drawn out one by one and not replaced. What is the probability that they are alternatively of different fruits?

A. 7/51

B. 8/53

C. 9/55

D. None of these

10) In an interview the probability of Pavithra to got selected is 0.5. The probability of Gayathri to got selected is 0.6. The probability of Supriya to got selected is 0.7. The probability of Somesh to get selected is 0.9. What is the probability that at least 2 of them got selected on that day?

A. 0.344

B. 0.911

C. 0.834

D. 0.734

PROFIT & LOSS

Some Important Definitions

- Cost Price (C.P):** price of purchasing goods
- Selling Price (S.P):** price at which goods are sold
- Profit (P):** if $S.P > C.P$;
That is if you sold at a price higher than your purchase price
 $p=S.P-C.P$
- Loss (L):** if $S.P < C.P$;
That is if you sold at a price lower than your purchase price
 $L=C.P-S.P$
- Profit Or Loss Percentage:** it is expressed as percent profit ($p\%$) or percent loss ($l\%$) rather than speaking actual value of profit or loss.
- Marked Price:** the sellers after purchasing an item usually tag or label it with a price which is higher than the price at which they purchased it. This higher price marked up is called marked price (M.P)
- Discount:** it is a reduction in the marked price.
- Successive discount:** if two or more discounts allowed on M.P



Formulas:

Profit Or Gain = Selling Price - Cost Price

Loss = Cost Price - Selling Price

Note: profit or loss percent is always calculated on cost price(c.p).

$$\underline{\% \text{ Of Profit}} = \left(\frac{\text{S.P} - \text{C.P}}{\text{C.P}} \right) \times 100$$

$$\underline{\% \text{ Of Loss}} = \left(\frac{\text{C.P} - \text{S.P}}{\text{C.P}} \right) \times 100$$

In case of Profit:

$$S.P = \left(\frac{100 + \%}{100} \right) C.P$$

$$C.P = \left(\frac{\cdot * 100}{100 + \%} \right)$$

In case of loss:

$$SP = CP * \frac{(100 - \% \text{ of Loss})}{100}$$

$$CP = \frac{SP * 100}{100 - \% \text{ of Loss}}$$

$$MP = CP * \frac{100 + \text{Profit \%}}{100 - \text{Discount \%}}$$

$$M.P = \frac{\cdot}{100 - \text{Discount \%}} * (100 - \text{Loss \%})$$

$$\text{Successive / Net discount} = \frac{X+Y - XY}{100}$$

Example 1:

Ms. Sonia and Mr. Rahul bought a shirt, the price is Rs. 5000. And then Ms. Sonia sold it again with price rs.7500. Mr. Rahul's shirt has little broken and he sold it too but with the price of Rs.4000. Find how much more Sonia got than Rahul.

Solution:

C.P of Rahul and Sonia is 5000 each,

But S.P of Sonia is 7500 and S.P of Rahul is 4000,

Then Sonia got $7500 - 4000 = 3500$ more than Rahul

Example 2.

Ms. Pallavi brought 1 dozen of markers, the price is rs. 9000, then she sold the marker, the price is rs.1125 for 1 marker. Does she get profit or loss? How much percentage does she get profit or loss?

Solution:

C.P of 1 dozen markers is 9000, then C.P of 1 marker is

$9000/12 = 750$ S.P of 1 marker is 1125,

$S.P > C.P$, then she gets a profit of $1125 - 750 = 375$

$$\% \text{ of profit} = \left(\frac{375}{750} \right) * 100$$

$$\% p = \frac{375 * 100}{750}$$

$$= 50\%$$

Example 3.

A dealer gives a 50% off on shoes and still makes a profit of 50%. Find the actual cost of the article to the dealer if he marks the price of the article at Rs. 3000.

Sol:

M.P of the article,

M.P=Rs.3000 Discount=50%,

Discounted amount = 50% of

M.P=(50/100*3000)=Rs.1500

S.P of the article, S.P=M.P-discount=(3000-1500)=1500

Let C.P be x. Since the gain% is given as 50%,

Thus gain=50% of x=(50/100*x)=x/2

Also we know that S.P=C.P+ gain→1500=x+x/2

Thus, the C.P of the article is 1000



Example 4.

The successive-discount of 20% and 40% are given on the purchase of a water bottle. If the price of the bag is \$ 4500, find the selling price.

Sol:

As the successive discount is 20% and 40%

Total discount = (x + y - xy / 100) %

X = 20% and y = 40%

Total discount = [20 + 40 - (20 x 40) / 100] %

= (60-800/100)%

Total discount = 52%

Discount = 52% of 4500

Discount = rs. 2340 S.p

= m.p - discount

= 4500 - 2340

Selling price = rs. 2160



Example 5: A dealer offered a discount of kit kat chocolates "buy 4 get 1 free is ". The percentage discount offered for?

Sol:

Total items = 5

Discounted = 1 Percentage = $1/5 * 100 = 20\%$



Practice Questions Set I:

1. Find the cost price of an article which is sold at Rs. 720 at a profit of 12.5%.
 A. Rs. 640
 B. Rs. 540
 C. Rs. 850
 D. Rs. 700
2. A shopkeeper sells an article at $16 \frac{2}{3}\%$ profit on selling price. Find his actual profit%
 A. 20%
 B. 25%
 C. 30%
 D. 15%
3. Find the selling price of an article a shopkeeper allows two successive discount 15% each on marked price of Rs. 120.
 A. Rs. 85.6
 B. Rs. 84.3
 C. Rs. 87.4
 D. Rs. 86.7
4. A salesman expects a gain of 14% on his cost price. If in a month his sale was Rs. 9,12,000. What was his profit?
 A. Rs. 85,659
 B. Rs. 88,300
 C. Rs. 1,12,000
 D. Rs. 97,786
5. The cost price of 15 articles is equal to the selling price of 10 articles. Find the gain percent.
 A. 65%
 B. 60%
 C. 15%
 D. 50%
6. A person sold an article at a profit of Rs.26% if he had sold it for Rs.50 more, he would have gained 36%. What is the cost price?
 A. Rs. 100
 B. Rs. 360
 C. Rs. 260
7. Two houses are sold Rs.5,04,000 each. One house brings the owner a gain of 12 % and the other at a loss of 4%. The gain or loss percent by selling both houses.
 A. Loss 3 3 / 5 %
 B. Profit 3 5/13 %
 C. Profit 4 3/5 %
 D. Loss 41 3/5 %
8. By selling 27 toys, a vendor loses the selling price of 3 toys. Find loss percent.
 A. 15%
 B. 17%
 C. 10%
 D. 12%
9. The percentage profit made when an article is sold for Rs 64 is thrice as when it is sold for Rs.50. The cost price of an article is:
 A. Rs. 38
 B. Rs. 43
 C. Rs. 25
 D. Rs. 39
10. 50% Goods are sold at 5% loss while rest is sold at 10 % profit. If there is a total profit of Rs 250, then the worth of goods sold is?
 A. Rs. 10,000
 B. Rs. 10,250
 C. Rs. 20,000
 D. Rs. 20,250

Practice Questions Set II:

1. If the mark-up percentage of an article is 65% and discount percentage is also 30%, then the profit percentage will be:
 A. 12.5%
 B. 15.5%
 C. 16.5%
 D. 13.5%
2. A shopkeeper promises to sell his goods at 34% loss, but he uses 30% less weight. Find the actual loss%
 A. 20%
 B. 15%
 C. 54%
 D. 57%
3. A man purchases some number of articles at 5400 and he sells $\frac{4}{3}$ of them at 5% profit. At what profit % did he sell the remaining to gain 13% overall.
 A. 29%
 B. 28%
 C. 27%
 D. 26%
4. A cash payment that will settle a bill for 350 chairs are Rs. 10 per chair less 10% and 15% with a further discount of 20% on cash payment is:
 A. Rs. 2145
 B. Rs. 2142
 C. Rs. 3142
 D. Rs. 3145
5. A manufacturer of a certain item can sell all he can produce at the selling price of Rs. 55 each. It cost him Rs. 35 in materials and labour to produce each item and he has overhead expenses of Rs. 3000 per week in order to operate the plant. The number of units he should produce and sell in order to make a profit of at least Rs. 2000 per week, is:
 A. 200
 B. 250
 C. 300
 D. 400
6. CP of 12 toys is equal to the SP of 9 toys and the discount of 10 toys is equal to the profit on 5 toys. What is the percentage point difference between the profit percentage and discount percentage?
 A. 20
 B. 22.22
 C. 16.66
 D. 15
7. Anil asked the shopkeeper the price of a watch. Anil found that he had just the required sum of money. When the shopkeeper allowed me a discount of 50%. Anil could bring another watch worth Rs. 640 for Ajay. What is the price which Anil have paid for his watch?
 A. Rs. 1280
 B. Rs. 640
 C. Rs. 1588
 D. Rs. 1720
8. When a Bike manufacturer reduced its SP by 50%, the number of Bikes sold radically increased by 600%. Initially the manufacturer was getting only 140% profit. What is the percentage increase of his profit?
 A. No profit no loss
 B. 10% profit
 C. 20% profit
 D. 5% profit
9. Charan marks his product 25% above the CP and offers a discount of 10%. Find the loss percentage incurred by the trader.
 A. 8%
 B. 12.5%

C. 12%
D. 6%

10. Buy 5 get 4 free" what is the
actual discount.

- A. 44%
- B. 50%
- C. 52%
- D. 36

PERCENTAGES

Percentage:

Percentage is a number or ratio expressed as a fraction of 100. A percent can also be expressed as a Decimal or a Fraction. Percent is an abbreviation for the Latin word Per centum. The meaning is per hundred or hundredths.

We typically use percentage in three ways.

- To compare a quantity to the whole
- To compare one quantity to another
- To compare a quantity to an increased or decreased amount of the same quantity

A Half can be written...

As a Percentage	50%
As a Decimal	0.5
As a Fraction	$\frac{1}{2}$

Application of percentage:

- Profit calculation
- Loss calculation

Examples:

- Calculate 25% of 80
 - $25\% = 25/100$
 - And $25/100 \times 80 = 20$
 - So 25% of 80 is **20**
- 15% of 200 apples were bad. How many apples were bad?
 - $15\% = 15/100$
 - And $15/100 \times 200 = 15/100 \times 200 = 15 \times 2 = 30$ apples
 - 30 apples** were bad
- A Skateboard is reduced 25% in price in a sale. The old price was \$120. Find the new price.
 - First, find 25% of \$120:
 - And $25 / 100 \times \$120 = \30
 - 25% of \$120 is \$30
 - So the reduction is \$30
 - Take the reduction from the original price
 - $\$120 - \$30 = \$90$
 - The Price of the Skateboard in the sale is **\$90**



Formulae

1. Percentage Formula:

$$\text{Percentage Formula} = (\text{Value} / \text{Total Value}) * 100$$

Example: There are 120 students in a class. Out of them 70 are girls. Find the percentage of girls in the class?

Answer: Total students in the class = 120
 Girls in the class = 70
 % of girls in the class = $70/120 \times 100$
 $= 350/6$
 $= 58.33\%$

2. Percentage Increase Formula:

Percentage increase gives us an idea about how much percentage of amounts has increased from the initial amount. That is here the amount increased is represented in percentage format.

Percentage increase = Increase amount / Initial amount $\times 100$
 Where,
 Increased amount = New amount – Initial amount

Example: The cost of the bag is increased from Dollar 50 to Dollar 75. Calculate increase percentage.

Answer: Increased amount = $75 - 50 = \$25$
 Percentage increase = Increase amount / Initial amount $\times 100$
 $= (25/50) \times 100$
 $= (1/2) \times 100$
 Correct answer is 50%

3. Percentage Change Formula:

Percentage change gives us an idea about change in a value with respect to time. The concept of percentage change is widely used in business field.

Percentage change = $V_2 - V_1 / V_1 \times 100$.
 V_1 - represents the old value
 V_2 - represents the new value

Example: Tom scored 6 runs in the cricket match on Sunday. On Wednesday he scored 3 runs. Determine the Percentage of change?

Answer: Given,
 V_2 = new value = 3 runs.
 V_1 = old value = 6 runs.

Percentage of change = ?

The formula used to calculate the percentage change is

Percentage change = $(V_2 - V_1) / V_1 \times 100$

By plugging in the given values in to the formula we get

Percentage change = $(3 - 6) / 6 \times 100$

(Here we get a negative value because we have -3 in the numerator i.e., $(3-6)$) therefore the percentage change indicates reduction in Tom's score which is 50%)

The fraction gives us 50 %

The percentage of change is 50 percent.



4. Percentage Difference:

The percent value of difference of values is called as percent difference. It is also called as percentage change.

Difference Percentage = Newer Value – Older Value / Older Value

Note:

- If the result is negative, avoid the minus sign and it is considered as a decrease.
- If the result is positive, it is considered as an increase.

Methods for Calculating Difference Percentage:

Method 1:

- Step 1: Find out the difference.
- Step 2: The subtracted value is divided by old value.
- Step 3: The resultant value is changed into percentage.

Example: Find out the difference percentage from older value 300 and newer value 350.

Answer: Formula for difference percentage is:

$$= \text{Newer value} - \text{older value} / \text{older value} * 100$$

Step 1: Find the difference as $350 - 300 = 50$.

Step 2: Divide the 50 by old value 300 as $50 / 300 = 0.166$

Step 3: Multiply the value 0.166 with 100 as $0.166 \times 100 = 16.66\%$.

Therefore, the difference percentage is **16.66 %**.

Method 2:

Step 1: The new value is divided by the old value.

Step 2: The resultant value is converted into percentage.

Step 3: Subtract 100 from that percentage value.

Example: Find out the difference percentage from older value 300 and newer value 350.

Answer: Formula for difference percentage is:

$$= \text{Newer value} / \text{older value} * 100$$

Step 1: The new value is divided by the old value = $350/300 = 1.167$

Step 2: The resultant value is converted into percentage = $1.167 * 100 = 116.7$

Step 3: Subtract 100% from that percentage value = $116.7 - 100 = 16.7$

Therefore, the difference percentage is **16.7 %**.

5. Profit Percentage:

Profit percentage tells us how much of sales income is profit. Profit is the sales amount of the cost of particular items that was sold. Profit will be gained whenever the selling price of one thing is greater than cost price.

Profit Percentage is if the selling price of any material is greater than its cost price, then a profit is seemed to have occurred. Profit percentage is calculated by the ratio of profit to the cost price. Therefore to know how much the percentage of profit is we use profit percentage formula.

Profit percentage is one of the ways to evaluate financial performance of a person or company. It helps to see what percentage of their earning after costs is profit.

$$\text{Profit Percentage} = \text{Gross Profit} / \text{CP}$$

$$\text{Gross Profit} = \text{Net Sales Price} - \text{Cost of Goods}.$$

$$\text{Profit Percentage} = (\text{Selling Price} - \text{Cost Price})/\text{Cost Price} \times 100$$

Find the gross

Cost Price of a fan = 3.30

Selling price = 5.90

Step 1: Find Gross Profit

$$\text{Gross Profit} = \text{Sell Price} - \text{Cost Price} = 5.90 - 3.30 = 2.60$$

Step 2: Find the gross profit percentage

$$\text{Gross Profit Percentage} = \text{Gross Profit} / \text{CP}$$

$$= 2.60 / 3.30 = 0.78$$

$$= 0.78 * 100 = \mathbf{78\%}$$

Example: Calculate gross profit ratio from the following particulars:

Particulars	Sales	Opening Stock	Purchases	Closing Stock
\$	1,66,000	50,000	90,000	20,000

Answer: Cost of goods sold = Opening stock + Purchases - Closing stock
 $= 50,000 + 90,000 - 20,000 = 1,20,000$

Gross profit = 1,66,000 - 1,20,000 = 46,000

Gross profit ratio = Gross Profit / Selling Price $\times 100$

Gross profit ratio = 46,000 / 1,66,000 $\times 100 = 27.71\%$

6. Successive Percentage Changes:

If in the first year, A's salary increases by 10% and in the second year, the salary increases by 20% again, what would be the net increase over the two years?

Is it $10 + 20 = 30\%$?

If A's salary was 100 at start, after 1st year it would be 110. In the second year, it would be $110 \times 120/100 = 132$. Thus the net increase is 32 i.e., 32% and not 30%.

The 10% increase and the 20% increase in the above problem are called successive increases, as the second percentage's increase is affected on the new base and not on the initial value.

Let's consider the above calculation with two successive percentage increases of a% and b%.

$$\begin{aligned}\text{Net multiplying Factor} &= (1+a/100) (1+b/100) \\ &= [1 + a/100 + b/100 + ab/10000] \\ &= [1 + (a+b+ab)/100]\end{aligned}$$

Therefore the net increase in percentage is:

$$\boxed{a + b + ab / 100}$$

Example 1:

In a rectangle, if the length increases by 10% and the breadth increases by 20% the net percentage change in area will be?

Answer:

$$a+b+ab/100$$

$$10 + 20 + (10*20 / 100) = 30.5\%$$

Example 2:

In a rectangle, if the length increases by 10% and the breadth decreases by 5% the net percentage change in area will be?

Answer:

$$a+b+ab/100$$

$$10 - 5 - (10*5 / 100) = 4.5\%$$

Here,

We use -ve sign as there is a decrease in the breadth of the rectangle

You may come across the following types of questions in the examination

Types of questions	Examples	Approach to the question
1. Convert $x\%$ into fraction	Express 12% as a fraction	$x\% = x/100 = 12/100$ = 3/25
2. Convert fraction or decimal into percentage	Express $5/11$ as percentage	Multiply the fraction by 100. $= 5/11 * 100 = 45.45\%$
3. If A's income is $x\%$ of B's income and B's income is given, then find A's income	A's income is 40% of B's income. If B's income is Rs. 10,000, what is A's income?	$A = x/100 * P$ $= 40/100 * 10,000$ = Rs. 4,000
4. If A's income is $r\%$ less than B's income, then by how much % is B's income more than A's income?	X's income is 40% less than Y's. By how much % is Y's income more than X's income?	Difference = $r/(100-r) * 100$ $= 45/65 * 100$ = 66.67%
5. If A's income is $r\%$ more than B's income, then by how much % is B's income less than A's income?	X's income is 25% more than Y's income, then by how much % is Y's income less than X's income?	Difference = $r/(100+r) * 100$ $= 25/125 * 100$ = 20%
6. If the price of a commodity increases by $r\%$, find the % decrease in the consumption given that expenditure remains same	If the price of potato is increased by 20%, by how much should the consumption be decreased so as to maintain the same expenditure?	Expenditure = Price * Consumption Decrease = $r/(100+r) * 100$ $= 20/120 * 100 = 50/3$ = 16.67%

Shortcuts

Method 1: Multiplication through base method

This method is applicable only when the numbers are close to a base like 10, 100, 1000 or so on. Both the numbers in the questions should be close to the base.

Note: 1 represents 100 for example (1 + 5) meaning 100 + 5

Example 1: 105*107

Base = 100

Surplus = 5 & 7 (Difference of the numbers from the base)

105 1 + 5

107 1 + 7

Step 1: $(105 + 7) = 112$

Step 2: $(5 * 7) = 35$

Therefore the answer is **11235**

Example 2: 112*113

Base = 100

Surplus = 12 & 13 (Difference of the numbers from the base)

112 1 + 12

113 1 + 13

Step 1: $(112 + 13) = 125$

Step 2: $(12 * 13) = 156$

The number of digits allowed in step two will be equal to the number zero's in the base. Our base is 100 which has 2 zero's. This means that we can display only 2 digits from step 2 which is 56.

What about the left out digit?

We add it to the number we get in step 1. i.e., $125 + 1 = 126$

Therefore the answer is 12656

Base Not Equal To 100:**Example 1: 209*211**

Base = 200

Surplus is 9 & 11

209 1 + 9

211 1 + 11

Step 1: $(209 + 11) = 220$

Step 2: $220 * 2 = 440$ (Because base is 200)

Step 3: $(11 * 9) = 99$

Therefore the answer is 44099

QUESTIONS

1. $33 \frac{1}{3}\%$ of 240
 A. 72
 B. 16
 C. 80
 D. 78
2. Express $\frac{2}{5}$ as rate percent
 A. 80%
 B. 60%
 C. 40%
 D. 50%
3. An inspector rejects 0.08% of the meters as defective. How many meters he examine to reject 2 meters
 A. 1200
 B. 2400
 C. 1400
 D. 2500
4. $x\% \text{ of } y + y\% \text{ of } x = ?$
 A. 2% of xy
 B. 50% of xy
 C. $2xy$
 D. 100% xy
5. If the salary of an employee is first increased by 20% and the decreased by 12%, then what is the change in his salary in percent?
 A. 16% Increase
 B. 16% Decrease
 C. 5.6% Increase
 D. 8 % Increase
6. For a candidate to clear an examination, he/she must score 55% marks. If he/she gets 120 and fails by 78 marks, the total marks for the examination is
 A. 300
 B. 360
 C. 400
 D. 320
7. The price of a commodity is increased by 15%. By what percent should its consumption be decreased so as to not increase the expenditure?
 A. 90%
 B. $300/23\%$
 C. $100/3\%$
 D. $100/11\%$
8. To increase the area of a circle by 44%, by what percent should its radius be increased?
 A. 22%
 B. 6.63%
 C. 20%
 D. 44%
9. In 2012, Ajay's salary was Rs.36000 and Vijay's salary was Rs.24000. In 2011, Vijay's salary was 20% of the sum of their salaries in 2012. What is the percent change in Vijay's salary from 2011 to 2012?
 A. 50%
 B. 20%
 C. 100%
 D. 10%
10. If the population of a city increases by 5 % annually, what will be the population of the city in 2 years time if its current population is 78000?
 A. 81900
 B. 85995
 C. 85800
 D. 90000
11. $\frac{3}{5}$ th of the voters in Varanasi promised to vote for A and the remaining promised to vote for B. On the day of election, 15% of the voters who had promised to vote for A did not cast their votes while 20% of the voters who had promised to vote for B did not turn up. What is the total number of votes polled if B got 128 votes?
 A. 400
 B. 240
 C. 204
 D. 332

12. Suresh's salary is 60% more than Ramesh's salary. Suresh got a 50% increment in his salary while Ramesh got 20% increment. By what percent is Suresh's salary more than Ramesh?
- A. 90%
B. 96%
C. 92%
D. 100%
13. In a class, 20% of the students were absent for an exam. 30% failed by 20 marks and 10% just passed. Find the average score of the class if the remaining students scored an average of 65 marks and the cut off for passing the exam is 40.
- A. 45
B. 49
C. 36
D. 40
14. Ram lends 15% of his pocket money to Shyam and spends 30 % of the remainder. He is left with Rs.714. How much did he lend to Shyam?
- A. 1200
B. 486
C. 180
D. 1020
15. If a candidate who got 60% of vote cast won by 144 votes during an election, find total number of voters on the voting list if 75% of people cast their votes and there were no invalid votes.
- A. 960
B. 720
C. 240
D. 1000
16. In an examination, A got 10% marks less than B, B got 25% marks more than C and C got 20% less than D. If A got 360 marks out of 500. The percentage of marks obtained by D was :
- A. 70%
B. 75%
C. 80%
D. 85%
17. Raman's salary was decreased by 50% and subsequently increased by 50%. He has a loss of :
- A. 0%
B. 25%
C. 0.25%
D. 2.5%
18. $\sqrt{5}$ percent of $5\sqrt{5}$ =
- A. 0.05
B. 0.25
C. 2.5
D. 25
19. The price of a cycle is reduced by 25 per cent. The new price is reduced by a further 20 per cent. The two reductions together are equal to a single reduction of
- A. 45%
B. 40%
C. 35%
D. 32.5%
20. A shopkeeper sells 1 quintal of wheat for Rs. 460 and he gets a profit of 15% by selling a quintal of rice for the same amount, he loses 8%. Find the CP per kg of wheat and rice. Find the total gain or loss.
- A. Rs. 20
B. Rs. 17
C. Rs. 25
D. Rs. 30
21. The population data for India and Delhi are given below. Using rounded figures.
- A. Find the increase % in India's population from 1961 to 1971
B. Find the increase % in Delhi's population from 1971 to 1981

Growth of population:

	1961	1971	1981	
Ind	43,92,3	54,81,5	68,51,8	B. 45 5/11%
	4,771	9,652	4,892	C. 54 6/11%
	(43	(54	(68	D. 55
	crores)	crores)	crores)	
Del	26,58,6	40,65,6	62,20,4	
hi	12	98	06	
	A. 25% and 50%			
	B. 30% and 52.5%			
	C. 25% and 52.5%			
	D. 30% and 54%			

22. A grocer sells rice at a profit of 10% and uses weights which are 20% less than the marked weight. The total gain earned by him will be

- A. 32.5%
- B. 37.5%
- C. 40%
- D. 30%

23. In an acoustics class, 120 students are male and 100 students are female. 25% of the male students and 20% of the female students are engineering students. 20% of the male engineering students and 25% of the female engineering students passed the final exam. What percentage of engineering students passed the exam?

- A. 5%
- B. 10%
- C. 16%
- D. 22%

24. If b equals 10% of a and c equals 20% of b, then which one of the following equals 30% of c?

- A. 0.6%
- B. 5.9%
- C. 10.2%
- D. 40.8%

25. A batsman scored 110 runs which included 3 boundaries and 8 sixes. What percent of his total score did he make by running between the wickets?

- A. 45%

Simple Interest and Compound Interest

What is **Interest**?

The amount of money borrowed from a person will lead us to pay a service charge to the person. This amount is paid back to the lender along with the original amount borrowed. This is sometimes known as the cost of money which does not belong to you but you have used it. This extra amount is called the "Interest".

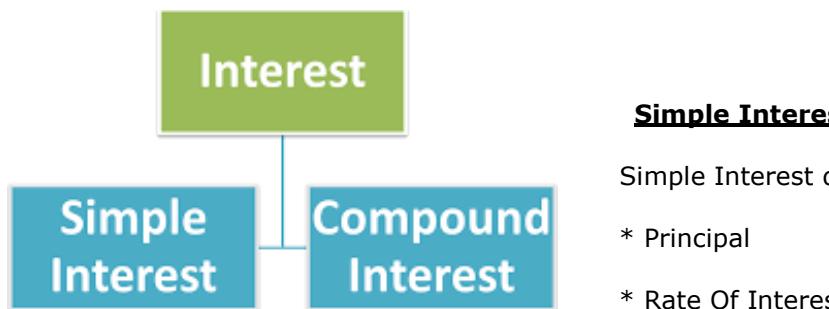


The original sum of money borrowed is known as the "Principal" or "Capital". The sum of both Principal and the Interest is known as "Amount".

Principal is denoted by 'P' , Rate of Interest is denoted by 'R' and Time period is denoted by 'T'.

There are two types of Interest. They are:

- * Simple Interest
- * Compound Interest



Simple Interest:

Simple Interest depends on

- * Principal
- * Rate Of Interest
- * Time period

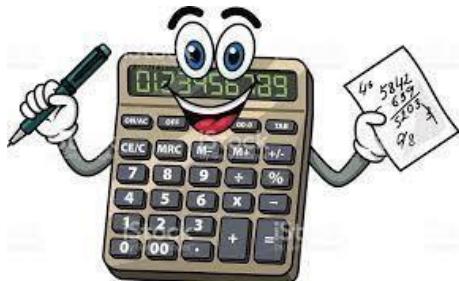
Simple Interest is denoted by SI.



*Principal remains the same at the beginning of all the periods.

*Simple Interest remains the same every year.

Compound Interest:



Compound Interest depends on:

- * Principal
- * Rate of Interest
- * Time period

Compound Interest is denoted by CI.

- * Principal increases by the interest amount at the end of each period

- * Interest for the next period is calculated on the Principal amount that is increased.
- * This principal amount that we get in the previous point becomes the principal for the next period.
- * The process of calculation goes on till the time period given. This process is called compounding or compounding interval.
- * The compounding can be
 - Yearly
 - Half yearly
 - Quarterly
 - Monthly
 - Weekly
 - Daily

To calculate Interest:

To calculate Interest we need the following inputs:

- *A rate known as the Rate of Interest (RI or ROI) which is expressed in percent per year
- *Time period expressed in years or months or days
- *Principal amount on which Interest is to be calculated.
- * Type of interest is required (whether Simple Interest or Compound Interest) because the way of calculating interest in each (SI and CI) is different.

Calculation of SI [P=Rs.100, R=5%, T = 2 years]

- *Let the Principal be Rs. 100

For two years ;

Formula

$$SI = P \cdot R \cdot T / 100 = 100 \cdot 5 \cdot 2 / 100 \\ = Rs. 10$$

$$P = 100 \cdot SI / R \cdot T$$

$$R = 100 \cdot SI / P \cdot T$$

$$T = 100 \cdot SI / P \cdot R$$

For each year it will be Rs. 5.

Thus the SI remains the same for each year till 2 years is completed.

Calculation of CI [P=Rs.100, R=5%, T = 2 years]

* **Calculation can be done in two methods**

*** Method 1 (Formula based)**

$$\text{Amount} = \text{Principal} (1 + \text{Rate}/100)^{\text{Time period}}$$

Amount is denoted by A, Principal is denoted by P, Rate is denoted by R. Time period is denoted by

$$\text{Compound Interest (CI)} = \text{Amount} - \text{Principal} = A - P.$$

Let the principal be Rs. 100,

$$\text{For 2 years; } A = 100(1+5/100)^2 = \text{Rs. } 110.25$$

$$CI = 110.25 - 100 = \text{Rs. } 10.25$$

*** Method 2**

Let principal be Rs. 100 and Rate of Interest be 5%, Time period be 2 years.

$$\text{Compound Interest for the first year} = R/100 \cdot P = 5/100 \cdot 100 = 5$$

Add this 5 to the Principal, we get Rs. 105

$$\text{Then we do } 105 \cdot 5/100 = \text{Rs. } 5.25$$

$$\text{So the total CI will be } 5 + 5.25 = \text{Rs. } 10.25$$

From the above two calculations for CI and SI we can conclude that:

* SI and CI is the same for first year

* After 1st year, CI > SI

Some important points:

- * If the compounding interval is not mentioned then it is assumed yearly
- *The compounding interval can never be more than a year.
- *The less the compounding interval, the more is the effective annual interest.

Important formula:



- * When Compound Interest is calculated yearly,

$$A = P(1+R/100)^T$$
- * When interest is compounded half-yearly

$$A = P(1+(R/2*100))^2T$$
- * When interest is compounded quarterly

$$A = P(1+(R/4*100))^4T$$
- * When interest is compounded annually but time is in fraction say 3 1/4 years

$$A = P(1+(R/100))^3[1+(1/4R)/100]$$
- * When rates are different for different years say R1%, R2% and R3% for 1st,2nd and 3rd year respectively

$$A = P(1+R1/100)(1+R2/100)(1+R3/100)$$

Examples:

- * An amount of Rs. 25000 is invested at 10% rate of Compound interest for a year half-yearly. Find the amount and the Compound Interest?

Solution:

$$\begin{aligned}
 A &= P(1+R/100)^T \\
 A &= 25000(1+10/100)^2 \\
 &= 25000 * 11/10 * 11/10 = \text{Rs. } 30250
 \end{aligned}$$

$$CI = A - P = 30250 - 25000 = \text{Rs. } 5250$$

*An amount of Rs. 20000 is invested at 12% rate of Compound Interest for a year quarterly. Find the amount and the Compound Interest?

Solution:

$$A = P(1+R/100)^{4T}$$

$$= 20000(1+12/100)^{4*1}$$

$$= 20000 * 112/100 * 112/100 * 112/100 * 112/100 =$$

Rs. 31470.39

$$CI = 31470.39 - 20000 = \text{Rs. } 11470.39$$

*An amount of Rs. 10000 is invested at 6% ROI for a time period of 3 1/2 years. What is the amount and the CI?

Solution:

$$P(1+(R/100))^3[1+(1/2R)/100]$$

$$= 10000(1+6/100)^3[1+(1/2*6)/100]$$

$$10000 * 106/100 * 106/100 * 106/100 * 103/100 = \text{Rs. } 12267.46$$

$$CI = 12267.46 - 10000 = \text{Rs. } 2267.46$$

* An amount of Rs.2000 is invested for 5 years at the rates of 1% for the first two years, 3% for the third year and 5% for the next two years all compounded continuously. What is the total amount at the end of the 5 years?

Solution:

$$A = P(1+R1/100)(1+R2/100)(1+R3/100)$$

$$= 2000(1+1/100)^2(1+3/100)(1+5/100)^2$$

$$= \text{Rs. } 2316.8$$

$$\text{* Difference between SI and CI for 2 years} = PR^2/100^2$$

where P is the principal, R is the rate of interest.

$$\text{* Difference between SI and CI for 3 years} = P(R/100)^2 / (R/100) + 3$$

where P is the principal and R is the rate of interest.

Present Value:**Present value under CI:**

Till now we have been finding the future value of money by adding interest. The reverse process of finding the present value of a sum of money to be received in future implies discounting the fund flows to the extent of the interest rate.

The assumption is that a sum of money received today has more value than a similar sum sometime in the future. We can discount the money by doing the reverse of what we do in compound interest.

$$A = P(1+R/100)^n$$

So present value of a sum A due after n years = $A/(1+R/100)^n$

Example:

What is the present value of a property which would be valued at Rs. 300000 at the end of 2 years, the rate of interest being 4% compound?

$$PV \text{ of a sum } A \text{ due after } n \text{ years} = A/(1+R/100)^n = 3/(1.04)^2 = \text{Rs. } 2,77,777$$

Present value under SI:

Let the principal be P, amount be A and the time period be n years. From this we get

$$A = P[1+(nr/100)]$$

$P = \{A/[1+nr/100]\}$ which is the present value under SI.

Example

*An amount of Rs. 30000 under 10% SI and 2 years gives a Present value of?

Solution:

$$P = 30000/[1+(2*10/100)]$$

$$=30000/12/10 = \text{Rs. } 25000$$

Continuous Compounding:

*If we keep increasing the value of n to a very high number or infinity; the interval gets lesser and lesser to say zero. This is known as infinitely compounding for which the formula is:

$$A = P \cdot e^{(i \cdot t)}$$

where e=base of natural logarithm = 2.71828

The i is expressed in decimal - eg:11% is 0.11

The t is expressed in multiples of the period of interest rate

Eg: If ROI is per annum then 4 yrs 3 months is 4.3 years.

Example: P=100 and i = 11% pa. If t=1 year, what is Amount and CI?

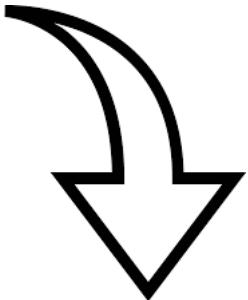
$$A = P \cdot e^{(i \cdot t)}$$

$$= 100 \cdot 2.71828^{(0.11 \cdot 1)}$$

$$= 100 \cdot 1.1163 = \text{Rs. } 111.63$$

$$\text{CI} = 111.63 - 100 = \text{Rs. } 11.63$$

Benefits of SI and CI:



*Simple Interest avoids the fact that it needs to be compounded or in other words, calculation of interest on interest is not required.

*The biggest advantage of Simple Interest is that it is comparatively easy to calculate compared to Compound interest because we calculate simple interest on the principal amount rather than on principal + interest like we do in Compound interest.

*Compound Interest is advantageous when you are an investor but not when you are a borrower.

*Simple Interest is paid only on the principal amount invested while compound interest is paid on the principal + interest already earned.

*If a certain sum becomes 36 times in 2 years ,what will be the rate of compound interest?

Solution:

Using the formula derived above:

$$r = 100[(x)^{1/t}-1]$$

where x is the amount and t is the time period in years.

$$=100[(36)^{1/2}-1] = 100[6-1]=500\%$$



Practice Questions – Set I

1. Mr. Harley invested an amount of Rs. 14,000 divided in two different schemes A and B at the simple interest rate of 15% p.a. and 12% p.a. respectively. If the total amount of simple interest earned in 2 years be Rs. 3515, what was the amount invested in Scheme B?

A. Rs. 1437.37

B. Rs. 1438.49

C. Rs. 1498.87

D. Rs. 1416.67

2. A sum of Rs. 12,800 amounts to Rs. 16000 in 3 years at the rate of simple interest. What is the rate of interest?

A. 6%

B. 7%

C. 8%

D. 9%

3. A financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 15%, the effective rate of interest becomes?

A. 16.3484%

B. 15.5625%

C. 17.3344%

D. 15.3445%

4. What will be the ratio of simple interest earned by a certain amount at the same rate of interest for 7 years and that for 10 years?

A. 6/7

B. 3/7

C. 7/9

D. 7/10

5. Akshay borrows Rs. 5500 for 2 years at 5% p.a. simple interest. He immediately lends it to another person at 7% pa for 2 years. Find his gain in the transaction per year?

A. 220

B. 115

C. 110

D. 225

6. On a sum of money, the simple interest for 2 years is Rs.600, while the compound interest is Rs.700, the rate of interest being the same in both the cases. The rate of interest is ?

A. 42.34%

B. 33.33%

C. 35.43%

D. 42.44%

7. Ram took a loan of Rs. 1500 with simple interest for as many years as the rate of interest. If he paid Rs. 440 as interest at the end of the loan period, what was the rate of interest?

A. 5.48%

B. 5.35%

C. 5.67%

D. 5.41%

8. A man invests an amount of Rs. 15800 in the names of his three sons A, B and C in such a way that they get the same amount after 1,2 and 3 years respectively. If the rate of simple interest is 10% the ratio of amounts invested among, A, B and C will be?

A. 6:3:2

B. 6:4:3

C. 6:3:4

D. 6:2:3

9. The rate at which a sum becomes five times of itself in 10 years at S.I will be?
- A. 30%
B. 45%
C. 40%
D. 35%
10. A sum of money doubles itself in 4 years. In how many years will it become four fold (if interest is compounded)?
- A. 9 years
B. 10 years
C. 7 years
D. 8 years
- Practice Questions - Set II**
1. What is the rate of simple interest for the first 3 years if the sum of Rs.300 becomes Rs.500 in 8 years and the rate of interest for the last 5 years in 6%?
- A. 12.2 %
B. 13.4%
C. 14.3%
D. 12.5%
2. Find the principal of the interest compounded at the rate of 5% per annum for the two years is Rs.300.
- A. Rs.2845.56
B. Rs.2934.56
C. Rs.2376.89
D. Rs.2926.83
3. Find the present worth of Rs. 70000 due in 3 years at 10% interest per year?
- A. Rs. 6000.45
B. Rs. 6153.85
- C. Rs. 6473.28
D. Rs. 6128.34
4. Find the compound interest on Rs. 15000 at 10% rate of interest for 1 year, compounded half-yearly?
- A. Rs. 1376.7
B. Rs. 1537.5
C. Rs. 1487.3
D. Rs. 1626.2
5. The difference between SI and CI compounded annually on a certain sum of money for 2 years at 7% per annum is Rs. 13.50. Find the principal?
- A. Rs. 2755.1
B. Rs. 2834.2
C. Rs. 2647.4
D. Rs. 2893.2
6. A certain amount becomes Rs. 5700 in 2 years and Rs. 6900 in 3 years. What is the principal amount and the rate of interest?
- A. Rs. 3874.34, 21.5%
B. Rs. 3284.38, 21.2%
C. Rs. 3345.89, 20.5%
D. Rs. 3889.96, 21.05%
7. A sum of Rs. 500 was taken as a loan. This is to be paid back in two equal installments. If the rate of interest be 10% compounded annually, then the value of each installment is ?
- A. Rs. 288.09
B. Rs. 266.56
C. Rs. 237.39
D. Rs. 219.39

8. The simple interest on a certain sum of money for 3 years at 10% per annum is half the compound interest on Rs. 5000 for 2 years at 12% per annum. The sum placed on simple interest is?
- A. Rs. 2349
B. Rs. 2139
C. Rs. 2322
D. Rs. 2120
9. A Woman took a loan of Rs. 10,000 to purchase a mobile. She promised to make the payment after three years. The company charges CI at 10% per annum for the same. But, suddenly the company announces the rate of interest as 20% per annum for the last one year of the loan period? What extra amount she has to pay due to the announcement of new rate of interest?
- A. Rs. 1200
B. Rs. 1180
C. Rs. 1290
D. Rs. 1210
10. The ratio of the amount for two years under compound interest annually and for one year under simple interest is 7:6. When the rate of interest is same, then the value of rate of interest is?
- A. 16.67%
B. 15.34%
C. 17.64%
D. 12.89%

Time Speed and Distance

Speed: The rate at which someone or something moves or operates or is able to move or operate is called Speed. Speed refers to the distance travelled by an object during a specific amount of time. Speed is a very fundamental and important concept in motion which is all about how fast or slowly any object moves. We define speed as distance divided by time.



Distance: Distance is a numerical measurement of how far apart objects are. Distance is a scalar quantity or a magnitude. It usually refers to physical space in between two objects, like the distance between your house and the school. It can also mean an interval in time, like a distance of three years since you graduated.

Time: Time is the unspecified continued process of existence and events that occur in succession from the past through the present to the future. Time is a quantity of various measurements used to sequence events, to compare the duration of events or the intervals between them.

Important formula:

$$\text{Speed} = \text{Distance} / \text{Time}$$

$$\text{Distance} = \text{Speed} * \text{Time}$$

$$\text{Time} = \text{Distance} / \text{Speed}$$



Conversions:

Speed:

Km/hr to m/s : Multiply with $5/18$

m/s to km/hr: Multiply with $18/5$

Km/hr to miles/hr: Multiply with $5/8$

miles/hr to km/hr: Multiply with $8/5$

km/hr to m/min: Multiply with $1000/60$

m/min to km/hr: Multiply with $60/1000$

Distance:

Km to metres: Multiply with 1000
metres to km: multiply with 10^{-3}
km to miles: multiply with $5/18$
miles to km: multiply with $1 \frac{8}{5}$

Time:

Hours to minutes: Multiply with 60
Minutes to Seconds: Multiply with 60
Hours to seconds: Multiply with 3600
Minutes to hours: Divide by 60
Seconds to minutes: Divide by 60
Seconds to hours: Divide by 3600

Example:

A car moves with a speed of 25 km/hr covering a distance of 1500metres. What is the time taken in minutes and the distance covered in miles?

Ans:

Speed = 25 km/hr

Distance = 1500metres

$$\text{Speed} = 25 * 1000 / 60 = 5000 / 12 = 2500 / 6 = 1250 / 3 \text{ m/min}$$

$$\text{Time} = \text{distance} / \text{speed} = 1500 / (1250 / 3) = 4500 / 1250 = 450 / 125 = 90 / 25 = 18 / 5 = \mathbf{3.6 \text{ minutes.}}$$

Distance in miles:

$$1500 \text{ m} = 1.5 \text{ km} = 1.5 * \frac{5}{8} \text{ miles} = \mathbf{0.625 * 1.5 = 0.9375 \text{ miles}}$$

Relative Speed:

When two bodies are moving in same direction, Relative speed = $|A - B|$ where A and B are the individual speeds of the two bodies.

When two bodies are moving in opposite directions, Relative Speed = $|A+B|$ where A and B are the individual speeds of the two bodies.



Examples:

1) Two athletes are running from the same place at the speed of 8 km/hr and 6km/hr. find the distance between them in miles after 20 minutes?

Ans. Here the athletes are running in the same direction so relative speed = $8-6=2$ km/hr.

Time = 20 minutes = $20/60 = 1/3$ hours. Distance = $2/3$ km

Distance in miles = $2/3 * 5/8 = 5/12 = \mathbf{0.416 \text{ miles}}$

2)Two athletes are running towards each other at the speeds of 10m/s and 20 m/s respectively. Find the time in hours if distance covered by them is 50 miles.

Ans: Relative speed = 30 m/s since the athletes are moving in opposite directions.

Distance = 50 miles = $50*8/5$ km= 80 km = 80000 metres

Time = Distance/Speed = $80000/30 = 8000/3$ seconds = $(8000/3)/3600 = 80/3*36 = 80/108= \mathbf{0.74 \text{ secs}}$

Problems are divided into three categories: Trains, Boats and Streams and Races

Trains:



1) When train is crossing a man or a body which is stationary

Distance travelled by train = Length of the train+ Length of the object

The object is very small compared to the train so the length of the object becomes negligible compared to that of the train

In other words, Distance travelled by train = Length of train

Example:

* A man is standing on the platform waiting for a train whose length is 0.2 kms. Train crosses the man in 55 seconds. What is the speed of the train in km/hr?

Ans) Length of the train = 0.2 kms

Time = 55 seconds=55/3600 hours

Speed = $0.2/55/3600 = 0.2*3600/55 = 720/55=144/11=\mathbf{13.09 \text{ km/hr.}}$

2) When train is crossing a bridge or a platform

Distance travelled by train = Length of train+ Length of bridge or platform



Note: Here the bridge or platform is quite big so we can't make it negligible.

Example:

* A train 600metres long crosses a bridge and speed of the train is 50 km/hr. It crosses the bridge in 50 seconds. What is the length of the bridge in miles?

Ans) Length of the train = 600 metres = 0.6 kms

Speed = 50 km/hr

Time = 50/3600 hours

Distance travelled by train = Speed * Time = $50 \times 50 / 3600 = 25 / 36$ km

Length of bridge = $0.7 - 0.6 = 0.1$ Kms = $0.1 \times 5 / 8 = 0.5 / 8 = 5 / 80 = 0.0625$ miles.

3) When train crosses a man who is moving

When train crosses a man who is moving, relative speed occurs.

Example

* A train crosses a man who is moving with a speed of 50 km/hr in the same direction as the man. Relative speed is 60 km/hr. Distance covered by train is 1000m. What is the time taken by train to cross the man in minutes?

Ans) Speed of man = 50 km/hr

Relative speed = 60 km/hr

Speed of train= 110 km/hr. Distance = 1000m

Time taken = $1000 / 110 = 60 / 110 = 6 / 11 = 0.545$ minutes.

4) When two train cross each other



- **When two trains cross each other in the same direction**

Distance travelled = $L_1 + L_2$ (Sum of the lengths of the individual trains)

Relative Speed = $S_1 - S_2$ (Difference between the individual speeds)

- When two trains cross each other in opposite directions

Distance travelled = $L_1 + L_2$ (Sum of the lengths of the individual trains)

Relative speed = $S_1 + S_2$ (Sum of the individual speeds)

Example:

Two trains of lengths 500m and 600m cross each other travelling in opposite directions taking a time of 150 seconds. What is the relative speed in km/hr?

Ans Distance travelled = 1100 m

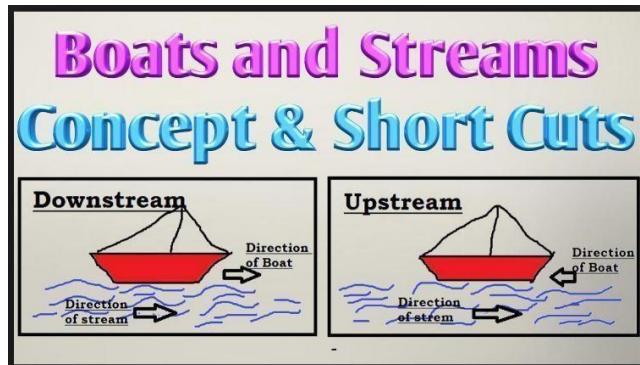
Time = 150 seconds

$$\text{Speed} = 1100/150 = 110/15 = 7.33 \text{ m/s} = 7.33 * 18/5 = \mathbf{26.388 \text{ km/hr}}$$

Boats and Streams:

In this concept we consider the speed of the boat in still water and the speed of the stream

If the boat and stream are moving in same direction, it is called downstream speed and when they are moving in opposite directions it is called upstream speed.



Case 1:

Let speed of boat in still water be U km/hr

Let speed of stream be V km/hr

Downstream speed = $U+V$ km/hr

Upstream speed = $|U-V|$ km/hr

Case 2:

Let downstream speed = a km/hr

Upstream speed = b km/hr

Speed of boat in still water = $(a+b)/2$

Speed of stream = $|a-b|/2$



Examples:

* A boat travels with a speed of 50 km/hr in opposite direction to the stream which is moving with a speed of 60 km/hr. What is the downstream or upstream speed in m/s?

Ans) Speed of boat in still water = 50 km/hr

Speed of stream = 60 km/hr

Upstream speed = 10 km/hr = $10 \times 5/18 = 50/18 = 2.77$ m/s

* The downstream and upstream speed of boat and stream are 45 km/hr and 60 km/hr respectively. What is the rate of boat in still water and what is the rate of current(in m/s)?

Ans) Rate of boat in still water = $(45+60)/2 = 105/2 = 52.5$ Km/hr * $5/18 = 14.58$ m/s

Rate of current = $60-45 = 15/2 = 7.5 \times 5/18 = 2.08$ m/s

Races:

It is a contest of speed in running, riding, driving, rowing etc over a particular distance.



Starting Point - Starting Point is the point from which a race starts.

Winning Point (or Goal) - Winning Point (or Goal) is the point where a race finishes.

Dead-heat Race

A race is said to be a dead-heat race if all the persons taking part in the race, reach the winning point (goal) exactly at the same time.

Giving a start:

Giving a start to someone means If there are two people A and B and A gives a start to B; then A asks B to stand some distance in front of him or starts later than B in the race.

For eg:

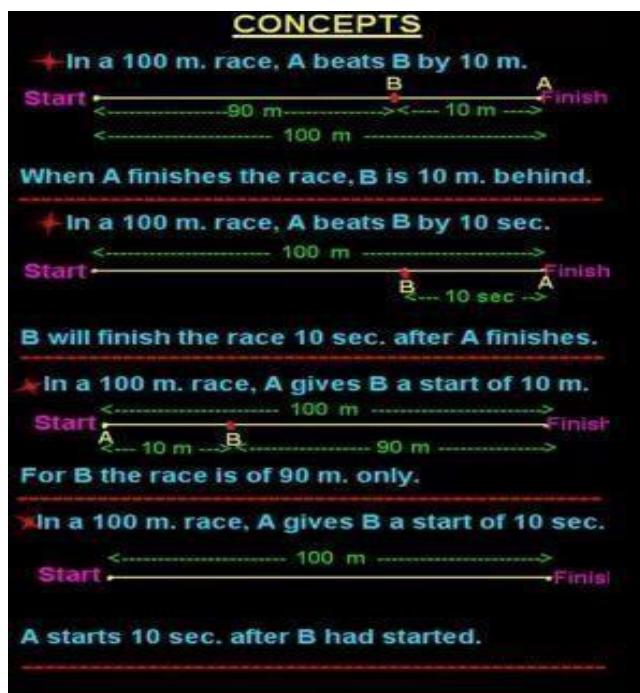
* In a race of 100m A gives B a start of 15m.

Here it means A allows B to stand 15metres in front of him so that B can run 85 m only while A has to run 100m.

*In a race of 100m, A gives B a start of 20 seconds

Here it means A allows B to start 20 seconds before himself so that B gets an advantage over A and can start 20 seconds before A starts.

Concepts:



Example:

In a 100M race A gives B a start of 20m and A gives C a start of 30 m How much start does B give C?

Ans) In the first 100m race B will cover 80m while in the second 100m race C covers 70m.

B will give C 10m start if it covers 80M What will be the start for B covering 100m.

$$\text{Start} = 100 * 10 / 80 = 1000 / 80 = 100 / 8 = \mathbf{12.5 \text{ m}}$$

Apart from the above three concepts i.e. Trains, Boats and Streams and Races there are some more important things to remember

***Average Speed**

*Average speed of two bodies moving with two different speeds A and B is $(2*A*B)/(A+B)$ when the distance covered is the same.

***If distance covered is not the same, Average Speed = Total distance/ Total time**

*Average speed of three bodies moving with three different speeds A, B and C is $(3*A*B*C)/(AB+BC+AC)$ when the distance covered is the same.

***When one of the terms is constant**

*When distance is constant, Speed = Distance/ Time. Speed and time are inversely proportional.

As speed increases time decreases and as speed decreases time increases

*When speed is constant, Distance and Time are directly proportional. As distance increases time increases and as distance decreases time decreases.

*When time is constant, Speed and Distance are directly proportional. As Speed increases time increases and as speed decreases time decreases

Distance covered in 5 minutes = $5 * 5 / 60 = 5 / 12 \text{ km} = 5000 / 12 = 2500 / 6 = 1250 / 3 = 416.67 \text{ m}$

Distance between thief and policeman is $(500 - 416.67) \text{ m} = 83.33 \text{ m}$.

Finding Speed or Time Required after Crossing Each Other

Theorem: If two persons or trains A and B start their journey at the same time from two points P and Q towards each other and after crossing each other they take a and b hours in reaching Q and P respectively, then

A's speed/B's speed=Square root of (b/a)

Example: Two, trains, one from Bangalore to Patna and the other from Patna to Bangalore, start simultaneously. After they meet, the trains reach their destinations after 16 hours and 9 hours respectively. The ratio of their speeds is:

Ans:

$$S_A/S_B=\text{Sqrt } (9/16) = 3/4.$$

Important Problems:

1. A thief is noticed by a policeman from a distance of 500 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 15 km and 20 km per hour respectively. What is the distance between them after 5 minutes?



Ans:

$$\text{Relative speed of the thief and policeman} = (20-15)=5 \text{ km/hr}$$

2. A man takes 7 hours 15 minutes in walking a distance and riding back to the starting place. He could walk both ways in 8 hours 45 minutes. The time taken by him to ride both ways, is?

Ans:

$$\text{Time taken in walking both ways} = 8 \text{ hours } 45 \text{ minutes} \text{ --- (i)}$$

$$\text{Time taken in walking one way and riding back} = 7 \text{ hours } 15 \text{ minutes} \text{ --- (ii)}$$

$$W_{\text{front}}+W_{\text{back}} = 8*60+45 = 525 \text{ minutes} \text{ --- (i)}$$

$$W_{\text{front}}+R_{\text{back}} = 7 *60+15 = 435 \text{ minutes} \text{ --- (ii)}$$



By equation (ii)*2-(i), we have

Time taken to man riding both ways = 14 hours 30 minutes - 8 hours 45 minutes = 870 minutes - 525 minutes = 345 minutes/60 = 5 hours 45 minutes.

3. In a one km race A gives B a start of 120m and in a one km race B gives a start of 90 m to C. In a 1 km race who will win and by how much distance from the worst performer between two losers?

Ans:

Ratio of speeds of A:B = 1000:880=100:88

Ratio of speeds of B:C = 1000:910 = 100:91

Thus when A moves 1000m, B moves 880m and when B moves 880m, C moves 800.8m

Since, C moves 9% less than B in the same time. Thus, C is the worst performer and A will win him by 199.2 m

Using shortcuts:

* A boy goes to school at a speed of 5 km/hr and returns to village at 3 km/hr. If he takes 6 hours, what is the distance between the school and the village?

Ans: One way distance = Total time taken x (Product of two speeds) / (Addition of two speeds) = T *

$$\{S_1 \cdot S_2 / (S_1 + S_2)\}$$

Distance between school and village is $6 * (5*3) / (5+3) = 6 * 15 / 8 = 90 / 8 = 11.25$ km

Thus, from Time, Speed and Distance problems we learn to calculate one term using the other two given terms at basic level and finding the different terms using the data given in the question at advanced level.

Practice questions Set I:

1) Excluding stoppages, the speed of a bus is 56 kmph and including stoppages, it is 44 kmph. For how many minutes does the bus stop per hour?

A.4.84

B.6.68

C.8.42

D.12.86

2) Two trains starting at the same time from 2 stations 250 km apart and going in opposite direction cross each other at a distance of 120 km from one of the stations. What is the ratio of their speeds?

A.12:13

B.7:3

C.18:4

D. None of these

3) A train covers a distance in 40 min, if it runs at a speed of 50 kmph on an average. The speed at which the train must run to reduce the time of journey to 30 min will be.

A.45.45 kmph

B.66.67 kmph

C.75.46 kmph

D. None of these

4) Samuel can cover a distance in 1hr 25 min by covering $\frac{3}{4}$ of the distance at 5 kmph and the rest at 6 kmph. The total distance is?

A.5.49 km

B.7.39 km

C.7.41 km

D.8.26 km

5) A passenger train takes two hours less for a journey of 350 km if its speed is increased by 6 km/hr from its normal speed. The normal speed is?

A.35.64 km/hr

B.50.26 km/hr

C.29.54 km/hr

D.30.46 km/hr

6) If the wheel of a bicycle makes 550 revolutions in travelling 1.5 km, what is its radius?

A.43 cm

B.47 cm

C.45 cm

D.42cm

7) Rajesh travelled from city A to city B covering twice as much distance in the second part as he did in the first part of this journey. His speed during the second part was twice as that of the speed during the first part of the journey. What is his average speed of journey during the entire travel?

A. His average speed is the harmonic mean of the individual speeds for the two parts.

B. His average speed is the arithmetic mean of the individual speeds for the two parts.

C. His average speed is the geometric mean of the individual speeds for the two parts

D. Cannot be determined.

8) The speed of a motor boat itself is 22 km/h and the rate of flow of the river is 5 km/h. Moving with the stream the boat went 110 km. What distance will the boat cover during the same time going against the stream?

- A. 69.26 km
- B. 72.34 km
- C. 60.45 km
- D. 74.32 km

9) A motorboat, whose speed in 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is:

- A. 10
- B. 6
- C. 5
- D. 4

10) A boatman goes 4 km against the current of the stream in 3 hour and goes 4 km along the current in 30 minutes. How long will it take to go 6 km in stationary water?

- A. 1.29 hours
- B. 2.34 hours
- C. 1.36 hours
- D. 2.15 hours

Practice Questions Set II

11) Rajesh walks to and fro to a shopping mall. He spends 30 minutes shopping. If he walks at speed of 11 km an hour, he returns to home at 19.00 hours. If he walks at 15 km an hour, he returns to home at 18.30 hours. How fast must he walk in order to return at 18.15 hours?

- A. 27.3 km/hr
- B. 28.5 km/hr
- C. 29.6 km/hr
- D. 27.5 km/hr

12) Starting from my office, I reach the house 25 min late if I walk at 4kmph. Instead, if I walk at 5 kmph, I reach the house 15 min early. How far is my house from my office?

- A. 14.23 km
- B. 12.43 km
- C. 13.33 km
- D. 16.23 km

13) A passenger train covers the distance between stations X and Y, 60 minutes faster than a goods train. Find this distance if the average speed of the passenger train is 70 kmph and that of goods train is 30 kmph?

- A. 2.45 kms
- B. 5.25 kms
- C. 4.5 kms
- D. 4 kms

14) A is $3 \frac{1}{3}$ times as fast as B. If A gives B a start of 70 m, how long should the race course be so that both of them reach at the same time?

- A. 69 metre
- B. 75 metre
- C. 49 metre
- D. 59 metre

15) A can run 224 metre in 29 seconds and B in 33 seconds. By what distance A beat B?

- A. 32.12 metre
- B. 24.12 metre
- C. 32.14 metre
- D. 27.12 metre

16) In a 550 m race, the ratio of the speeds of two contestants A and B is 4 : 5. A has a start of 150 m. Then, A wins by:

- A. 160 m
- B. 180 m
- C. 140 m
- D. 110 m

17) Jim travels the first 3 hours of his journey at 70 mph speed and the remaining 6 hours at 26 mph speed. What is the average speed of Jim's travel in mph?

- A. 42 mph
- B. 36 mph
- C. 40.67 mph
- D. 42.5 mph

18) A man and a woman 82 miles apart from each other, start travelling towards each other at the same time. If the man covers 6 miles per hour to the women's 5 miles per hour, how far will the woman have travelled when they meet?

- A. 27.5
- B. 37.25
- C. 35.45

D. None of these

19) Three friends A, B and C run around a circular track of length 120 metres at speeds of 5 m/s, 7 m/sec and 15 m/sec, starting simultaneously from the same point and in the same direction. How often will the three of them meet?

- A. Every 60 seconds
- B. Every 120 seconds
- C. Every 30 seconds
- D. None of these

20) In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30minutes. What is the duration of the flight?

- A. 2 hour
- B. 1 1/2 hour
- C. 1/2 hour
- D. 1 hour

TIME AND WORK

Time and Work is an important concept of quantitative aptitude. Generally, time and work problems are time consuming where students feel difficult to solve. Now here we are providing easier methods and different kinds of problems in time and work.

INTRODUCTION:

What is Time?

Time is a component quantity of various measurements used to sequence events, to compare the duration of events or the intervals between them. But for this topic time is defined as the duration for the completion a task or a part of a task.

What is Work?

Technically speaking, **work** is the quantity of energy transferred from one system to another system to another but for questions based on this topic, work is defined as the amount of the job assigned or amount of the job actually done.

So, here in this topic going to work on the problems where time and work are related to each other.

Important points:

Rule of Flip:

The complete concept of time and work is based on this Rule of Flip.

The total work is assumed to be 1.

If a person can complete a piece of work in 'n' days, then

By rule of flip, work done in 1 day = $1/n$

From this rule, it is clear time is always a reciprocal of work done.

Example: If Ram and Raman can do a job in 10 days and 15 days independently, how many days would they take to complete the same job working simultaneously?

Solution: Time taken by Ram to finish the task = 10 days

By rule of flip, work done = $1/10$

In the same way time taken by Raman = 15 days

Work done = $1/10 + 1/15 = 1/6$

Again by rule of flip, time taken = 6 days.

Efficiency Method:

Efficiency and work done are directly proportional. If the efficiency of a person is more, then the work done is also more and vice-versa.

For example, the efficiency of two persons A and B are in the ratio of 2:1 i.e, B is twice as good workman as A.

The ratio of work done = 1:2

Example: A is as good workman as B and both can finish a piece of work in 18 days. In how many days will A alone finish the work?

Solution: A:B = 1:2 [Time Taken/ Efficiency}

A: B= 2:1 [Work done]

So, work done by A = $2/3$

A and B can finish the task in 18 days, work done in 1 day = $1/18$

A's work per day = $1/18 * 2/3 = 1/27$
A alone can finish the task in 27 days.

JOINING AND LEAVING THE JOB:

These problems are most important, time consuming and critical to solve. Hence, here we are giving a simple shortcut of verifying option.

Example: A can do a piece of work in 10 days while B can do the same work in 15 days. They begin together but 5 days before the completion of job B leaves off. Find the number of days for the work to be completed?

- A) 6days B) 8 days C) 7 days D) 9 days

Solution: Total work done = 1

Persons who started the work = A & B

Persons till the end of the work = A

As given, A can finish the task in 10 days and B can complete in 15 days.

Here, should calculate how many days A worked out of 10 days to complete the task

Consider option A

Out of 10 days A worked for 6 days = $6/10$

Then, B left 5 days before the completion of the task. From this, B worked for 1 day out of 15 days = $1/15$

$6/10 + 1/15 = 1$ [as total work is considered as 1]

$5/30 \neq 1$ [So eliminate option A]

Option B

$8/10 + 3/15 = 1$

$1=1$ [Therefore, L.H.S = R.H.S]

PRACTICE QUESTIONS SET I

1. Akbar and Birbal can complete a piece of work in 8 days. If A alone can complete the same work in 16 days, in how many days can B alone complete the work?
A. 10 days B. 16 days
C. 17 days D. 18 days
2. Ram takes 12 hours to do a job. Kiran takes 18 hours to do a Job. How long will it take for both working together to finish the task?
A. $34/5$ B. $22/5$
C. $36/5$ D. $41/5$
3. Karthik can do a certain job in 15 days. Bhanu is 80% more efficient than A. How many days does Bhanu alone take to do the same job?
A. 8 days B. 7.5 days
C. 8.3 days D. 9 days
4. 30 girls can repair a road in 15 days, working 5 hours a day. In how many days will 25 girls, working 6 hours a day, complete the work?
A. 15 days B. 13 days
C. 14 days D. 17 days
5. Raju and Mahesh undertake to do a piece of work for Rs.800. Raju alone can do it in 8 days while Mahesh alone can do it in 6 days. With the help of Kyathi they finish it in 3 days. Find the share of Kyathi?
A. 200 B. 100
C. 75 D. 85
6. Keerthi can complete a work in 20 days while Sruthi in 30 days and Megha in 60 days. Keerthi works continuously on every third day. In how many days work will be completed?
A. 16 days B. 15.5 days
C. 15 days D. None of these
7. A tyre has two punctures. The first puncture alone would have made the tyre flat in 12 minutes and second alone would have done it in 15 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat?
A. $6 \frac{1}{5}$ min B. $6 \frac{2}{5}$ min
C. $6 \frac{2}{3}$ min D. $6 \frac{4}{5}$ min
8. A, B and C can do a piece of work in 11 days, 20 days and 55 days respectively working alone. How soon can the work be done if A is assisted by B & C on alternate days?
A. 4 days B. 2 days
C. 8 days D. 6 days
9. Ajay can complete a piece of work in 8 days. After working for 5 days, Rahul joins him and they finish the remaining work in next 1 day. In how many days can Rahul finish the work alone?
A. 4 days B. 3 days
C. 10 days D. 5 days
10. Arun can do a piece of work in 15 days and Bhanu in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is
A. $\frac{1}{4}$ B. $\frac{1}{10}$
C. $\frac{7}{15}$ D. $\frac{8}{15}$
- 11.

PRACTICE QUESTIONS SET II

1. Kiran is thrice as good workman as Ambika and therefore is able to finish the job in 60 days less than Ambika. Working together they can do it in
 - A. 20 days
 - B. 22 1/2 days
 - C. 25 days
 - D. 30 days
2. A alone can do a piece of work in 6 days and B alone in 8 days. A and B undertook to do it for Rs. 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?
 - A. Rs. 375
 - B. Rs. 400
 - C. Rs. 600
 - D. Rs. 800
3. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be:
 - A. 4 days
 - B. 5 days
 - C. 6 days
 - D. 7 days
4. A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it?
 - A. 8 hours
 - B. 10 hours
 - C. 12 hours
 - D. 24 hours
5. A and B can do a piece of work in $6 \frac{2}{3}$ days and 5 days respectively. They work together for 2 days and then A leaves. In how many days after that B will complete the work alone.
 - A. 2 days
 - B. 1 1/2 days
 - C. 3 days
 - D. 4 days

6. A can do a piece of work in 30 days. He works at it for 5 days and then B finishes it in 20 days. In what time can A and B together it?
 - A. 10 days
 - B. 13 1/3 days
 - C. 14 1/4 days
 - D. 16 days
7. 4 men and 6 women working together can complete the work within 10 days. 3 men and 7 women working together will complete the same work within 8 days. In how many days 10 women will complete this work?
 - A. 4 days
 - B. 12 days
 - C. 5 days
 - D. 6 days
8. To complete a piece of work A and B take 8 days, B and C takes 12 days. A, B and C takes 6 days. Then A and C will take ?
 - A. 8 days
 - B. 7.5 days
 - C. 7 days
 - D. 6.5 days
9. Mr. Raman is on tour and he has Rs 360 for his expenses. If he exceeds his tour by 4 days he must cut down daily expenses by Rs 3. Find the number of days ?
 - A. 15 days
 - B. 20 days
 - C. 25 days
 - D. 30 days
10. Amit takes twice as much time as Rithik or thrice as much time as Solmon to finish a piece of work. All working together they can finish the work in 2 days. Rithik can do the work alone in
 - A. 4 days
 - B. 6 days
 - C. 8 days
 - D. 12 days

Permutation and Combination



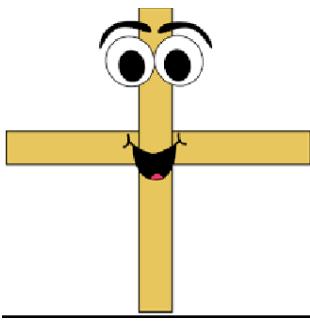
1. Multiplication rule:

If an event can occur in m ways, a second event in n ways and a third event in r ways then the three events can occur in $m \times n \times r$ ways.

Eg. Herald has 6 tops, 7 skirts and 3 caps from which to choose an outfit. In how many ways can she select one top, one skirt and one cap?

Solution: Number of ways = $6 \times 7 \times 3 = 63$ ways.

2. Addition Theorem (Fundamental Principles Of Counting)



If an operation can be performed in m different ways and a second independent operation can be performed in n different ways, either of the two operations can be performed in $(m+n)$ ways.

Eg: Chandan wants to go to Washington. He can choose from 4 bus services or 3 train services to head from home to downtown New York. From there, he can choose from 3 bus services or 4 train services to head to Washington. How many ways are there for Cinderella to get to Washington?

Solution: He has $4+3=5$ ways to get to New York.(Rule of Sum)

From there he has $3+4 = 7$ ways to get to Washington.(Rule of Sum)

Hence he has $5 \times 7 = 35$ ways to get to Washington in total.(Rule of Product).

Repetition of an Event

If one event with n outcomes occurs r times with repetition allowed, then the number of ordered arrangements is n^r .

Eg: What is the number of arrangements if a die is rolled

(a) If a die is rolled twice then the permutation is $6^2=36$

(b) If a die is rolled thrice then the permutation is $6^3 = 216$

(c) If a die is rolled r times then the permutation is 6^r times



Eg

How many different car number plates are possible with 3 letters followed by 4 digits?



Ans: $26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10 = 26^3 \times 10^4$

Factorial Representation:

$$n! = n(n-1)(n-2) \dots 3 \times 2 \times 1$$

$$\text{For eg: } 4! = 4 \times 3 \times 2 \times 1$$

$$\text{Note: } 0! = 1$$

Eg:

a) In how many ways can 6 people be arranged in a row?



Solution: $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

b) How many arrangements are possible if only 3 of them are chosen?

Solution: $6 \times 5 \times 4 = 120.$

Permutations:

Permutation means setting the elements of a set into an order or resetting the ordered elements.

If I buy a dish for lunch, it may contain a mixture of onions, raddish, cucumber and brinjals. The orders in which the vegetables are mixed doesn't really matter. All that matters is that the dish that contains onions, raddish, cucumber and brinjal can consist of the vegetables in any order.|| It's still the same dish.


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Eg: All arrangements that can be formed with the letters a,b and c by taking three at a time are (abc,acb,bac,bca,cab,cba)

All arrangements that can be formed with the letters a,b,c by taking two at a time are (ab,ac,ba,bc,ca,cb)

The order matters in permutation and is extremely important. Thus it needs to be preserved.

Eg: In how many ways can the letters of the word ANDHRA be arranged?

Solution: ANDHRA has total 6 words. So we will arrange 6 letters at 6 places in 6! ways = 720 ways.

But in this question A is coming twice. Whenever any letter is more than once in the word then we have to divide by the number of repetitions of the letter. So we have to divide the total 120 ways by $2! = 2$. So total different words that can be made will be $120/2 = 60$

Eg: How many different words can be made using letters of RAMBA starting with R

Solution: RAMBA has total 5 letters. According to the question, R is fixed at first place so we will arrange remaining 4 letters at 4 places in $4! = 24$ ways. But in this question A is coming twice so we have to divide the total 24 ways by $2!=2$ making it $24/2 = 12$ ways.

The number of permutations of n objects taken r at a time is given by

$$nPr = n!/(n-r)!$$

where n=number of objects

r = number of positions.

Eg:

$$* {}^6P_4 = 6!/2! = 3*4*5*6 = 360$$

$$* {}^4P_2 = 4!/2! = 3*4 = 12$$

$$* {}^5P_3 = 5!/2! = 3*4*5 = 60$$

Permutation formula proof:

- There are n ways to choose the first element
- $n-1$ ways to choose the second
- $n-2$ ways to choose the third
- $n-r+1$ ways to choose the r^{th} element

By the product rule, that gives us:

$$P(n,r) = n*(n-1)*(n-2)-----*(n-r+1)$$



Special cases

$${}^n P_0 = 1$$

$${}^n P_r = 0 \text{ for } r > n. \quad {}^n P_r \text{ is also denoted by } P(n,r).$$

Number of permutations of n distinct things taking all at a time = ${}^n P_n = n!$

Remember By definition, $0! = 1$.

*The number of permutations of n objects where p_1 objects are of one kind, p_2 objects are of second kind----- p_k objects are of k^{th} kind is $n! / p_1! p_2! \dots p_k!$

Eg: A team consists of 5 speakers



a) In how many ways can all 5 speakers be arranged in a row?

Solution: $5 * 4 * 3 * 2 * 1 = 5!$ or $5P5$

Eg: Joe has 8 different items. Find the number of orders in which the 8 items can be arranged in a shelf?

Solution: The number of items is 8

$8P8 = 8!/(8-8)! = 8!/0! = 8*7*6*5*4*3*2*1/1 = 40320$. The items are arranged 8 at a time.

There are 40320 permutations. This means there are 40320 orders in which 8 items can be arranged on the shelf.

Combinations:

A Combination is a selection of things in any order. Whenever we deal with combinations order is not important.

The number of ways in which a subset of objects can be selected from a given set of objects, where order is not important is a Combination.

Eg: Suppose we want to select 3 out of 4 boys A,B,C,D then possible combinations are AB,BC,CD,DA.(Note that BC and CB represent the same selection)



$$C(n,r) = n!/r!(n-r)! \text{ (where } 0 < r \leq n).$$

Eg:

$$* {}^6C_4 = 6!/2!4! = 5*6/2 = 15$$

$$* {}^7C_3 = 7!/4!3! = 210/6 = 70/2 = 35$$

$$* {}^5C_4 = 5!/1!4! = 5$$

Combination formula derivation:

Let there be 'n' objects from which you have to form selections or groups of 'r' objects at a time. Let the total number of combinations, or selections of these 'n' objects taken 'r' at a time be $C(n,r)$. Now each of these $C(n,r)$ combinations contains r objects, and hence gives rise to $r!$ different permutations. Thus the total number of permutations of all of these n objects, taken r at a time is equal to $C(n,r) \cdot r!$.

Now, we know that the formula for permutations of n objects taken r at a time is $n!(n-r)! \cdot r!$. Hence $C(n,r) \cdot r!$ is equal to $n!(n-r)! \cdot r!$. Thus we obtain the equation, $C(n,r) \cdot r = n!(n-r)! \cdot r!$. Solving this equation for $C(n, r)$, we obtain:

$C(n,r)=n!(n-r)! \cdot r!$ which is the formula for combinations of n objects taken r at a time.

Eg: A committee of 4 persons is to be formed from a group of 3 men and 4 women. In how many ways can this be done ?

Solution: Here order does not matter. Therefore we need to count Combinations. There will be as many committees as there are combinations of 7 different persons taken 4 at a time. Hence the required number of ways = ${}^7C_4 = 7!/3!(4!) = 5*6*7/6 = 35$.

* Eg: On a circle there are 7 points selected. How many triangles with edges in these points exist?

Solution:

$${}^7C_3 = 7!/4!3! = 35.$$

Special cases:

$${}^nC_{0}=1$$

$${}^nC_r = 0 \text{ for } r > n. {}^nC_r \text{ is also denoted by } C(n,r).$$

Let n and r be non-negative integers with $r \leq n$. Then $C(n,r) = C(n,n-r)$

Question: A five-digit number is formed using digits 2, 3, 4, 6 and 8 without repeating any one of them. What is the sum of all such possible numbers?

Solution: The sum of all the numbers formed by the digits $a_1, a_2, a_3, .an$, without repetition of the digits is given by:

$$(n-1)!(a_1 + a_2 + a_3 ++a_n) (10^{n-1})/9$$

Hence, the sum of the given numbers = $4!(2 + 3 + 4 + 6 + 8) \times 11111 = 6133272$

Determining whether a question is of Permutation or Combination

*If a problem says it is about arrangement or number of ways they can be lined up or made in, then it is a problem on Permutations

*If the problem says find the number of ways it can be selected/chosen/drawn/taken/grouped then it is a problem on Combinations.

Differences/Similarities between Permutations and Combinations

PERMUTATIONS	COMBINATIONS
Number of ways of arranging r items out of n items	Number of ways of selecting r items out of n items
Repetitions are not allowed	Repetitions are not allowed
Order is important	Order is not important
Arrangements of n items taken r at a time	Subsets of n items taken r at a time
${}^nP_r = n!/(n-r)!$	${}^nC_r = n!/(n-r)!r!$
Clue words: arrangement, schedule, order	Clue words: group, sample, selection

Practice Questions Set I:

1. There are 3 questions in a question paper. If the questions have 5, 4 and 3 solutions respectively, find the total number of solutions?
 - A. 50
 - B. 40
 - C. 60
 - D. 70

2. In a city, the bus route numbers consist of a natural number less than 200, followed by one of the letters A, B, C, D and E. How many different bus routes are possible?
 - A. 1005
 - B. 995
 - C. 985
 - D. 975

3. Consider the word LEVEL. Whichever way you read it, from left to right or from right to left, you get the same word. Such a word is known as palindrome. Find the maximum possible number of 5-letter palindromes using alphabet?
 - A. 18756
 - B. 17345
 - C. 18236
 - D. 17576

4. How many 3-digit numbers can be formed with the digits 1,3,5,7 and 8 if the digits are not repeated?
 - A. 60
 - B. 50
 - C. 40
 - D. 30

5. If you have 7 New Year greeting cards and you want to send them to 5 of your friends, in how many ways can this be done?
 - A. 2398
 - B. 2520
 - C. 2734
 - D. 2434

6. In how many ways can a trainer arrange 5 wolves and 4 elephants in a row so that no two wolves are together?
 - A. 2870 ways
 - B. 2780 ways
 - C. 2880 ways
 - D. 2890 ways

7. In how many ways can 5 girls and 7 boys be arranged in a row so that all the five girls are together?
 - A. 4838400
 - B. 4325300
 - C. 4465400
 - D. 4523500

8. How many arrangements of the letters of the word "ORANGE" can be made if the vowels are never together?
 - A. 676
 - B. 734
 - C. 466
 - D. 576

9. Find the number of subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ having 3 elements?

- A. 74
- B. 84
- C. 94
- D. 104

10. 14 points lie on a circle. How many cyclic quadrilaterals can be drawn by using these points?

- A. 1001
- B. 2002
- C. 1401
- D. 1301

Practice Questions Set II

1. In a box there are 6 black pencils, 4 white pencils and 5 red pencils. In how many ways can 2 black pencils, 3 white pencils and 2 red pencils can be chosen?

- A. 500
- B. 400
- C. 600
- D. 700

2. A question paper consists of 16 questions divided into two parts A and B. Each part contains eight questions. A candidate is required to attempt five questions in all of which at least 1 should be from part A and at least 2 from part B. In how many ways can the candidate select the questions if he can answer all questions equally well?

- A. 2565
- B. 2636
- C. 2128
- D. 3696

3. A committee of 4 persons is to be formed from 5 men and 4 women. In how many ways can this be done when at least 2 women are included?

- A. 71
- B. 61
- C. 81
- D. 91

4. In a classroom there are 5 seats in a bench. In how many ways we can allocate 5 students in the same bench in such a way that Neethu and Neha should not be together

- A. 72
- B. 61
- C. 56
- D. 66

5. How many three letter words are formed using the letters of the word MAIN without repetition?

- A. 12 words
- B. 24 words
- C. 32 words
- D. 28 words.

6. Using all the letters of the word "FRIDAY", how many different words can be formed?

- A. 620
- B. 700
- C. 600
- D. 720

7. Using all the letters of the word "APRIL", how many words can be formed, which begin with A and end with L?

- A. 4 ways
 - B. 5 ways
 - C. 6 ways
 - D. 7 ways
8. The number of arrangements that can be made with the letters of the word MANGOES so that the vowels occupy the even places?
- A. 164
 - B. 144
 - C. 154
 - D. 174
9. The number of permutations of letters of the word 'AMALGAM' is?
- A. 420
 - B. 300
 - C. 520
 - D. 440
10. A committee has 6 men and 5 women. What is the number of ways of selecting a group of seven persons?
- A. 220 ways
 - B. 300 ways
 - C. 330 ways
 - D. 400 ways.

HCF AND LCM



What is Prime number?

Consider this number 12. This number can be found in many multiplication tables for example

$$1 \times 12 = 12.$$

$$2 \times 6 = 12$$

$$3 \times 4 = 12$$

That means, 12 has many factors (1,2,3,4,6,12). Such number is called a composite number.

On the other hand, consider the number, 29. You cannot find it in any table except $29 \times 1 = 29$. Such number is called a prime number.

Prime	Non-prime (composite)
2,3,5,7,11,13,17,19,23,29.....	4,6,8,9,10,12,14,15.....

Now hold this prime number thought in your mind for a while.

How to find number of factors of a number

We have to first write the number as a product of prime factors. Then we have to add 1 to each power and multiply the numbers obtained to get the number of factors of the number.

Example

$$\underline{84 = 2 \cdot 42}$$

$$= 2 * (21 * 2)$$

$$= 2 * (3 * 7 * 2) = 2^2 * 3 * 7.$$

$$(2+1) * (1+1) * (1+1) = 12 \text{ factors}$$

Thus, there are 12 factors for the number 84 which are as follows:

1,2,3,4,6,7,12,14,21,28,42,84.

What is HCF?



Highest Common Factor(HCF) of two or more numbers is the greatest number which divides each of them exactly.

Greatest Common Measure(GCM) and Greatest Common Divisor(GCD) are the other terms used to refer HCF.

Example HCF of 60 and 75 = 15 because 15 is the highest number which divides both 60 and 75 exactly.

We can find out HCF using prime factorization method or by dividing the numbers or division method.

How to find HCF?

Prime Factorization method



Let us take two numbers 45 and 54:

$$45 = 15 \times 3 \\ = 3 \times 5 \times 3 = 3^2 \times 5$$

$$54 = 27 \times 2 \\ = 3 \times 3 \times 2 \times 2 = 3^3 \times 2$$

$$\text{HCF} = 3^2 = 9$$

Let us take three numbers 24, 36, 40

$$24 = 3 \times 2^3 \\ 36 = 3 \times 3 \times 4 = 3^2 \times 2^2 \\ 40 = 2^3 \times 5 \times 2 = 2^4 \times 5 \\ \text{HCF} = 2^2 = 4$$

From the above two methods we conclude that to find HCF using prime factorization method we should find the common factor in all the numbers and raise it to its lowest power and multiply them.

HCF by division method

Let us take the two numbers 45 and 54.

We should keep dividing the numbers by their prime factors till they both cannot be divided by a common prime factor.

3 divides 45 and 54; 15 times and 18 times respectively.

3 divides 15 and 18; 5 times and 6 times respectively.

5 and 6 can't be divided further by a common prime factor. Thus, the HCF of 45 and 54 is $3 \times 3 = 9$.

Let us take three numbers 24, 36 and 40

24, 36 and 40 can be divided by 2 giving 12, 18 and 20

12, 18 and 20 can be further divided by 2 giving 6, 9 and 10.

Thus, the HCF is $2 \times 2 = 4$

LCM

The Least Common Multiple (LCM) of a group of numbers is the smallest number that is a multiple of all the numbers. For instance, the LCM of 16 and 20 is 80; 80 is the smallest number that is both a multiple of 16 and a multiple of 20. You can find the LCM of two or more numbers through various methods.



LCM by prime factorization method:

Let us take two numbers 36 and 42

$$36 = 3 \times 2 \times 3 \times 2$$

$$42 = 3 \times 2 \times 7$$

$$36 = 3^2 \times 2^2$$

$$42 = 3 \times 2 \times 7$$

$$\text{LCM} = 3^2 \times 2^2 \times 7 = 36 \times 7 = 252$$

Let us take three prime numbers 24, 38 and 42

$$24 = 3 \times 2^3$$

$$38 = 19 \times 2$$

$$42 = 2 \times 21 = 2 \times 3 \times 7$$

$$\text{LCM} = 3 \times 2^3 \times 19 \times 7 = 3192$$

By using prime factorization method, we conclude that to find LCM we have to take all the factors once and raise it to their highest power and multiply them.

LCM by division method:

Let us take the same two numbers 36 and 42.

We have to continue dividing the numbers by prime factors till we get 1,1 in the end

36 and 42 can be divided by 2 to get 18 and 21.

18 and 21 can be divided by 3 to get 6 and 7

6 and 7 can be divided by 2 to get 3 and 7.

3 and 7 are prime numbers so we can multiply them

Thus, the final LCM will be $3 \times 7 \times 2 \times 3 \times 2 = 252$.

Let us take three numbers 24, 38 and 42.

24, 38 and 42 can be divided by 2 first to get 12, 19 and 21

12, 19 and 21 can be further divided by 3 to get 4, 19 and

7. Final LCM will thus be $4 \times 19 \times 7 \times 3 \times 2 = 3192$.

Important points:



Product of HCF and LCM = Product of two numbers
HCF always divides LCM

HCF and LCM of Decimals:

Steps to solve H.C.F. and L.C.M. of decimals:

Step I: Convert each of the decimals to like decimals.

Step II: Remove the decimal point and find the highest common factor and least common multiple as usual.

Step III: In the answer (highest common factor /least common multiple), put the decimal point as there are a number of decimal places in the like decimals.

Now we will follow the step-by-step explanation on how to calculate the highest common factor and the least common multiple of decimals.

Examples

- Find the H.C.F. and the L.C.M. of 1.40 and 22.2

Sol: 1.40 is written as 140 and 22.2 as 2220

$$140 = 7 \times 20 = 7 \times 5 \times 2^2$$

$$2220 = 2 \times 1110 = 2 \times 2 \times 555 = 2 \times 2 \times 5 \times 111 = 2 \times 2 \times 5 \times 3 \times 37 = 2^2 \times 5 \times 3 \times 37 = 15540$$

We converted them to like decimals by taking two decimal points, so LCM will be 155.40.

$$\text{HCF will be } 2^2 \times 5 = 160 = 1.60$$

HCF and LCM of fractions:

- * HCF of a/b , c/d and e/f is HCF of a , c and e /LCM of b , d , and f
- * LCM of a/b , c/d and e/f is LCM of a , c and e /HCF of b , d and f

Co-prime Numbers:

Co-prime numbers are those numbers whose HCF is 1.

Two co-prime numbers are called twin primes

Three co-prime numbers are called prime triplets.

Example of Twin primes -> (3,7), (3,4), (4,5).

Example of Prime Triplets -> (15,16,17), (3,4,5), (2,6,7).

Some important example problems

The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

Sol: Let the numbers 13a and 13b.

$$\text{Then, } 13a \times 13b = 2028$$

$$\Rightarrow ab = 12.$$

Now, the co-primes with product 12 are (1, 12) and (3, 4).

[Note: Two integers a and b are said to be co-prime or relatively prime if they have no common positive factor other than 1 or, equivalently, if their greatest common divisor is 1]

So, the required numbers are $(13 \times 1, 13 \times 12)$ and $(13 \times 3, 13 \times 4)$.
Clearly, there are 2 such pairs.

* The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is:

$$= \text{H.C.F. of } 1651 \text{ and } 2032 = 127.$$

* Three numbers are in the ratio of 3: 4: 5 and their L.C.M. is 2400. Their H.C.F. is?

Sol:

Let the numbers be $3x$, $4x$ and $5x$.

Then, their L.C.M. = $60x$

So, $60x = 2400$ or $x = 40$.

The numbers are (3×40) , (4×40) and (5×40) .

Hence, required H.C.F. = 40.

* Product of two co-prime numbers is 117. Their L.C.M should be

Sol:

H.C.F of co-prime numbers is 1. So, L.C.M = $117/1 = 117$

* The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is:

Sol:

L.C.M. of 6, 9, 15 and 18 is 90.

Let required number be $90k + 4$, which is multiple of 7. Least value of k for which $(90k + 4)$ is divisible by 7 is $k = 4$.

=> Required number = $(90 \times 4) + 4 = 364$.

Practice Questions Set I

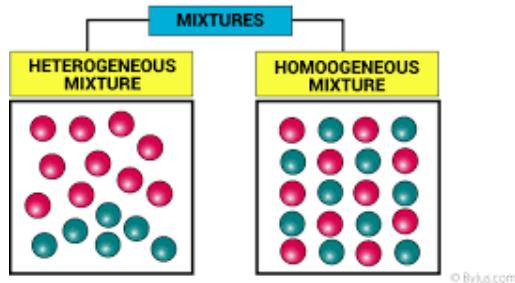
1. Find HCF of 0.24 and 3.6?
 - A. 0.14
 - B. 0.12
 - C. 0.004
 - D. 1.20
2. Find LCM of 2.46 and 0.04?
 - A. 5.92
 - B. 4.96
 - C. 5.94
 - D. 4.92
3. Find HCF of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$?
 - A. $\frac{1}{42}$
 - B. $\frac{2}{9}$
 - C. $\frac{1}{36}$
 - D. $\frac{2}{3}$
4. Find LCM of $\frac{4}{5}$, $\frac{6}{3}$, $\frac{8}{71}$?
 - A. $\frac{24}{1}$
 - B. $\frac{23}{6}$
 - C. $\frac{22}{4}$
 - D. $\frac{20}{5}$
5. HCF of two numbers is 11. If the numbers are in the ratio 5:6, find the numbers?
 - A. 66, 44
 - B. 55, 66
 - C. 44, 66
 - D. 55, 44
6. A rectangular courtyard 4.65 meters long and 5.45 meters wide is paved exactly with square tiles of same size. Find the largest size of the tile used for this purpose? A. 0.06
 - B. 0.07
 - C. 0.05
 - D. 0.04
7. Find the greatest number of 5 digits that will give us a remainder of 4, when divided by 5 and 6 respectively?
 - A. 99992
 - B. 99244
 - C. 99994
 - D. 99430
8. Find the greatest number of four digits which when divided by 8, 10, 12 and 18 leaves 2, 4, 6 and 12 as remainders respectively?

Practice Questions Set II

1. A rectangular courtyard 4.45 meters long and 5.65 meters wide is paved exactly with square tiles of same size. Find the largest size of the tile used for this purpose?
 A. 0.10
 B. 0.20
 C. 0.05
 D. 0.04
2. The GCD and LCM of two numbers are 64 and 386. If one of the numbers divided by 2 gives the results as 190, what is the second number?
 A. 65
 B. 64
 C. 75
 D. 74
3. 5 bells commence tolling together and toll at intervals 4, 6, 8, 10 and 12 seconds respectively. Find in 20 minutes, how many times do they toll together?
 A. 4
 B. 5
 C. 6
 D. 8
4. HCF of two numbers is 13. If the numbers are in the ratio 6:7, find the numbers?
 A. 91, 76
 B. 74, 92
 C. 76, 92
 D. 78, 91
5. There are 600 boys and 400 girls in a school that are to be divided into equal sections of either boys or girls alone. Fine the minimum total number of sections thus formed
 A. 10
 B. 30
 C. 20
6. Find the greatest number of four digits which when divided by 10, 12, 14 and 16 leaves 2, 4, 6 and 8 as remainders respectively?
 A. 8322
 B. 8392
 C. 8244
 D. 8334
7. What least number must be subtracted from 1900 so that the remainder when divided by 6, 8 and 10 will leave in each case the same remainder 6?
 A. 8
 B. 9
 C. 10
 D. 12
8. Find the HCF and LCM of $16xy$, $20x^2y$ and $24x^3y^4$?
 A. $4xy$, $480x^3y^4$
 B. $4x^2y$, $480x^3y$
 C. $4xy^2$, $480x^3y^3$
 D. $4x^3y^2$, $480x^3y^4$
9. A watch ticks 100 times in 105 seconds and another watch ticks 300 times in 320 seconds. If both the watches are started together, how many times will they tick together in the first hour?
 A. 52
 B. 53
 C. 55
 D. 56
10. If the product of two numbers is 84940 and their H.C.F. is 34, find their L.C.M.
 A. 2496
 B. 2498
 C. 2488
 D. 2486

MIXTURES AND ALLIGATIONS:

Mixtures:



A mixture is a substance formed by mixing two or more components together

The mixture obtained can be represented as a ratio or a percentage.

Eg: When two varieties of wheat are mixed to form a new variety of wheat it is called a mixture.

There are two kinds of mixtures:

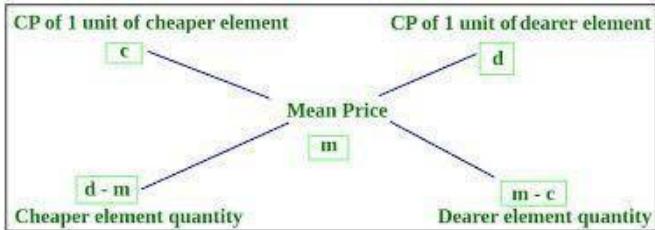
Simple Mixtures:-When two or more different substances are mixed together, a Simple mixture is formed.

Eg: Mixing water with alcohol etc.

Compound Mixture:-When two or more different mixtures are mixed together we get a Compound mixture.

Eg: Mixing two or more alloys containing two or metals mixed together in different ratios.

Alligation:



It is a rule or a process that enables us to mix two or more different ingredients at different prices and ratios and give us a mixture of the desired price.

Mean price:

Mean price is found for unit quantity of the mixture. It is the cost price of that unit quantity.

Weighted Average

Grade	Weight	
0.90	0.25	= 0.225
0.75	0.50	= 0.375
0.87	0.25	= 0.2175

Source: Wikipedia - Calculation of Weighted Average

Let the average weight of 30 boys be 45 kg and let the average weight of 20 girls be 35 kg

$$A = (30*45+20*35) / (30+20)$$

$$= (N1*A1+N2*A2) / (N1+N2)$$

$$\Rightarrow A(N1+N2) = (N1*A1+N2*A2)$$

$$\Rightarrow N1(A-A1) = N2(A2-A)$$

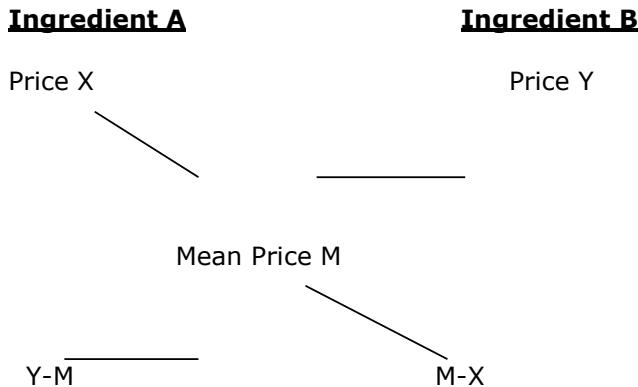
$\Rightarrow N1/N2 = A2-A/A-A1$ where N1 is the quantity of cheaper quantity; N2 is the quantity of dearer quantity; A2 is the Cost price of 1 kg dearer quantity, A is the Mean price and A1 is the cost price of 1kg cheaper quantity

For three ingredients, $A = \frac{(N1*A1+N2*A2+N3*A3)}{(N1+N2+N3)}$

Alligation:

If two ingredients at different prices are mixed together in different ratios to give a mixture of the desired price, the ratios obtained after mixing is called Alligation.

The formula for Alligation can be represented in a diagrammatic form:



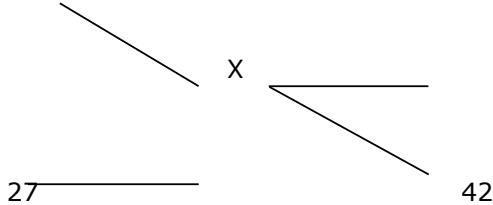
Eg: If 24 kg of rice costs Rs. 42 per kg and 12kg of rice costs Rs. 27 per kg then what is the cost of resultant mixture



Solution:

Ingredient A

12



Ingredient B

24

42

$$27/42 = 24-X/X-12$$

$$27(X-12) = 42(24-X)$$

$$27X-324 = 1008-42$$

$$69X = 1332; X = 1332/69 = 19.3 \text{ kg}$$

Replacement of Part of Solution formula:

If a container contains a solution and part of the solution is taken out and replaced with one of the ingredients and this process is repeated x times then the following formula can be used:

Final Amount of ingredient that is not replaced = [Initial Amount \times (Vol. after removal/Vol. after replacing)] x

Above formula is not only true for absolute amounts but for ratios as well. So following formula is also valid:

Final ratio of ingredient not replaced to total = [Initial ratio \times (Vol. after removal/Vol. after replacing)] x

Eg: Find the quantity of wine left in a cask containing 100 litres of wine if 10 litres are withdrawn and replaced with water, again 10 litres of resultant mixture is withdrawn and replaced with water?

Solution: Quantity of original liquid left = $x (1-y/x)^n$

x is the initial amount of original liquid

y is the quantity of mixture withdrawn each time

n is the number of times the process is done

$$=100(1-10/100)^2 = 100*9/10*9/10 = 81 \text{ litres of wine and } 19 \text{ litres of water.}$$

Mixture containing three ingredients:

In what ratio must a person mix three kind of tea each of which has a price of 60, 70 and 130 rupees per kg, in such a way that the mixture costs him 90 rupees per kg?

Solution:

Here the prices of tea are 60, 70 and 130 and mean price is 90.

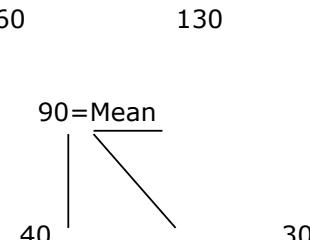
Here prices lower than the mean are 60 and 70 and prices higher than the mean is 130.

Thus possible pairs which give mean value of 90 is: {60,130} and {70,130}

Let us denote tea of Rs. 60 with t_{60} , tea of Rs. 70 with t_{70} and tea of Rs. 130 with t_{130}

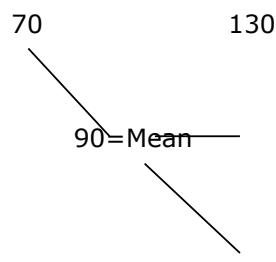
We apply the old alligation rule to all(two in this case) the pairs

For the 1st pair (t_{60} , t_{130})



$$t_{60}:t_{130} = 40:30$$

Similarly for the second pair (t_{70} , t_{130})



$$t_{70}:t_{130} = 40:20$$

Thus Final ratio:

$$t_{60}:t_{70}:t_{130} = 40:40:(30+20) = 4:4:5.$$

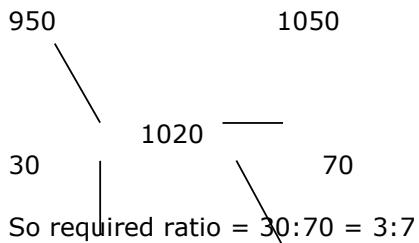
Examples:

1) In what ratio must rice at Rs 9.50 per Kg be mixed with rice at Rs 10.50 per Kg so that the mixture be worth Rs 10.20 per Kg?

Solution: CP of 1 kg rice of 1st kind - 950 Paisa

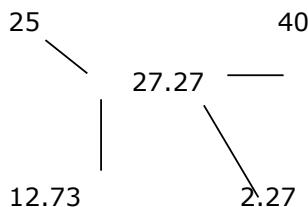
CP of 1 kg rice of 2nd kind - 1050 paisa

Mean price - 1020 paisa



2) How many kg. of salt at 40 P per kg. must a man mix with 20 kg. of salt at 25 P per kg. so that he may , on selling the mixture at 30 P per kg, gain 10% on the outlay?

Solution: Cost price of mixture = $30*100/110 = 300/11 = 27.27$



Ratio = $12.73/2.27$

Thus for every 2.27 kg of salt at 25p, 12.73 kg of salt at 40p is used.

Thus the required no. of kg = $20*12.73/2.27 = 112.15\text{kg.}$

3) A container contained 70 kg of milk. From this container 6 kg of milk was taken out and replaced by water . This process was further repeated two times. How much milk is now contained by the container?

Solution: Amount of liquid left after n operations, when the container originally contains x units of liquid from which y units is taken out each time

$$x(x-y/x)^n \text{ units}$$

Thus, in the above case amount of milk left

$$= 70(70-6/70)\text{kg} = 64 \text{ kg.}$$

4) A vessel is filled with liquid, 2 parts of which are water and 4 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

Solution: Suppose the vessel initially contains 6 litres of liquid

Let x litres of this liquid be replaced with water

Quantity of water in new mixture = $(2 - 2x/6 + x)$ litres

Quantity of syrup in new mixture = $(4 - 4x/6)$ litres

$$(2 - 2x/6 + x) = (4 - 4x/6)$$

$$x = 4.24$$

$$\text{So part of mixture replaced} = 4.24 * 1/6 = 424/600 = 53/75.$$

5) A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 30%. The percentage of water in the mixture is:

Solution:

Let C.P. of 1 litre milk be Rs. 1

Then S.P. of 1 litre of mixture = Rs. 1, Gain = 30%

C.P. of 1 litre mixture = Rs. $(100/130 * 1) = 10/13$

By the rule of alligation we have

CP of 1 litre of milk

Rs. 1

CP of 1 litre of water

0

Mean price = $10/13$ _____

$10/13$

$3/13$

Ratio of milk to water = $10/13 : 3/13 = 10:3$

Hence percentage of water in the mixture = $3/13 * 100 = 300/13 = 23.07\%$

Some Shortcuts:

Quantity of ingredient to be added to increase the content of ingredient in the mixture to $y\%$

If there is $x\%$ ingredient in P litres of a mixture, find the quantity of ingredient to be added to the mixture to make the content $y\%$

Let the quantity of ingredient to be added = Q liters

Quantity of ingredient in the given mixture = $x\% \text{ of } P = x/100 * P$

Percentage of ingredient in the final mixture = Quantity of ingredient in final mixture / Total quantity of final mixture.

$$\text{Quantity of ingredient in final mixture} = [x/100 * P] + Q = [P*x + 100 * Q] / 100$$

$$\text{Total quantity of final mixture} = P + Q$$

$$\rightarrow y/100 = [[P*x + 100 * Q] / 100] / [P + Q]$$

$$\rightarrow y[P + Q] = [P*x + 100 * Q]$$

***If n different vessels of equal size are filled with the mixture of P and Q**

If n different vessels of equal size are filled with the mixture of P and Q in the ratio $p_1 : q_1, p_2 : q_2, \dots, p_n : q_n$ and content of all these vessels are mixed in one large vessel, then

Let x liters be the volume of each vessel,

$$\text{Quantity of P in vessel 1} = p_1 * x / (p_1 + q_1)$$

$$\text{Quantity of P in vessel 2} = p_2 * x / (p_2 + q_2)$$

$$\text{Quantity of P in vessel } n = p_n * x / (p_n + q_n) \dots \text{and so on}$$

Similarly,

$$\text{Quantity of Q in vessel 1} = q_1 * x / (p_1 + q_1)$$

$$\text{Quantity of Q in vessel 2} = q_2 * x / (p_2 + q_2)$$

$$\text{Quantity of Q in vessel } n = q_n * x / (p_n + q_n) \dots \text{and so on}$$

Therefore, when content of all these vessels are mixed in one large vessel, then

$$\text{Quantity of P / Quantity of Q} = \text{Sum of quantities of P in different vessels} / \text{Sum of quantities of Q in different vessels.}$$

Practice Questions Set I

1. In what ratio trader mix two varieties of ground nut costing Rs.23 and Rs.28 per Kg to get a mix worth Rs.25.
 A) 3:2
 B) 4:9
 C) 11:4
 D) 4:5
 E) None of these
2. A merchant has 2500 kg of rice, part of which he sells at 11% profit and the rest at 16% Profit .He gains 14% overall. The quantity sold at 16% profit is
 A) 1450
 B) 1500
 C) 1510
 D) 1600
 E) None of these
3. A mixture of certain quantity of milk with 20 liters of water is worth Rs.10 per liter. If pure milk is of worth Rs. 15 per liter, how much milk is there in the mixture.
 A) 60
 B) 70
 C) 80
 D) 90
4. A sum of Rs.10000 is lent out in two parts, one at 13% simple interest and the other at 18% simple interest. If the annual interest is Rs. 1500, the sum lent at 12% is
 A) 5000
 B) 6000
 C) 7000
 D) 8000
5. A vessel contains 77 liters of a mixture of milk and water. The ratio of milk to water is 3:4. If 21 liters of mixture is taken out from that vessel and then 12 liters of water added to it, what will be the percentage of milk in the final mixture?
 A) 40%
 B) 41%
 C) 43%
 D) 46%
6. How many Kg of sugar costing Rs.35/kg must be mixed with 40 Kg of sugar costing Rs.42 per Kg, so that there may be gain of 10% by selling the mixture of Rs.44 per Kg?
 A) 16 kg
 B) 17 Kg
 C) 18 Kg
 D) 19 Kg
7. A milkman mixes 6 liters of free tap water with 18litres of pure milk. If the cost of pure milk is Rs.28 per liter the % Profit of the milkman when he sells all the mixture at the cost price is
 A) 25 %
 B) 16.5 %
 C) 33 (1/3) %
 D) 16 (1/3) %
8. In what ratio must a grocer mix two varieties of tea worth Rs.40 a kg and Rs.52 a kg so that by selling the mixture at Rs.55 a kg he may gain 10%?
 A) 1:4
 B) 1:5
 C) 5:1
 D) None of these
9. A certain quantity of water is mixed with milk priced at Rs.12 per liter. The price of mixture is Rs.8 per liter. Find out the ratio of water and milk in the new mixture.
 A) 1:8
 B) 4:5
 C) 10:1
 D) 6:7
10. In a Zoo, there are lions and peacocks. If heads are counted there are 400. If legs are counted there are 850. Find the number of lions in the forest.
 A) 30
 B) 40
 C) 25

D) 35

Practice Questions Set II

1. Oil worth Rs.56 per liter and another worth Rs.60 per liter are mixed with a third variety in the ratio of 1:1:2. If the mixture is worth Rs.72 per liter, the price of the third variety per liter will be

- A) 77
- B) 80
- C) 88
- D) 86

2. In a mixture of alcohol and water, there is only 26% of water. After replacing the mixture with 7 liters of alcohol, the percentage of alcohol in the mixture becomes 76%. The quantity of mixture is:

- A) 65 liters
- B) 91 liters
- C) 38 liters
- D) None of these

3. Two vessels A and B contain spirit and water in the ratio 2:5 and 6:7 respectively. Find the ratio in which these mixtures be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 5:8?

- A) 3:4
- B) 4:3
- C) 9:7
- D) 7:9

4. 8 liters are drawn from a cask full of mango pulp and is then filled with water. This operation is performed three more times. The ratio of quantity of mango pulp now left in the cask to that of water is 16:65. How much mango pulp did the cask hold initially.

- A) 30 liters
- B) 26 liters
- C) 24 liters
- D) 32 liters

5. Praveen bought 20 kg of sugar at 10 per kg and 35 kg at 12 per kg. He mixed the two varieties. Approximately at what price per kg

should she sell the mixture to make 30% profit at the cost price?

- A) 15
- B) 16
- C) 19
- D) 17

6. A merchant mixes three variants of rice costing Rs 30/kg, Rs 34/kg and Rs. 40/kg. How many kg of second variant will be in the mixture if 10 kg's of the third variant is there in the mixture

- A) 21 Kg
- B) 25 Kg
- C) 23 Kg
- D) 26 Kg

7. The total number of people in the village is 6000. If the male and female increases by 10% and 15% respectively, the population of the village becomes 6800. What is the number of female in the village initially?

- A) 3000
- B) 6000
- C) 4000
- D) 8000

8. The ratio in which tea costing Rs205/kg is to be mixed with tea costing Rs150/kg so that the mixed tea when sold for Rs.240/kg gives a profit of 20%.

- A) 10:1
- B) 5:6
- C) 7:3
- D) 4:9

9. Milk and water in the two vessels P and Q are in the ratio 7:3 and 2:3 respectively. In what ratio the liquids in both the vessels are mixed to obtain a new mixture in vessel C containing half milk and half water?

- A) 4:3
- B) 3:1
- C) 7:2
- D) 5:3

10. A 20 liter mixture contains phenyl and water in the respective ratio of 2:3. Then 10 liters of mixture is removed and replaced with pure phenyl and the operation is repeated once. At the end of the two removals and replacements, what is the ratio of phenyl and water in the resultant mixture respectively?

- A. 17:3
- B. 9:1
- C. 12:3
- D. 19:3

MENSURATION

Mensuration is a branch of mathematics which deals with the measurements of lengths of lines, areas of surfaces and volumes of solids.

Mensuration may be divided into two parts.

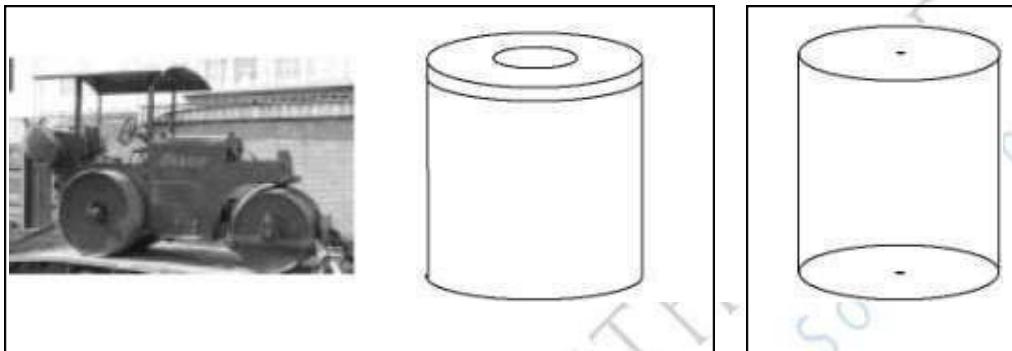
1. Plane mensuration
2. Solid mensuration

Plane mensuration deals with perimeter, length of sides and areas of two dimensional figures and shapes.

Solid mensuration deals with areas and volumes of solid objects.

CYLINDER

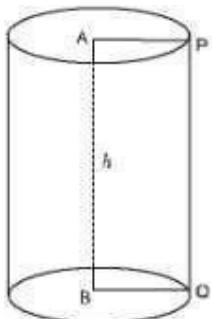
Observe the following figures



Wheels of a road roller, a circular based storage tank etc will suggest you, the concept of a right circular cylinder.



The right circular cylinder



A right circular cylinder is a solid described by revolution of a rectangle about one of its sides which remains fixed.

AP = Radius of the circular plane

AB = Axis of the cylinder

PQ = Height of the cylinder

Features of a right circular cylinder

- 1) A right circular cylinder has two plane surfaces, circular in shape.
- 2) The curved surface joining the plane surfaces is the lateral surface of the cylinder.
- 3) The two circular planes are parallel to each other and also congruent.
- 4) The line joining the centres of the circular planes is the axis of the cylinder.
- 5) All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
- 6) Radius of circular plane is the radius of the cylinder.

Two types of cylinders:

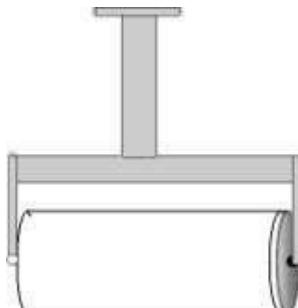
1. Hollow cylinder and
2. Solid cylinder

A hollow cylinder is formed by the lateral surface only.



Example: A pipe

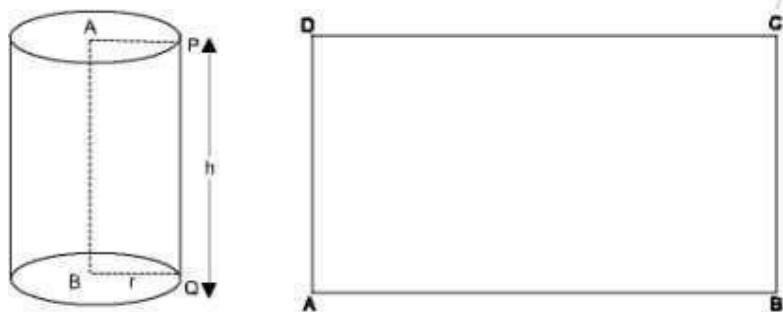
A solid cylinder is the region bounded by two circular plane surfaces and also the lateral surface.



Example: A garden roller

2. Surface of a circular cylinder

A. Lateral Surface area:



Activity:

1. Take a strip of paper having width equal to the height of the cylinder.
2. Wrap the strip around the lateral surface of the cylinder and cut the overlapping strip along the vertical line. (say PQ)
3. You will get a rectangular paper cutting which exactly covers the lateral surface.
4. Area of the rectangle is equal to the area of the curved surface of the cylinder.

Expression for the lateral surface area:

- (i) Length of the rectangle is equal to the circumference, $l = 2 \pi r$
- (ii) Breadth of the rectangle b is the height of the cylinder = h Area of the rectangle $A = l \times b$

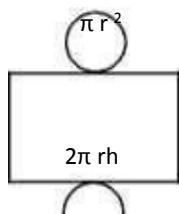
$$\text{Lateral surface area of the Cylinder } A = 2 \pi r h$$

$$A = 2 \pi r h \text{ sq. units}$$

Observe:

Surface area of a thin hollow cylinder having circumference P and height is 'h' = Ph or $= 2 \pi r h$ ($P = 2 \pi r$)

B. Total surface area of a cylinder :



$$\pi r^2$$

The total surface area of the cylinder

$$\begin{aligned}
 & \text{Area of two circular bases} + \text{Lateral surface area of cylinder} \\
 &= \pi r^2 + \pi r^2 + 2\pi rh \\
 &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi r(r + h) \text{ sq. units}
 \end{aligned}$$

Lateral surface area of a cylinder = $2\pi rh$ sq. units. Total surface area of a cylinder = $2\pi r(r+h)$ sq. units.

Remember: Area is always expressed in square unit



Practice Questions Set I

1. A rectangle carpet has an area of 125 square metres and a perimeter of 45 m. The length of its diagonal is?
- 10m
 - 15m
 - 20m
 - 25m
2. The whole surface area of a cuboid 25 cm long, 18 cm broad and 7 cm height, is?
- 1502 cm^2
 - 1480 cm^2
 - 1500 cm^2
 - 1450 cm^2
3. Find the cost of carpeting a room 10m long and 8m broad with a carpet 70cm broad at the rate of Rs.25 per meter?
- Rs. 2645
 - Rs. 3120
 - Rs. 2857
 - Rs. 3257
4. The length of a rectangle is thrice its breadth. If its length is decreased by 6 cm and breadth is increased by 6 cm, the area of the rectangle is increased by 70 sq. cm. Find the length of the rectangle?
- 25.39 cm
 - 28.34 cm
 - 29.12 cm
 - 26.49 cm
5. A man walking at the rate of 8km per hour crosses a square field diagonally in 10 seconds, the area of the field is?
- 252.64 sq.metres
 - 287.12 sq.metres
 - 246.865 sq.metres
 - 222.22 sq.metres
6. A lawn is in the form of a rectangle having its sides in the ratio 4:5. The area of the lawn is $(1/5)$ hectares. Find the length and breadth of the lawn?
- 40m, 50m
 - 50m, 40m
- C. 60m, 70m
D. 70m, 60m
7. A rectangular grassy plot 120 m. by 80 m has a gravel path 3.5 m wide all round it on the inside. Find the cost of gravelling the path at 100 paise per sq. metre?
- Rs.1450
 - Rs. 1325
 - Rs. 1456
 - Rs. 1351
8. A room 10 m 50 cm long and 6m 70 cm broad is to be paved with square tiles. Find the least number of square tiles required to cover the floor?
- 7000
 - 7035
 - 7234
 - 7156
9. If the length of a certain rectangle is decreased by 5 cm and the width is increased by 4 cm, a square with the same area as the original rectangle would result. Find the perimeter of the original rectangle?
- 84 cm
 - 90 cm
 - 86 cm
 - 92 cm
10. Length and Breadth of a rectangle is 8 m and 4 m respectively. Find the area of circle of maximum radius?
- 12.57 sq. meters
 - 13.23 sq. meters
 - 14.12 sq. meters
 - 15.67 sq. meters

Practice Questions Set II

1. A person wants to make a cylindrical box which is open from the top. If the height of that box is 15 cm and radius is 6 cm find the area of sheet which is required to make it?

- A. 634.45 cm^2
- B. 678.86 cm^2
- C. 628.92 cm^2
- D. None of these

2. The perimeter of a square is equal to the perimeter of a rectangle of length 15 cm and breadth 22 cm. Find the circumference of a semicircle (approx.) whose diameter is equal to the side of the square?

- A. 23.45cm
- B. 25.67 cm
- C. 29.07 cm
- D. 28.79 cm

3. There are two circles of different radius such that radius of the smaller circle is two – seventh that of the larger circle. A square whose area equals 4000 sq cm has its side as twice the radius of the larger circle. What is the circumference of the smaller circle?

- A. 56.76 cm
- B. 54.56 cm
- C. 58.24 cm
- D. 53.45 cm

4. A Birthday cap is in the form of a right circular cone which has base of radius as 10 cm and height equal to 14 cm. Find the approximate area of the sheet required to make 6 such caps?

- A. 3340.54 cm^2
- B. 3243.43 cm^2
- C. 3456.34 cm^2
- D. None of these.

5. The barrel of a fountain pen is cylindrical in shape which radius of base as 0.6 cm and is 7 cm long. One such barrel in the pen can be used to write 250 words. A barrel full of ink which has

a capacity of 15 cu cm can be used to write how many words approximately?

- A. 460 words
- B. 450 words
- C. 473 words
- D. 480 words.

5. A car has wheels of diameter 80 m. How many revolutions can the wheel complete in 10 minutes if the car is travelling at a speed of 100 m/s?

- A. 235
- B. 236
- C. 239
- D. 240

6. The diameters of the internal and external surfaces of a hollow spherical shell are 5cm and 12 cm respectively. If it is melted and recasted into a solid cylinder of length 3 cm, find the diameter of the cylinder?

- A. 52.34 cm
- B. 53.38 cm
- C. 54.34 cm
- D. 55.23 cm

7. The radii of two cylinders are in the ratio 3:4 and their curved surface areas are in the ratio 4 : 5. What is the ratio of their volumes?

- A. 4:7
- B. 5:7
- C. 3:5
- D. 9:10

8. If a copper wire is bend to make a square whose area is 361 cm^2 . If the same wire is bent to form a semicircle, then find the radius of semicircle?

- A. 13.45 cm
- B. 14.78 cm
- C. 15.23 cm
- D. 16.12 cm

9. A man wants to make small sphere of size 2 cm of radius from a large sphere of size of 8 cm of radius. Find out how many such sphere can be made?

- A. 70
- B. 68

- C. 65
- D. 64

10. A conical cup is filled with ice cream. The ice cream forms a hemispherical shape on its top. The height of the hemispherical part is 8 cm. The radius of the hemispherical part equals the height of cone then the volume of ice cream is?

- A. 1734.34 cm³
- B. 1645.47 cm³
- C. 1678.73 cm³
- D. 1609.14 cm³

Logarithms

Logarithm: It is the power to which number must be raised in order to get some other number.

Common logarithm: Logarithms with base 10 are common logarithms. Common logarithm are written as $\log_{10}x$ and if any expression is not indicated with a base, then base 10 is considered.

Natural Logarithm: Logarithms with base e are natural logarithms. Natural logarithms are written as $\log_e x$ and denoted as ln x.

Logarithm Concepts & Rules

The logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number.

Example:

If $a^x=N$ then $x=\log_a N$

This is read as "log N to the base a".

In the equation, 'N' is a positive number and 'a' is a positive number other than 1.

Exponential Laws	Logarithm Laws
$x^a \cdot x^b = x^{a+b}$	$\log(ab) = \log(a) + \log(b)$
$\frac{x^a}{x^b} = x^{a-b}$	$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
$(x^a)^b = x^{ab}$	$\log(a^b) = b \cdot \log(a)$
$x^{-a} = \frac{1}{x^a}$	$\log_x\left(\frac{1}{x^a}\right) = -a$
$x^0 = 1$	$\log_x 1 = 0$

Logarithm of a number contains 2 parts:

1. Characteristic: Integral part of logarithm.
2. Mantissa: It is the decimal part of logarithm. Log table is used to find the Mantissa.

1.1. Case 1: If number is greater than 1.

In this condition, characteristic is considered as one less than the number of digits in the left of decimal point in the given number.

Example: 359.36

Number of digits to the left of decimal point are 3. Hence, the characteristic is one less than number of digits before decimal points i.e 2.

1.2. Case 2: If number is less than 1.

In this condition, characteristic is considered as one more than the number of zeros between decimal point and first digit of the number. It is negative and is denoted as
 (One bar) 1 or (Two bar) 2

Example: 0.000598

Number of zeros between decimal point and first significant digit 5 are 3. Hence the characteristic is one more than number of zeros i.e 4.

Quick Tips and Tricks

Very Important: If base is not mentioned, then always remember to take it as base 10.

Properties of Logarithms

1. $y = \log_b x$ iff $b^y = x$
2. $\log_b b = 1$
3. $\log_b b^p = p$
4. $\log_b 1 = 0$
5. $b^{\log_b p} = p$
6. $\log_b m^p = p \log_b m$
7. $\log_b mn = \log_b m + \log_b n$
8. $\log_b \left[\frac{m}{n}\right] = \log_b m - \log_b n$
9. $\log a = \log_{10} a$
10. $\ln a = \log_e a$

Note: The base of logarithm can never be equal to 1, i.e $\log_1 x$ is undefined.

Important points to remember

1. Logarithms are opposite to exponentials which means **logs are inverses of exponentials**.
2. If a logarithm is given without mentioning the base, it is considered as a common logarithm. For Example, $49=7^2$ can be expressed as $\log_7 49 = 2$.
3. Given logarithmic form can be converted into exponential form as shown below:

b = base y = exponent x = answer

n=power (result obtained by raising b to the power of a)

$$\log_b n = a \text{ and } b^a = n$$

↓
 log_b
 ↑
 b=base ↑
 a=exponent

Exponential form: $b^y = x$

Logarithm form: $\log_b (x) = y$

Log of numbers (2 – 10) (Must Remember)

Log 2 = 0.301
 Log 3 = 0.477 = 0.48
 Log 4 = 0.60
 Log 5 = 0.698 = 0.7
 Log 6 = 0.778 = 0.78
 Log 7 = 0.845 = 0.85
 Log 8 = 0.90
 Log 9 = 0.954 = 0.96
 Log 10 = 1

Finding log without using calculator

These are very easy methods to find the log of numbers without calculator.

Step 1: You should know all the prime factors of given numbers.

Step 2: Method 2: Log of 4 to 9 can be easily determined if only the **value of log 2 and log 3 is remembered.**

Method 1: Log of composite number (x) = Sum of logarithms of its prime factors.

Example: Log of 4 =?

If you know the value of log 2, the value of log 4 can be easily determined.

Log 2 = 0.301

Log of 4 = **Sum of logarithms of its prime factors.**

Log of 4 = log 2 + log 2

= 0.301 + 0.301

= 0.602

Log of 9 = 0.602

Method 2: By remembering, Log 2 = 0.301

Log 3 = 0.477

Log 4 = $2 \times \log 2 = 0.60206$

Log 5 = $1 - \log 2 = 0.6989$

Log 6 = $\log 2 + \log 3 = 0.778$

Log 7 = 0.84510 --- (Remember)

PROBLEMS:

1. Solve the Equation
 $\log(18x-6) = \log(6x+30)$

- a) 2
- b) 3
- c) 1
- d) 0

2. solve the equation
 $\log(10x-5)=\log(5x+25)$

- a) $x=2$
- b) $x=2.5$
- c) $x=1$
- d) $x=1.5$

3. Solve the equation
 $\log_{10}(8x+15)=\log_{10}(-2x-25)$

- a) $x=3$
- b) $x=4$
- c) $x=-4$
- d) $x=-2$

4. solve $\log_2 x + \log_2 c(x-6)=4$

- a) $x=7$
- b) $x=3$
- c) $x=4$
- d) $x=8$

5. . Simplify

$\log 70/15 - 2\log 4/9 + \log 30/81?$

- a) $\log(4/35)$
- b) $\log(17/19)$
- c) $\log(35/4)$
- d) $\log(19/17)$

6. Simplify and find the value of x,

$\log_{10}2 + \log_{10}(2x+1) = \log_{10}(x+1)+1?$

- a) $2/3$
- b) $3/2$
- c) $3/4$
- d) $4/3$

7. What is $\log_8(1/512)$

- a) 3
- b) -3
- c) 2
- d) -2

8. solve the equation For x,
 $\log(20x-10)=\log(8x+20)$

- a) 2.5
- b) 2.25
- c) 2
- d) 1.5

9. solve for X, $\log_2 x + \log_2(x-14)=5$

- a) 10
- b) 12
- c) 16
- d) 20

10. Simplify the equation

$1/\log_{ab}(abc) + 1/\log_{bc}(abc) + 1/\log_{ca}(abc)$

- a) 2
- b) 3
- c) 1
- d) 0

11. Solve for x,

$\log_{10}(4) + \log_{10}(3x+1) = \log_{10}(x+2)+1$

- a) 10
- b) 5
- c) 8
- d) 4

12. Find the value of x?

$\log_{\sqrt{6}}x = 18/3$

- a) 256
- b) 125
- c) 36
- d) 216

13. Find the value of Y,

if $\log_xy=50$ and $\log_3x=20$.

- a) 3^{100}
- b) 3^{1000}
- c) 3^{10}
- d) 3^{10000}

14. $\log_{10}8=x$, then $\log_{10}(1/80)$ is equal to?

- a) $(1+x)$
- b) $-(1+x)$
- c) $(2+x)$
- d) $-(2+x)$

15. If $\log_3=0.4771$, find the number of digits in 3^{42}

- a) 19
- b) 20
- c) 21
- d) 22

16. $\log_{10}4=0.602$ and $\log_{10}2=0.301$, find
The value of $\log_{10}8$?

- a) 0.789
- b) 0.269
- c) 0.903
- d) 0.957

17. $\log_{10}4=0.602$, find the value of
 $\log_{10}25$

- a) 1.938
- b) 1.398
- c) 1.458
- d) 0.2458

18. $\log_{16}(4\log_2(1+\log_4(1+2\log_{2x}))=1/2$.
Find x?

- a) $3^{3/2}$
- b) $2^{2/3}$
- c) $2^{3/2}$
- d) None of the above

19. Solve for x,
 $\log(x+4)+\log(x-4)=\log 64$

- a) $\sqrt{48}$
- b) $\sqrt{32}$
- c) $\sqrt{39}$
- d) $\sqrt{28}$

20. $\log_7 m + \log_7(1/5)=4/5$, find m?

- a) $5*7^{4/9}$
- b) $4*7^{4/9}$
- c) $5*7^{2/9}$
- d) $5*7^{4/5}$

PIPES AND CISTERNS

Cistern:

Cistern is a container for holding liquids.

Pipe:

Pipe is used for fluid motion control.

In pipes and cisterns we will come across the following terminologies:

Inlet: A pipe connected with a tank or reservoir for filling is called as inlet. A pipe can fill a tank with water, at this time the nature of the pipe is positive.

Outlet: A pipe connected with a tank and used for emptying it is called outlet. A pipe can empty a tank, at this time the nature of the pipe is negative.

Rule: If a pipe can fill a tank in x hours, then the part filled in 1 hour = $1 / x$.

Rule: If a pipe can fill/empty a tank in x hours and another pipe can fill/empty the full tank in y hours, then the net part filled/emptied in 1 hour, when both the pipes are opened:

$$(1/x) - (1/y) \text{ - If X fills and Y empties}$$

$$(1/x) + (1/y) \text{ - If both fills}$$

the pipes are opened:

$$xy / (y - x) \text{ - If X fills and Y empties}$$

$$xy / (y + x) \text{ - If both fills}$$

The time taken to fill (or empty) the cistern in x, y and z hours while working alone. If all the three pipes are opened together, the net part filled/emptied in 1 hr:

$$(1/x) + (1/y) + (1/z) \text{ - If all three fill}$$

$$(1/x) - (1/y) - (1/z) \text{ - If X fills and Y, Z both empty}$$

So the time taken to fill (or empty) the cistern is given by:

$$xyz / (xz + yz + xy) \text{ - If all three fill}$$

$$xyz / (xz - yz - xy) \text{ - If X fills and Y, Z both empty}$$

Rule: If a pipe fills a tank in x hrs and another fills the same tank in y hrs, but a third empties the full tank in z hrs and all of them are opened together, the net part filled in 1 hr:

$$(1/x) + (1/y) - (1/z)$$

So time taken to fill the tank:

$$xyz / (yz + xz - xy)$$

Rule: 'A' pipe can fill a tank in ' x ' hours & 'B' pipe can fill in ' y ' hours. Both opened for ' a ' hours, then, A is shut-off. Alone 'B' pipe fill the remaining tank in:

$$y - [a * (x + y)] / x \text{ hours}$$

Pipes and Cisterns problems are similar to time and work problems.

Examples

1. A tank is filled by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. If all three pipes are open, the tank is filled in 5 hours. How much time will pipe A alone take to fill the tank?

Solution:

Let Pipe A take x hours to fill the tank, then pipes B will take $x / 2$ hrs and pipe C will take $x / 4$ hours respectively.

Then part to be filled by all three pipes in 1 hr will be:

$$(1/x) + (2/x) + (4/x) = 1/5$$

$$x = 25 \text{ hrs}$$

2. A cistern is normally filled in 8 hrs, but it takes four hrs longer to fill because of a leak in the bottom. If the cistern is full, how much time the leak will empty it?

Solution:

Let the leak empty the tank in x hrs.

Then part of cistern filled in 1 hr:

$$= (1/8) - (1/x) = (x - 8) / 8x$$

So cistern will completely filled in $8x / (x - 8)$

As given in the question, 8 hrs is the time taken by the cistern to be filled and 4 hrs is the additional time needed because of the leakage

$$8x / (x - 8) = 8 + 4 = 12$$

$$x = 24 \text{ hrs}$$

3. A tank can be filled by a tap in 20 minutes and by another tap in 60 minutes. Both the taps are kept open for 10 minutes and then the first tap is shut off. After this, how much time required to fill the tank?

Solution:

The part of the tank filled when both the taps are open per minute is:

$$= (1/20) + (1/60) = (1/15)$$

The part of the tank filled when both the taps are open per 10 minutes is:

$$= 10 \times (1/15) = 2/3$$

Now as the first tap is turned off, remaining $1/3$ rd of the tank should be filled by second tap:

$$= (1/3) \times 60 = 20 \text{ minutes.}$$

Problems

1. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank? This is the same as Example 4 above
 - A. 10 min. 20 sec.
 - B. 11 min. 45 sec.
 - C. 12 min. 30 sec.
 - D. 14 min. 40 sec.

2. Pipe A fills a tank of 700 litres capacity at the rate of 40 litres a minute. Another pipe B fills the same tank at the rate of 30 litres a minute. A pipe at the bottom of the tank drains the tank at the rate of 20 litres a minute. If pipe A is kept open for a minute and then closed and pipe B is kept open for a minute and then closed and then pipe C is kept open for a minute and then closed and the cycle repeated, how long will it take for the empty tank to overflow?
 - A. 42 minutes 20 seconds
 - B. 14 minutes 18 seconds
 - C. 39 minutes
 - D. 40 minutes 20 seconds

3. 3 taps A and B can fill a bucket in 12 minutes and 15 minutes respectively. If both are opened and A is closed after 3 minutes, how much further time would it take for B to fill the bucket?
 - A. 7 min. 45 sec.
 - B. 8 mm. 5 sec.
 - C. 7 mm. 15 sec.
 - D. 8 mm. 15 sec.

4. A tank is fitted with 8 pipes, some of them that fill the tank and others that are waste pipes meant to empty the tank. Each of the pipes that fill the tank can fill it in 8 hours, while each of those that empty the tank can empty it in 6 hours. If all the pipes are kept open when the tank is full, it will take exactly 6 hours for the tank to empty. How many of these are fill pipes?
 - A. 2
 - B. 4
 - C. 6

5. Pipe A usually fills a tank in 2 hours. On account of a leak at the bottom of the tank, it takes pipe A 30 more minutes to fill the tank. How long will the leak take to empty a full tank if pipe A is shut?
 - A. 2 hours 30 minutes
 - B. 5 hours
 - C. 4 hours
 - D. 10 hours

6. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
 - A. 6 min to empty
 - B. 7 min to full
 - C. 6 min to full
 - D. 7 min to empty

7. A tank can be filled by a tap in 20 minutes and by another tap in 60 minutes. Both the taps are kept open for 10 minutes and then the first tap is shut off. After this, the tank will be completely filled in what time?
 - A. 10 mins
 - B. 15 mins
 - C. 20 mins
 - D. 25 mins

8. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:
 - A. 30 hours
 - B. 15 hours
 - C. 10 hours
 - D. 6 hours

9. Two pipes A and B can fill a tank in 15 minutes and 40 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank?

- A. 10 min 10 sec
 B. 25 min 20 sec
 C. 14 min 40 sec
 D. 20 min 10 sec
10. Two pipes A and B can fill a tank in 8 hours. If only pipe A is open then it would take 4 hours longer to fill the tank. Find how much longer it would take if only pipe B is open.
 A. 6 hours
 B. 8 hours
 C. 16 hours
 D. 12 hours
11. There are two inlets and one outlet to a cistern. One of the inlets takes 3 hours to fill up the cistern and the other inlet takes twice as much time to fill up the same cistern. Both of the inlets are turned on at 9:00 AM with the cistern completely empty, and at 10:30AM, the outlet is turned on and it takes 1 more hour to fill the cistern completely. How much time does the outlet working alone take to empty the cistern when the cistern is full?
 A. 2.5 hours
 B. 3 hours
 C. 3.5 hours
 D. 4 hours
12. Pipe P can drain the liquid from a tank in $\frac{3}{4}$ th the time that it takes pipe Q to drain it and in $\frac{2}{3}$ rd the time that it takes pipe R to do it. If all 3 pipes operating simultaneously but independently are used to drain liquid from the tank, then pipe Q drains what portion of the liquid from the tank?
 A. $\frac{9}{29}$
 B. $\frac{8}{23}$
 C. $\frac{3}{8}$
 D. $\frac{17}{29}$
13. A tank is filled in 10 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?
 A. 70 hours
 B. 30 hours
 C. 35 hours
 D. 50 hours
14. **Pipe A, B and C are kept open and together fill a tank in t minutes. Pipe A is kept open throughout, pipe B is kept open for the first 10 minutes and then closed. Two minutes after pipe B is closed, pipe C is opened and is kept open till the tank is full. Each pipe fills an equal share of the tank. Furthermore, it is known that if pipe A and B are kept open continuously, the tank would be filled completely in t minutes. Find t?**
 A. 18
 B. 36
 C. 27
 D. 24
15. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is
 A. 10
 B. 12
 C. 14
 D. 16
16. A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 liters a minute. When the tank is full, the inlet is opened and due to the leak, the tank is empty in 24 hours. How many liters does the cistern hold?
 A. 4010 litre
 B. 2220 litre
 C. 1920 litre
 D. 2020 litre
17. Srivari Mansion is a big housing complex in Tirupathi. Giant tanks are placed in every complex building to cater to the needs of the residents. In a block named Annapurna, three taps P, Q and R can fill a tank in 12, 15 and 30 hours respectively. The caretaker of the complex has instructions to keep tap P open all the time and Q and R are to be opened for one hour alternately. When will the tank in Annapurna become full?
 A. 5 hrs
 B. 6 hrs
 C. 7 hrs
 D. 8 hrs

18. Sri Poojitha Enclave has 20 flats. Water from three different sources are fed into the cistern. Three taps can fill the cistern in 10mins, 15mins and 18mins respectively. The cistern being empty, all the three taps are kept open by the watchman of the enclave. After 3 mins, the watchman closes the third tap. After third tap is closed, how many minutes would be required by the first two taps to fill the cistern completely.

- A. 1 min
- B. 2 min
- C. 3 min
- D. 4 min

19. Fountain Square is one of the largest housing projects in Bangalore. It has over 300 flats, a swimming pool, play area, community hall, walking lawns etc. In one of the blocks named Arjun, there are two water tanks A and B. A is much smaller than B. Water fills at the rate of one litre every hour in A. At the end of first hour, tank B gets filled with 10 litres of water. At the end of second hour, the capacity raises to 20 litres. At the end of third hour the capacity raises to 40 litres and so on. If $\frac{1}{16}$ th of the total capacity of tank B is filled after 24 hours, what is the total time required for tank B to get completely filled?

- A. 26 hours
- B. 27 hours
- C. 29 hours
- D. 28 hours

DATA INTERPRETATION

Data Interpretation tests are used to determine a person's ability to extract the correct data from tables, graphs and charts. The data extracted has to be manipulated mathematically to find the correct answer. These types of test questions are often found as a part of numerical aptitude tests used during candidate's applicant screening.

These types of questions appear in four different formats.

- I. Bar Charts
- II. Pie Charts
- III. Table Charts
- IV. Line Charts

These are explained with examples as follows.

Type I - Bar Chart

1. Introduction

Bar charts are one of the easiest, graphically attractive and hence most commonly used methods of presenting all types of data. They are especially useful for representing various data series.

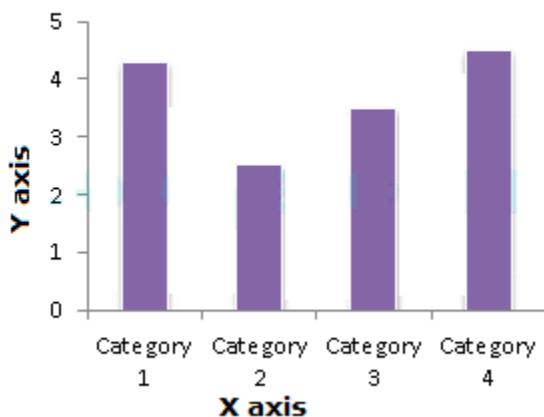
Presentation of data as bar charts makes the comparative study of the data very easy. A bar chart consists of a group of bars which are equidistant from each other. The values on the bar charts are read by the measurement of the length or the height of the bars. The width of the bars is largely inessential and is used only for the clarity of the presentation.

Now let's have a look onto the different kinds of bar charts and the kinds of data that can be represented on a bar-chart.

2. Simple Bar Chart

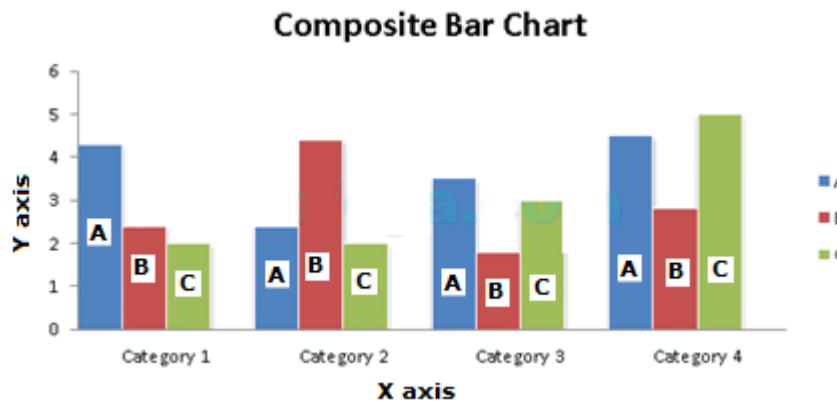
The simple bar chart is the 'simplest' bar chart which has one continuous variable charted along with one discrete variable (here x-axis has continuous values with equidistance spacing which are stated as continuous variables and y-axis consists of different values which are stated as discrete variables). Figure below shows an example of Simple Bar Chart.

Simple Bar Chart



3. Composite Bar Chart

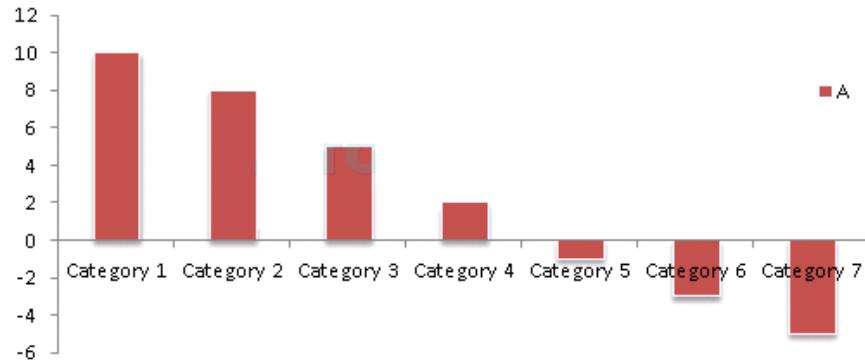
One of the primary limitations of the simple bar chart is that it can only be used to display a single continuous variable. If two or more sets of continuous variables are to be shown on the same bar chart, we use what is called a composite bar-chart. Figure below shows an example of the Composite Bar Chart.



4. The Use of Bar Charts to Show Deviations

Deviation bars are useful for graphic presentation of continuous variables which can have both positive and negative values, i.e., surplus or deficit, net profit or loss, net of imports and exports. In general, continuous variables which have both positive and negative values are best represented on bar charts.

Deviation Through Bar Charts



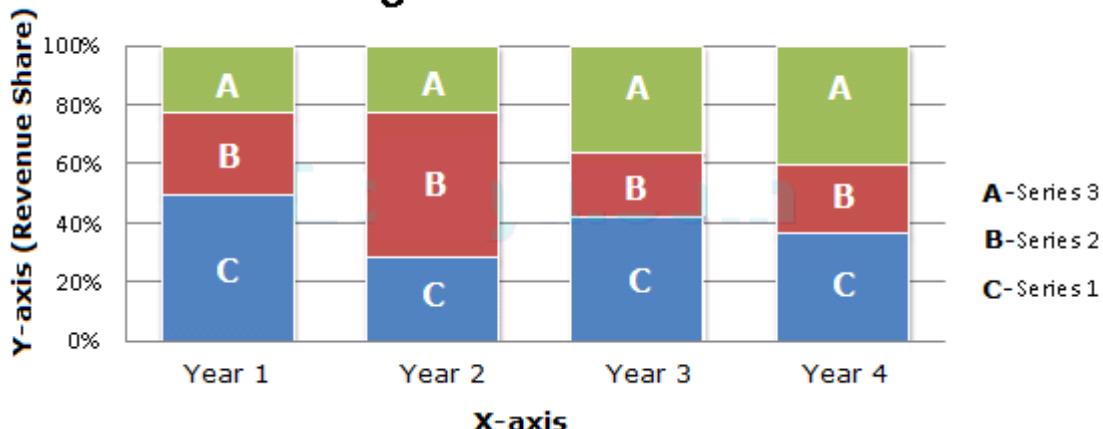
A base line is created and positive values (such as profit, surplus), etc., are represented by bars above the base line while negative deviations (loss or deficit) are represented by bars below the base line as shown in the figure above.

5. Representation of Percentage on a Stacked Bar Chart

Sometimes stacked bars can also be used to represent the break-up of some continuous variable. Figure below will make it clear.

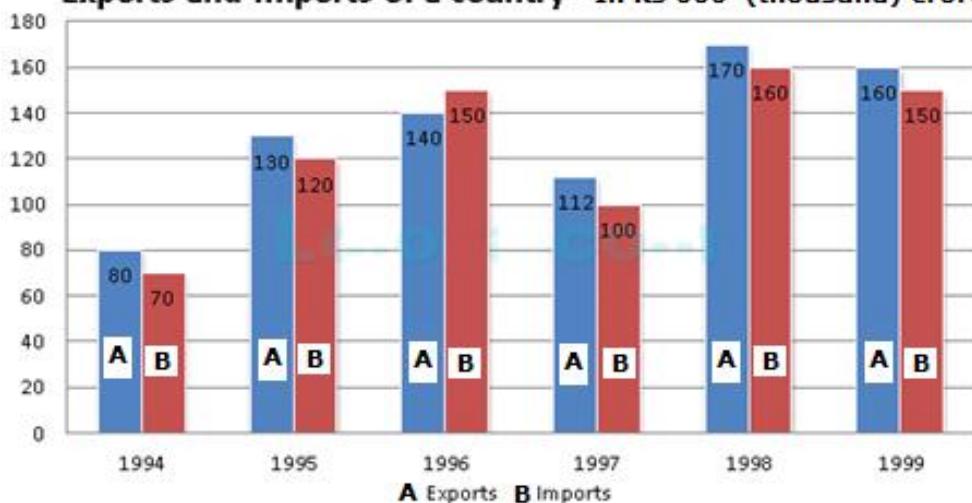
Such a use of bar charts is quite convenient for comparing two or more sets of data. Figure below shows the break-up of the various sources of revenues for the Government of India over a four-year period.

Percentage on a Stacked Bar Chart



Q. Answer the questions based on the chart below (1-5).

Exports and Imports of a country - In Rs 000' (thousand) crores



1. In how many of the given years was the exports at least 10% more than the imports?
 - A. 0
 - B. 1
 - C. 2
 - D. 3
 - E. 4

2. What was the average exports for the given period (in '000 crores)?
 - A. 145
 - B. 132
 - C. 126
 - D. 119
 - E. 138

3. From 1995 to 1999, in which year was the percentage growth in exports, when compared to the previous year, the highest?
 - A. 1995
 - B. 1996
 - C. 1997

- D. 1998
E. 1999

4. What is the simple average annual growth rate in the imports from 1994 to 1999?

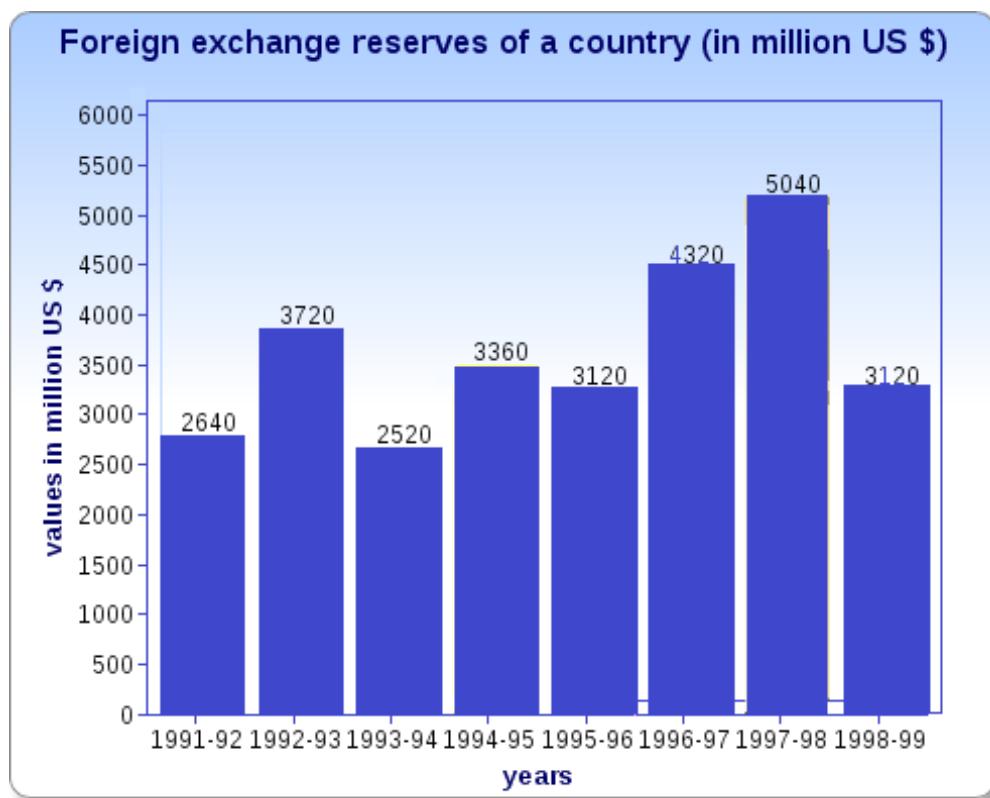
- A. 15
B. 18
C. 19
D. 21
E. 23

5. Among the years in which the imports as well as exports exceed those in the previous years, in how many years was the percentage increase in imports less than the percentage increase in exports?

- A. 0
B. 1
C. 2
D. 3
E. 4

Q. Answer the questions based on the chart below (6-10).

The bar graph given below shows the foreign exchange reserves of a country (in million US \$) from 1991-92 to 1998-99. Answer the questions based on graph.



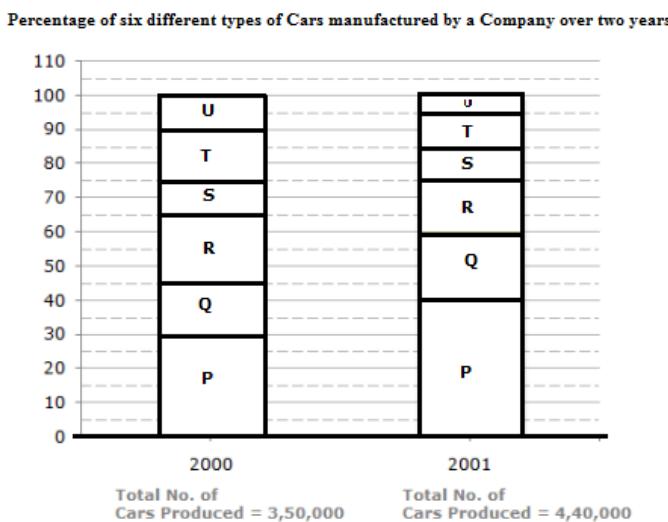
6. The foreign exchange reserves in 1997-98 was how many times that in 1994-95

- A. 1.5
B. 2
C. 3.5
D. 2.6

7. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?
- 80%
 - 90%
 - 100%
 - 110%
8. For which year, the percent increase of foreign exchange reserves over the previous year is the highest?
- 1994-95
 - 1995-96
 - 1998-99
 - 1992-93
9. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?
- 80%
 - 100%
 - 125%
 - 130%
10. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average is:
- 3:5
 - 2:3
 - 4:7
 - 3:7

Q. Answer the questions based on the chart below (11-15).

The bar graph given below shows the percentage distribution of the total production of a car manufacturing company into various models over two years.

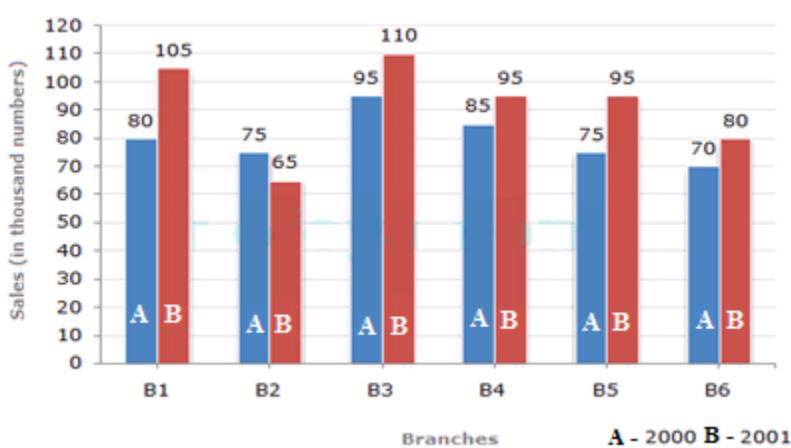


11. What was the difference in the number of Q type cars produced in 2000 and that produced in 2001?
- 35,500
 - 27,000
 - 22,500
 - 17,500

12. Total number of cars of models P, Q and T manufactured in 2000 is?
- 2,45,000
 - 2,27,500
 - 2,10,000
 - 1,92,500
13. If the percentage production of P type cars in 2001 was the same as that in 2000, then the number of P type cars produced in 2001 would have been?
- 1,40,000
 - 1,32,000
 - 1,17,000
 - 1,05,000
14. If 85% of the S type cars produced in each year were sold by the company, how many S type cars remain unsold?
- 7650
 - 9350
 - 11,850
 - 12,250
15. For which model the percentage rise/fall in production from 2000 to 2001 was minimum?
- Q
 - R
 - S
 - T

Questions: Common information for questions numbered 16 to 20

The bar graph given below shows the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001.



16. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
- 2:3
 - 3:5
 - 4:5
 - 7:9

17. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?

- A. 68.54%
- B. 71.11%
- C. 73.17%
- D. 75.55%

18. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?

- A. 75%
- B. 77.5%
- C. 82.5%
- D. 87.5%

19. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is?

- A. 250
- B. 310
- C. 435
- D. 560

20. What is the difference between total average sales of books from all six branches of publishing company in the years 2000 and 2001?

- A. 11.67
- B. 15.87
- C. 19.37
- D. 21.67

Type II - Pie Charts

1. Introduction

Pie charts are specific types of data presentation where the data is represented in the form of a circle. In a pie chart, a circle is divided into various sections or segments such that each sector or segment represents a certain proportion or percentage of the total.

In such a diagram, the total of all the given items is equated to 360 degrees and the degrees of angles, representing different items, are calculated proportionately. The entire diagram looks like a pie and its components resemble slices cut from a pie. The pie chart is used to show the break-up of one continuous variable into its component parts.

Tips and tricks:

#1 - EXAMINE THE PIE CHART FULLY BEFORE READING THE QUESTION:

Read the heading of the chart first, and then look at the labels for each "slice" of the pie. What can be readily inferred? Keep track of the units, and make sure there isn't any additional information printed below the chart that may be required to solve.

#2 - DON'T EQUATE PERCENTS WITH NUMBERS.

The pie chart may display a percentage only, while the question may ask about an actual, "real-world" number. Don't confuse the two.

#3 - USE ESTIMATION AND APPROXIMATION AS MUCH AS POSSIBLE.

Round numbers up or down to make the math easier. Rounding to the nearest whole number is usually the simplest way to solve, then proceed with the actual computations as you would normally.

Basics:

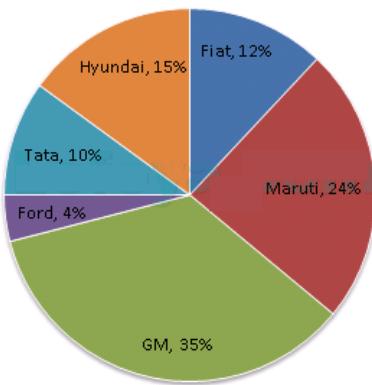
Total angle at the centre of a pie chart = 360°

To convert k% percentage into angle = $(k/100) \times 360^\circ$

To convert m degrees into percentage = $m/360 \times 100$

For example, chart below shows the distribution of the sales of the car industry between six car companies. Looking at the chart below, we can infer that Maruti accounts for 24 per cent of the market share, while GM accounts for 35 percent of the market share, Ford for 4 percent of the market share, Tata for 10 percent of the market share, Hyundai for 15 percent of the market share and Fiat for 12 per cent of the market share.

Distribution of car sales between six companies



The pie chart encompasses a circle of 360 degrees which represents 100 per cent of the value of the continuous variable. Thus, 3.6 degrees on the pie chart represent 1 percent of the total value of the continuous variable being represented.

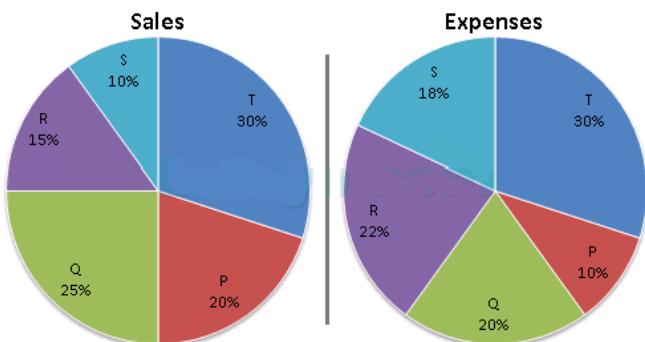
A single pie diagram can represent only one continuous variable. Hence, in terms of versatility of data representation, pie charts are less versatile than either of bar charts, x-y graphs or tables. However, their utility is in the fact that the representation of data is cleaner and it gives an immediate idea of the relative distribution of the continuous variable amongst different sectors.

Example below will make the things easier to understand.

Questions: Common information for questions numbered 21 to 24

Answer the questions on the basis of the information given below.

The following pie charts give the values of the sales and expenses of five companies — P, Q, R, S and T — as a percentage of the total sales and expenses of these five companies put together.



Profit = Sales – Expenses

Profit percentage= $(\text{Profit}/\text{Sales}) \times 100$

21. Which of the companies had the highest profit percentage?

- A. P
- B. Q
- C. R
- D. Cannot be determined

22. For which of the companies is the ratio of expenses to profit, the highest?

- A. P
- B. S
- C. T
- D. Cannot be determined

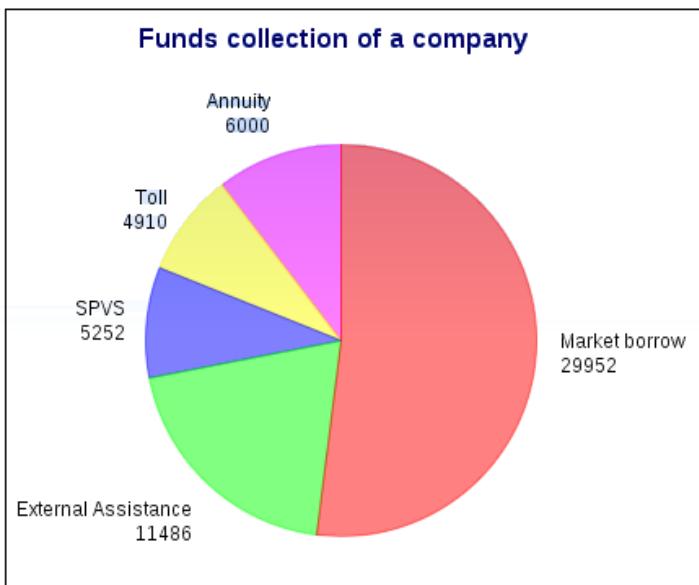
23. If the total sales were Rs.800 crore and the overall profit percentage of all the five companies put together was 50%, what were the expenses of company R?

- A. Rs.60 crore
- B. Rs.68 crore
- C. Rs.80 crore
- D. Rs.88 crore

24. If the overall profit percentage of all the five companies put together was 50%. How many companies had a profit percentage greater than 60%?

- A. 0
- B. 1
- C. 2
- D. 3

Questions (25-29): The following pie-chart shows the sources of funds(in crores) to be collected by a company. Study the pie-chart and answers the question that follow.



25. If company could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowing to arrange for the shortage of funds?

- A. 4%
- B. 6%
- C. 8%
- D. 10%

26. Near about 20% of the funds are to be arranged through?

- A. SPVS
- B. Annuity
- C. External Assistance
- D. Market borrowing

27. The central angle corresponding to Market Borrowing is?

- A. 187.2 degree
- B. 183.2 degree
- C. 181.2 degree
- D. 180.2 degree

28. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4,910 crores?

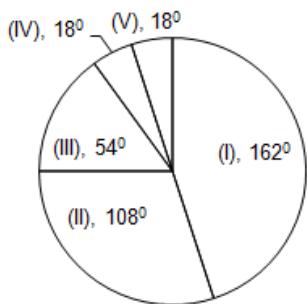
- A. Rs 5401 crore
- B. Rs 5301 crore
- C. Rs 5201 crore
- D. Rs 5101 crore

29. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is?

- A. 1:6
- B. 2:5
- C. 3:7
- D. 6:7

Questions (30-34): The various sections of the population are indicated below in the pie-chart. Study the pie-chart and answer the following questions:

The total population of a city is 5000



- I. Employees of the Public Sector
- II. Employees of the Private Sector
- III. Employees of the Corporate Sector
- IV. Self-Employed
- V. Unemployed.

30. What percentage of the employed persons is self-employed?

- A. 5%
- B. $5 \frac{5}{19} \%$
- C. 19%
- D. 20%

Note: From above concepts of Pie Charts, we know $3.6\text{degrees}=1\%$

Therefore, $1\text{ degree} = (5/18)\%$

Similarly the given degrees can be converted into percentages as follows,

- I. Employees of the Public Sector – 162 degrees – 45%
- II. Employees of the Private Sector – 108 degrees – 30%
- III. Employees of the Corporate Sector - 54 degrees 15%
- IV. Self-Employed - 18 degrees - 5%
- V. Unemployed - 18 degrees - 5%

31. Number of persons employed in the Corporate Sector is

- A. 250
- B. 500
- C. 750
- D. 1500

32. The number of Unemployed persons is

- A. 250
- B. 150
- C. 100
- D. 50

33. The number of persons employed in both the Public Sector and Corporate Sector is

- A. 3750
- B. 3000
- C. 2500

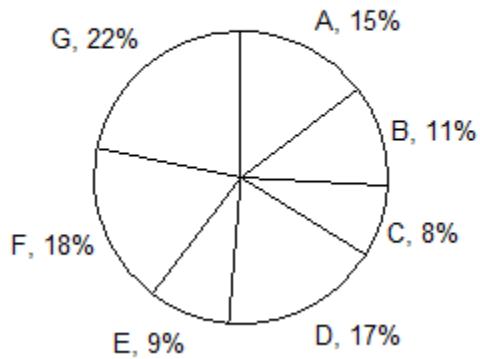
D. 2200

34. What percentage of the employed persons is employed in Private Sector?

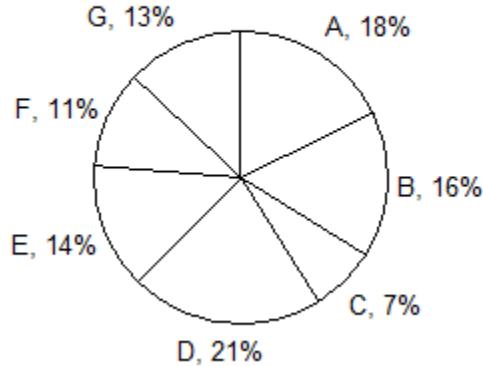
- A. 29%
- B. 31 ($11/19$)%
- C. 34%
- D. 31%

Questions (35-39): These questions are based on following graphs Classification of appeared candidates in a competitive test from different states and qualified candidates from those states.

Total Appeared Candidates =
45000



Total Qualified Candidates =
9000



35. What is the ratio between the number of appeared candidates from states C and E together and the appeared candidates from states A and F together?

- A. 17 : 33
- B. 11 : 13
- C. 13 : 27
- D. 17 : 27

36. In which State the percentage of qualified candidates to that of appeared candidates is minimum?

- A. C
- B. F
- C. D
- D. E

37. What is the difference between the number of qualified candidates of states D and G

- A. 690
- B. 670
- C. 780
- D. 720

38. What is the percentage of qualified candidates to that of appeared candidates from states B and C taken together?

- A. 23.11
- B. 24.21
- C. 21.24
- D. 23

39. What is the ratio between number of candidates qualified from states B and D together and the number of candidates appeared from state C respectively?

- A. 8:37
- B. 11:12
- C. 37:40
- D. 7:37

Type-III - Tabular presentation

1. Introduction:

A table is a set of data arranged in rows and columns and is one of the most common ways of putting information across to people. A table consists of several boxes with information inside. The first row and the first column are generally used to denote the titles. While any type of data can be presented in table form, that too in a very accurate manner, interpreting the data in table form is more difficult and time consuming than the other modes, all of which are basically pictorial or graphical in presentation.

2. Tips on Solving Table Chart Problems:

A: Read the data very carefully, as the smallest detail may change the meaning of the question completely. Similarly, the instructions have to be understood carefully to prevent wasting time in calculating data that is not required, and also to find out exactly what is the answer that is sought.

B: Try to understand the data provided carefully, before jumping to answer the questions. The questions are designed to be deceptive, and proper understanding of the requirements is a must. If the data provided is of the combined variety or if there are more than one data table/charts/graphs, try to understand the relation between the given tables.

For example, one table may talk about absolute sales figures, while the other table may talk of sales as a percentage of production. Hence, any question on excess production or Goods in stock, will require data from both tables.

C: Be very careful of the units used in the tables, and the units in which the answers (options) are provided. A mistake in the units may yield an entirely different answer. Also be careful of whether the answer is required in decimal or percentage. Such errors are common and easily avoidable.

Here is an example consisting tabular data:

Question(40-42):

Category of Assistance	Average number receiving per month		Total cost per help year (in crores of Rs.)		Cost paid by Centre for the year (in crores of Rs.)	
	1995	1996	1995	1996	1995	1996
A	36097	38263	38.4	34.8	18.4	17.4
B	6632	5972	5.0	3.2	2.6	1.6
C	32545	31804	76.4	59.4	13.0	10.0
D	13992	11782	26.4	42.6	6.6	10.6
E	21275	228795	216.6	242.8	55.0	62.6

40. The category receiving the least percentage help from the centre (in the entire data) is:

- A. Category B in 1995
- B. Category C in 1996
- C. Category B in 1996
- D. Category D in 1995

41. The difference between the average costs paid by the Centre during 1995 and 1996 is

- A. Rs. 66 lakh
- B. Rs. 13.2 crore
- C. Rs. 132 lakh
- D. Rs. 13.2 lakh

42. Monthly cost to the city receiving E category assistance in 1996 is most nearly:

- A. Rs. 1.8 crore less than that in 1995
- B. Rs. 2.1 crore more than that in 1995
- C. Rs. 2.1 crore less than that in 1995
- D. Rs. 1.8 crore more than that in 1995

Directions to solve (43-47):

The following table shows the number of new employees added to different categories of employees in a company and also the number of employees from these categories who left the company every year since the foundation of the Company in 1995.

Year	Managers		Technicians		Operators		Accountants		Peons	
	New	Left	New	Left	New	Left	New	Left	New	Left
1995	760	-	1200	-	880	-	1160	-	820	-
1996	280	120	272	120	256	104	200	100	184	96
1997	179	92	240	128	240	120	224	104	152	88
1998	148	88	236	96	208	100	248	96	196	80
1999	160	72	256	100	192	112	272	88	224	120
2000	193	96	288	112	248	144	260	92	200	104

43. What is the difference between the total number of Technicians added to the Company and the total number of Accountants added to the Company during the years 1996 to 2000?

- A.128
- B.112
- C.96
- D.88

44. What was the total number of Peons working in the Company in the year 1999?

- A.1312
- B.1192
- C.1088
- D.968

45. For which of the following categories the percentage increase in the number of employees working in the Company from 1995 to 2000 was the maximum?

- A. Managers
- B. Technicians
- C. Operators
- D. Accountants

46. What is the pooled average of the total number of employees of all categories in the year 1997?

- A.1325

- B.1195
 C.1265
 D.1235

47. During the period between 1995 and 2000, the total number of Operators who left the Company is what percent of total number of Operators who joined the Company?

- A.19%
 B.21%
 C.27%
 D.29%

Questions 48-52 are based on this Table Chart.

The following chart is a record of the performance of a baseball team for the first seven weeks of the season.

	Games Won	Games Lost	Total No. of Games Played (cumulatively)
First Week	5	3	8
Second Week	4	4	16
Third Week	5	2	23
Fourth Week	6	3	32
Fifth Week	4	2	38
Sixth Week	3	3	44
Seventh Week	2	4	50

48. How many games did the team win during the first seven weeks?

- A. 32
 B. 29
 C. 25
 D. 21
 E. 50

49. What percent of the games did the team win?

- A. 75%
 B. 60%
 C. 58%
 D. 29%
 E. 80%

50. According to the chart, which week was the worst for the team?

- A. second week
 B. fourth week
 C. fifth week
 D .sixth week
 A. seventh week

51. Which week was the best week for the team?

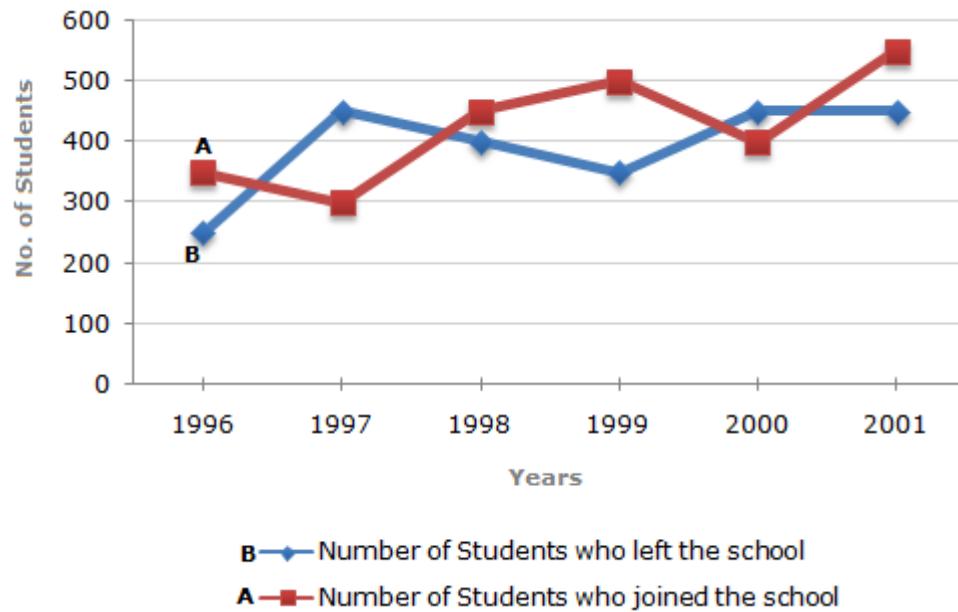
- A. first week

- B. third week
 C. fourth week
 D. fifth week
 E. sixth week
52. If there are fifty more games to play in the season, how many more games must the team win to end up winning 70% of the games?
- A. 39
 B. 35
 C. 41
 D. 34
 E. 32

Type-IV - Line Chart (53-57)

Study the following line graph which gives the number of students who joined and left the school in the beginning of year for six years, from 1996 to 2001.

Initial Strength of school in 1995 = 3000.



53. The number of students studying in the school during 1999 was?

- A. 2950
 B. 3000
 C. 3100
 D. 3150

54. For which year, the percentage rise/fall in the number of students who left the school compared to the previous year is maximum?

- A. 1997
 B. 1998
 C. 1999
 D. 2000

55. The strength of school increased/decreased from 1997 to 1998 by approximately what percent?

- A. 1.2%

- B. 1.7%
- C. 2.1%
- D. 2.4%

56. The number of students studying in the school in 1998 was what percent of the number of students studying in the school in 2001?

- A. 92.13%
- B. 93.75%
- C. 96.88%
- D. 97.25%

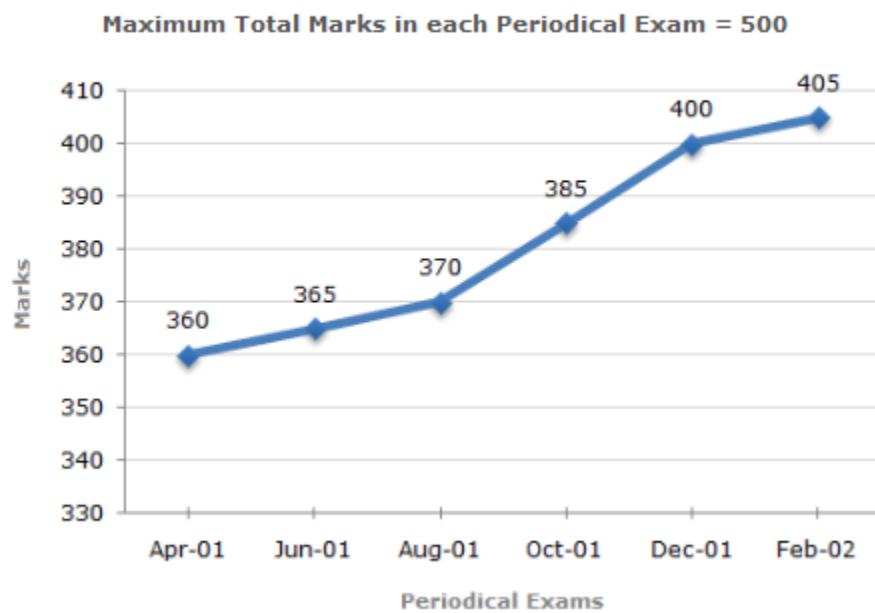
57. The ratio of the least number of students who joined the school to the maximum number of students who left the school in any of the years during the given period is?

- A. 7:9
- B. 4:5
- C. 3:4
- D. 2:3

Question(58-62):

In a school the periodical examination are held every second month. In a session during April 2001 - March 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below.

Marks Obtained by student in Six Periodical Held in Every Two Months During the Year in the Session 2001 - 2002.



58. In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams?

- A. June. 01
- B. August, 01
- C. Oct, 01
- D. Dec, 01

59. The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in April 01?

- A. 110%

- B. 112.5%
- C. 115%
- D. 116.5%

60. What is the percentage of marks obtained by the student in the periodical exams of August, 01 and Oct, 01 taken together?

- A. 73.25%
- B. 75.5%
- C. 77%
- D. 78.75%

61. What are the average marks obtained by the student in all the periodical exams during the last session?

- A. 373
- B. 379
- C. 381
- D. 385

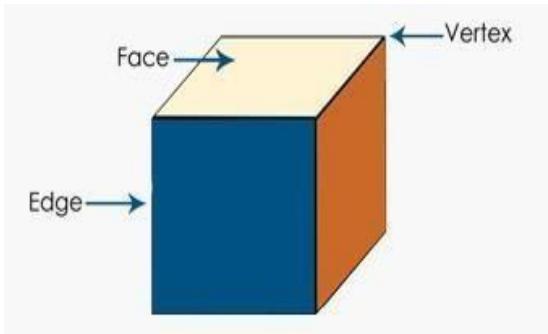
62. In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams?

- A. None
- B. June, 01
- C. Oct, 01
- D. Feb, 02

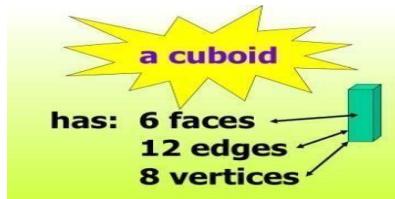
CUBES

In this topic, we are going to learn the concept of cube cutting or cutting the painted cube. While solving these questions, the most important part is to visualize the cube. Cube cutting is an important concept as questions are frequently asked from this topic in a number of competitive exams.

Let's first learn some basic terminologies i.e. face, vertex and edge of a cube.



In a cube, there are 6 faces, 8 vertices & 12 edges. Vertex means corners & edge means side.

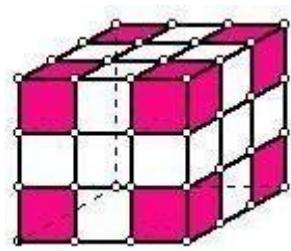


Generally, questions from this topic are of the type wherein, a cube with side measuring unit 'x' is painted on all faces and is cut into smaller cubes with sides measuring unit 'y'. You are then required to find the number of cubes having 'n' faces painted.

The first thing that you need to figure out is the number of smaller cubes. For this, you look at one particular edge of the big cube and figure out how many smaller cubes can fit into this. It will be x/y . So, the number of smaller cubes will be $(x/y)^3$.

Since all the smaller cubes will have at least one face facing inside i.e. not on the surface of the original cube, hence, none of the smaller cubes will have all faces painted. Further, since the maximum number of faces of the larger cube that intersect at a point are 3(at the corners), hence, the smaller cubes can have a maximum of 3 faces painted.

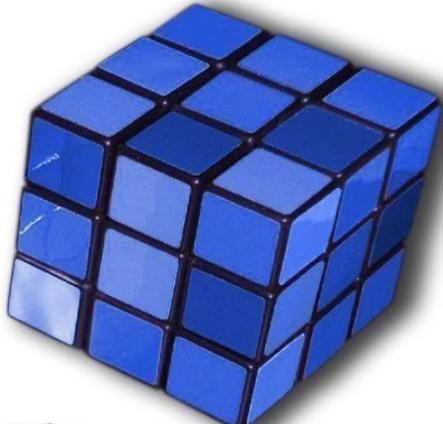
So, the number of smaller cubes with 3 faces painted = No of corners of larger cube = 8 (always), provided none of the faces of the larger cube is left unpainted.



Example 1

A cube having a side of 6 cm is painted Blue on all the faces and then cut into smaller cubes of 1 cm each. Find the total number of smaller cubes so obtained.

Solution:



As explained
(Here x=6 and y=1)

$$= (6/1)^3 = 216 \text{ smaller cubes.}$$

Example 2

In the above example, how many cubes will have three faces painted?

Solution:

As explained above, only the corner cubes i.e. the 8 cubes at the corners of the original cube will have three faces painted. Hence the answer will be 8 only.

To find the number of smaller cubes with only 2 faces painted, you need to consider the cubes where 2 faces of the bigger cube meet, i.e. the edges. Remember, this includes the cubes present at the corners as well, so you need to remove those 2 cubes from the number of cubes on each edge.

Example 3

In the above example, how many cubes will have only two faces painted?

Solution:

As discussed above, only the cubes at the edge of the bigger cube can have two faces painted.

The larger cube has 6 cm edge and smaller cube is 1 cm edge. Hence, there are 6 cubes on each edge. However, you need to consider 4 middle cubes only, as the 2 cubes on each corner will have 3 painted faces.

Hence, there are 4 such cubes on each edge. As there are 12 edges, there will be $4*12 = 48$ cubes

Example 4

In the above example, how many cubes will have only one face and no side painted?

Solution:

As discussed above, only the cubes at the face of the bigger cube can have only one painted face. Since the larger cube has 6 cm edge and smaller cube is 1 cm edge, hence, if you see one of the faces of the larger cube, you will see $6 \times 6 = 36$ cubes. Out of these, exclude the cubes which lie on the edges, as they have two or more faces which are painted. Thus, on each face of the original cube, there will be $4 \times 4 = 16$ cubes will have only one face painted. As there are 6 such faces, the number of such smaller cubes will be $16 \times 6 = 96$.

Lastly, the number of cubes having no faces painted can be found by subtracting the sum of the painted cubes from the total number of smaller cubes. Therefore, the required answer is $216 - (8 + 48 + 96) = 64$ cubes.

Example 5

A cube having an edge of 12 cm each. It is painted red on two opposite faces, blue on one other pair of opposite faces, black on one more face and one face is left unpainted. Then it is cut into smaller cubes of 1 cm each. Answer the following questions:

The total no. of smaller cubes

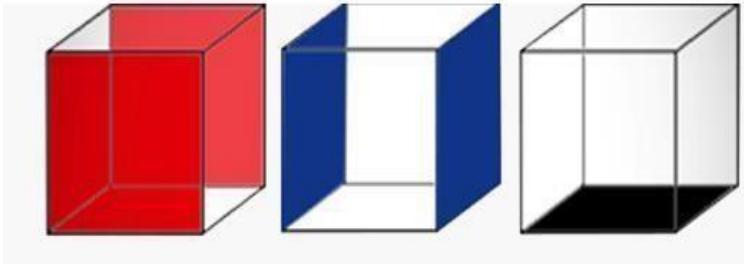
The no. of smaller cubes which are having three-faces painted.

The no. of smaller cubes which are having two-faces painted.

The no. of smaller cubes which are having one-face painted.

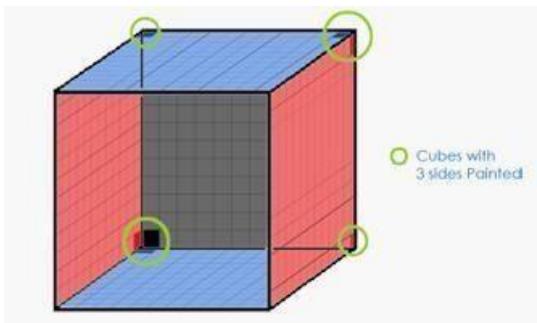
The no. of smaller cubes which are having zero-face painted.

Solution:



Total number of cubes = $(12 \times 12 \times 12) / (1 \times 1 \times 1) = 1728$

For a cube with all sides painted we have 8 cubes with 3 sides colored. But here we have 1 side unpainted. Therefore, we will have only 4 cubes with 3 sides painted. The other 4 cubes will have only 2 sides painted.



For 2 sides painted, we look for the edges.

cube has 12 edges.

8 edges, each edge having 10 cubes will have 2 sides painted. (4 edges of an unpainted side won't be included).

We'll also include those 4 cubes (which we didn't count while counting 3 colored sides, as they have 2 sides painted)

Cubes on 4 edges of the unpainted side of the cube will have 1 side painted (due to the unpainted side).

Therefore, total cubes with 2 sides painted= $8*10 + 4 = 84$ cubes.

For 1 side painted, we look for the faces of the cube.

cube has 6 faces.

5 faces each having $(12-2) * (12-2) = 100$ cubes will have one side painted.

We'll have to include those cubes on the edges linked with an unpainted face.

10 cubes on each of those edges will have 1 side painted.

Therefore, total cubes with 1 side painted= $5*100 + 4*10 = 540$ cubes.

According to the formula, cubes with no side painted= $(12-2)^3 = 1000$.

But we have to include the cubes from the unpainted side too. It will be $10*10 = 100$ So, total number of unpainted cubes= $1000 + 100 = 1100$.

Shortcut Formulae

For a cube of side $n*n*n$ painted on all sides which is uniformly cut into smaller cubes of dimension $1*1*1$,

Number of cubes with 0 side painted= $(n-2)^3$

Number of cubes with 1 sides painted = $6(n - 2)^2$

Number of cubes with 2 sides painted= $12(n-2)$

Number of cubes with 3 sides painted= 8(always)

For a cuboid of dimension, $a*b*c$ painted on all sides which is cut into smaller cubes of dimension $1*1*1$,

Number of cubes with 0 side painted= $(a-2)(b-2)(c-2)$

Number of cubes with 1 sides painted = $2[(a-2)(b-2) + (b-2)(c-2) + (a-2)(c-2)]$

Number of cubes with 2 sides painted= $4(a+b+c-6)$

Number of cubes with 3 sides painted= 8

Key Learning:

To summarize, the number of three faces painted cubes may be found with the help of vertices, two-faces painted with the help of edges and one-face painted with the help of faces. By using this learning, we can even solve complex questions on cube cutting for example cube painted with different colors.

Practice questions- Set I

Direction (Q.1-5): A cube is painted red and all its faces and is then divided into 43 equal cubes.

1. How many cubes will have three faces painted?

- A. 0
- B. 8
- C. 16
- D. 4
- E. None of these

2. How many cubes will have two faces painted?

- A. 55
- B. 70
- C. 45
- D. 50
- E. None of these

3. How many cubes will have one face painted?

- A. 112
- B. 148
- C. 162
- D. 150
- E. None of these

4. How many cubes have no face painted?

- A. 216
- B. 64
- C. 125
- D. 27
- E. None of these

5. How many cubes will have at least one face painted?

- A. 178
- B. 218
- C. 127
- D. 279
- E. None of these

Directions (Q.6-10): Read the following information_s carefully and answer the questions that follow:

There is a rectangular wooden block of length 8cm, height 6cm and breadth 5cm. The two opposite surfaces of 8cm x 6cm are painted from outside by yellow color. One surface of 8cm x 5cm is painted from outside by red color and the opposite surface brown. The remaining two surfaces of 6cm x 5cm are painted outside by orange and black respectively. Now, the block is cut in such a way that cubes of 1cm x 1cm x 1cm are created?

6. How many cubes have only one side painted?

- A. 82
- B. 122
- C. 114
- D. 96
- E. None of these

7. How many cubes have no side painted?

- A. 62
- B. 70
- C. 74
- D. 82
- E. None of these

8. How many cubes have combination of yellow and red only?

- A. 8
- B. 12
- C. 16
- D. 20
- E. None of these

9. How many cubes have the combination of brown with yellow and black?

- A. nil
- B. one
- C. two
- D. three
- E. None of these

10. How many cubes have two sides painted and remaining sides unpainted?

- A. 46
- B. 48
- C. 50
- D. 52
- E. None of these

D. Brown

5. The Lower face is?

- A. White
- B. Blue
- C. Black
- D. Red

Directions (Q.6-10): Read the following information_s carefully and answer the questions that follow:

There are 128 cubes with me which are colored according to two schemes viz.
 All the six faces of a cube of a cube are colored with six different colors - black, brown, green, red, white and blue. Red face is opposite to the black face. Green face is between red and black faces. Blue face is adjacent to white face.

- iv. Brown face is adjacent to blue face.
- v. Red face is in the bottom.

1. The upper face is _____

- A. White
- B. Black
- C. Brown
- D. None of these

6. How many cubes have at least two colored red faces each?

- A. 0
- B. 32
- C. 6
- D. 128

2. The face opposite to brown is _____

- A. Blue
- B. White
- C. Green
- D. Red

7. What is the total number of red faces?

- A. 32
- B. 0
- C. 320
- D. 128

3. Which of the following is adjacent to green?

- A. Black, white, brown, red
- B. Blue, black, red, white
- C. Red, black, blue, white
- D. None of these

8. How many cubes have two adjacent blue faces each?

- A. 64
- B. 32
- C. 0
- D. 128

4. Which face is opposite to green?

- A. Red
- B. White
- C. Blue

9. How many cubes have only one red face each?

- A. 128
- B. 32
- C. 64
- D. None

10. Which two colors have the same number of faces?

- A. Red and Yellow
- B. Blue and Green
- C. Red and Green
- D. Red and Blue

VENN DIAGRAMS

A Venn diagram is a drawing in which sets are represented by geometric figures such as circles and rectangles.

The terms used in Set Theory: Set, Universal Set and Subset

Set:

A well-defined collection of objects is called a set. Each member of a set is called an element. All elements of a set follow a certain rule and share a common property amongst them.

Consider the set of vowels in the English alphabet. Let this set be represented by the letter I. This set contains 5 elements – a, e, i, o, u. This can also be written as:

$$V = \{a, e, i, o, u\}$$

The following two conditions should be taken care of while writing a set –

A set must be denoted by a capital letter.

The elements of the set must be denoted in small letters.

Following are certain examples of sets –

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{a, b, c, d, e\}$$

In the above examples, A and B are sets while 1, 2, 3, 4, 5 and a, b, c, d, e are the elements belonging to these sets respectively.

The symbol \in is used to denote that an element belongs to a set.

Certain sets are used very frequently, like the set of all natural numbers. Following are certain standard letters used for particular sets – N = Set of all natural numbers = {1, 2, 3, 4 ...}

I = Set of all integers = {..., -3, -2, -1, 0, 1, 2, 3 ...}

= Set of all rational numbers

= Set of all real numbers P = Set of all prime numbers

Universal Set:

A set that contains all the elements and sets in a given scenario is called a Universal Set (U).

In our previous example of Football teams, the universal set can be considered as the set of all international teams that play Football.

For example,

U = Set of all numbers

The universal set for the set of all numbers is U = $\{-\infty, \dots, \infty\}$. It includes all natural numbers, integers, rational numbers, irrational numbers, etc.

Subset and Superset:

Set B is said to be a subset of another set A when all elements of set B are also elements of set A. Set A is said to be a superset of set B.

Using the same example, if A is the set of all teams that took part in the 2006 Football World Cup and B is the set of teams that reached the Quarterfinals, then B is said to be a subset of A. This simply means that all the teams present in B must definitely be present in A.

Let us consider another example:

$$A = \{1, 2, 3, 4, 5, 6\} \text{ and } B = \{1, 2, 3\}$$

We can see that every element of set B is an element of set A.

Hence, we can say that B is a subset of A. We can also say that B is contained in A.

A is said to be the superset of B.

In any given context or scenario all sets are subsets of the Universal set.

Remember:

A Null Set (a set that has no elements) is a subset of every set.

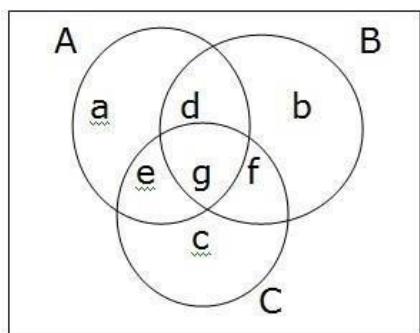
Every set is a subset of itself.

VENN DIAGRAMS

Now that we know what sets are, we can look at Venn Diagrams as an alternate way of depicting sets. Venn Diagrams consist of closed shapes, generally circles, which represent sets. The capital letter outside the circle denotes the name of the set while the letters inside the circle denote the elements of the set.

The various operations of sets are represented by partial or complete overlap of these closed figures. Regions of overlap represent elements that are shared by sets.

In practice, sets are generally represented by circles. The universal set is represented by a rectangle that encloses all other sets. Venn Diagrams are generally not drawn to scale.



The above figure is a representation of a Venn diagram. Here each of the circles A, B and C represents a set of elements.

Set A has the elements a, d, e and g.

Set B has the elements b, d, g and f.

Set C has the elements e, g, f and c.

Both A and B have the elements d and g.

Both B and C have the elements g and f.

Both C and A have the elements e and g.

A, B and C all have the element g.

OPERATION OF SETS

Let us now look at few basic set operations and ways of representing them using Venn diagrams. For understanding these operations, we will use a common example and perform operations on it.

Consider a class of students that form the universal set. Set A is the set of all students who were present in the English lecture, while Set B is the set of all the students who were present in the History lecture. It is obvious that there were students who were present in both lectures as well as those who were not present in either of the two lectures.

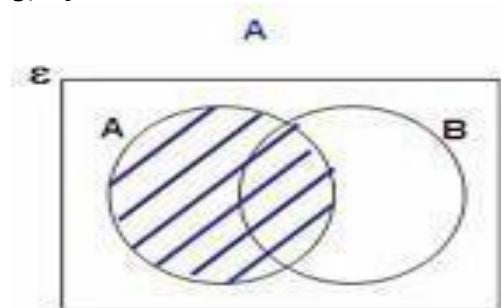
Complement of a Set:

Complement of a set A in the given context is the set having all elements that belong to the Universal set but not to A.

In our example, the complement of set A will be all the students who were absent in the English lecture.

Suppose,

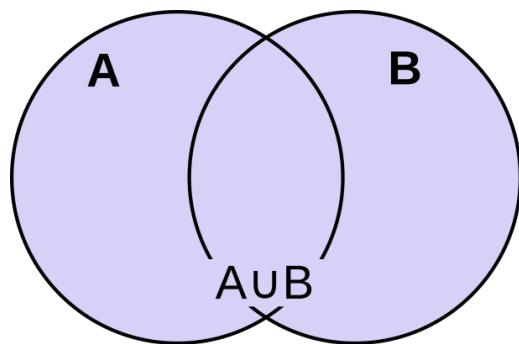
$U = \{a, b, c, d, e, f, g, h\}$ and $A = \{a, b, c, d, e\}$, Then A^c , or complement of the set A = {f, g, h}



Union of Sets:

The union of two sets A and B is defined as the set having all the elements which belong to either A or B or both A and B.

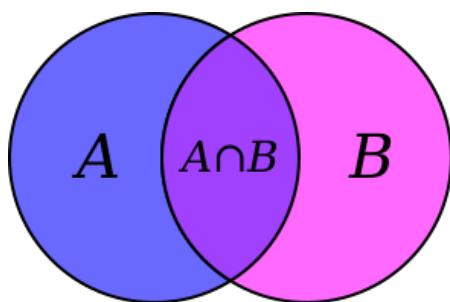
In our example, the union of sets A and B will contain all the students who were present in at least one of the two lectures. Only students who did not attend a single lecture will not be considered in the union.



Intersection of Sets:

The intersection of sets A and B is defined as the set having all elements which belong to both A and B.

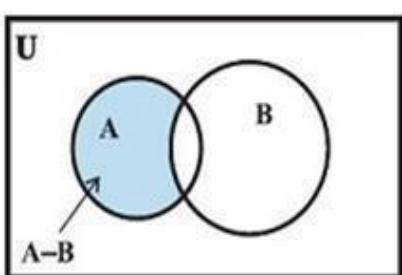
In our example, the intersection of A and B will contain all the students who sat for both, English as well as History lectures.



Difference of Sets:

The difference of two sets A and B, $A - B$, is defined as the set having all the elements of A that do not belong to B. Please note that $A - B$ is not always equal to $B - A$.

Example 1: In our example, $A - B$ will be all the students who have attended only the English lecture and not the history lecture while $B - A$ will be all the students who have attended just the History lecture and not the English lecture.



Example 2: Let us consider another example. In a room, there are 5 people a, b, c, d, e. Out of them, a, b and c are men while d and e are women. Also, a and e study science while b, c and d study commerce.

∴ The set of males is: $M = \{a, b, c\}$

The set of females is:

$F = \{d, e\}$

The set of science students is:

$S = \{a, e\}$

The set of commerce students is:

$C = \{b, c, d\}$

If we wish to find out all female students who have taken science, we need to find out what is common in set F and set S. This is called an intersection of set F and set S and is denoted by $F \cap S$.

Here, $F \cap S = \{e\}$

Thus, an intersection of two sets is formed by the elements which are common to both the sets.

Similarly, if we consider sets M and F, there is no common element between them.

Hence, $M \cap F = \emptyset$

Such sets which have no elements in common are called disjoint sets.

Now, let us find out those females who have not taken science. Here, we have to check the set F and remove all elements of set S present in this set. This is called the difference between two sets.

$S - F = \{a\}$

Thus, difference of set A and set B is defined as the set of all elements present in A but not in B.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Now, suppose you want to represent a set containing —either males or commerce students or both. This would mean taking all the elements from set M and set C together into one set. This is called the union of set M and set C and is denoted by $M \cup C$. Thus, $M \cup C = \{a, b, c, d\}$

Note:

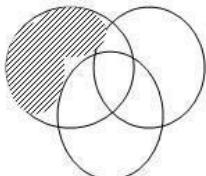
Though b and c exist in both sets, they are written only once while writing the union. This is because no element is ever written twice while writing a set.

Example 3:

Draw Venn diagrams for:

(1) $A - B - C$ Solution:

(1) $A - B - C$



Practice questions- Set I

Directions for Questions (1 to 4): Refer to the data below and answer the following:
 In an examination 43% passed in English, 52% passed in Kannada and 52% passed in Hindi. Only 8% students passed in all the three. 14% passed in English and Kannada and 21% passed in English and Hindi and 20% passed in Kannada and Hindi. Number of students who took the exam is 200. Let Set P, Set C and Set M denotes the students who passed in Kannada, Hindi and English respectively. Then

1. How many students passed in English only?
 A. 16
 B. 32
 C. 48
 D. 80
2. Find the ratio of students passing in English only to the students passing in Hindi only?
 A. 16:37
 B. 29:32
 C. 16:19
 D. 31:49
3. What is the ratio of the number of students passing in Kannada only to the students passing in either Kannada or Hindi or both?
 A. 4:46
 B. 49:32
 C. 26:84
 D. None of these
4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is?
 A. 33
 B. 66
 C. 39
 D. 78

Directions for Questions (5 to 7): Refer to the data below and answer the following:
 There were 3 sections in the Infosys campus recruitment paper. Out of them 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. 10 students cleared the cut-off in Section 1 and Section 2, 9 cleared the cut-off in Section 2 and Section 3, 8 cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

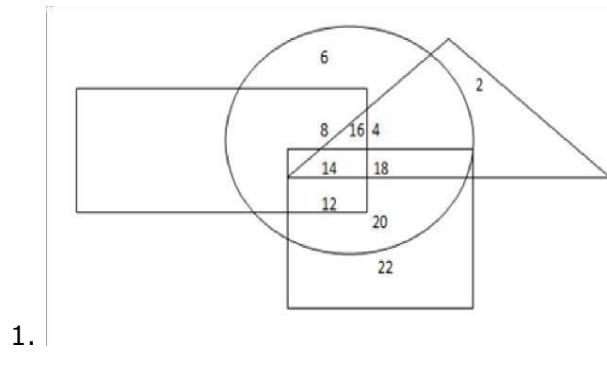
5. How many cleared all the three sections?
 A. 3
 B. 6
 C. 5
 D. 7
6. How many cleared only one of the three sections?
 A. 21
 B. 63
 C. 42
 D. 52
7. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cut off in Section 1 alone is?
 A. 78/21
 B. 3
 C. 73/21
 D. None of these

Directions for Questions (8 to 10):
 5% of the people went from Mysore to Mumbai on 08th February, 1991 do not like Apple, Orange and Grapes and 10% like all the three. 20% like Apple and Orange, 25% like Grapes and Apple and 25% like Grapes and Orange. 55% like Apple, 50% like Orange and 50 % like Grapes.

8. The number of passengers who like only Apple is greater than the passengers who like only Grapes by
- 50%
 - 100%
 - 25%
 - 0
9. The percentage of passengers who like both Orange and Grapes but not Apple is
- 15
 - 5
 - 10
 - 25
10. The percentage of passengers who like at least 2 of the 3 products is
- 40
 - 45
 - 50
 - 60

Practice questions- Set 2

Directions for Questions (1 to 5): In the following diagram, the Triangle represents Diamond, Square represents Silver, circle represents Platinum and Rectangle represents Gold selling showrooms.



Which showroom sells all four metals?

- 14
- 16
- 18
- 4

2. Which showroom sells only Gold and Platinum?

- 16
- 12
- 10
- None

3. Which showroom sells only Diamond?

- 10
- 2
- 4
- 8

4. Which showroom sells only platinum?

- 20
- 4
- 6
- 8

5. Which showroom sells only Diamond and Platinum and nothing else?

- 18
- 16
- 4
- 14

SYLLOGISMS

Syllogism_s is one of the easy to win questions in reasoning, but we have seen so many aspirants are finding difficult in solving these questions. Usually Venn diagram method (Easy) is used to solve these but they will consume time in case of NO/Possibility conclusion cases. So here we are explaining the concept of Syllogism with some examples by using some simple rules.

First and foremost, have a quick glance at the Main rules to solve Syllogism Problems.

All + All = All

All + No = No

All + Some = No Conclusion

Some + All = Some

Some + No = Some Not

Some + Some = No Conclusion

No + All = Some Not (Reversed)

No + Some = Some Not (Reversed)

No + No = No Conclusion

Some Not /Some Not Reversed + Anything = No Conclusion

RULE OF ALL+		
Statement 1	Statement 2	Conclusion
All	All	All
All	No	No
All	Some	No Conclusion

RULE OF NO+ & SOME NOT +		
Statement 1	Statement 2	Conclusion
No	All	Some Not
No	No	Some Not
No	Some	No Conclusion
Some Not	Anything	No Conclusion

RULE OF SOME+		
Statement 1	Statement 2	Conclusion
Some	All	Some
Some	No	Some Not
Some	Some	No Conclusion

THE QUESTION OF POSSIBILITY		
Statement	Possibility	
All A are B	Some B are A	
Some A are B	All B are A	
Some B are Not A	All A are B	

If the conclusion is in -Possibilityl case, then these rules must be applied.

If All A are B then we can say – Some B are Not A is a Possibility

If Some B are Not A then we can say - All A are B is a Possibility

If Some A are B then we can say - All A are B is a Possibility, All B are A is a possibility i.e.,

All \Leftrightarrow Some Not Reversed

Some \Rightarrow All

NO Conclusion = Any Possibility is true

When it is implemented (In case of Conclusion from Single Statement) All \Rightarrow Some that means if All A are B then Some B are A is true.

Some \Leftrightarrow Some that means if Some A are B then Some B are A is true. No \Leftrightarrow No that means if No A is B then NO B is A is true

How to use these Syllogism Rules to solve questions?

In order to solve Syllogism there are two types:

Cross Cancellation

Vertical Cancellatio

Let us see about Cross Cancellation with example:

Example 1:

Statements:

All Cows are Parrots

All Parrots are Birds

No Bird is Monkey

Conclusions:

No Parrot is Monkey

Some Cows being Monkey is Possibility

We know you might be able to solve it by using Venn diagram method that's good but this method won't help or a bit tough when it comes to No or possibility Conclusions.

Solution:

Let's take 1st conclusion, we have to make relation between Parrot and Monkey so we will take statements 2 and 3.



This is called Cross Cancellation, We have cancelled Bird from Bird so we have left with (ALL+NO) rule, and that leads to No Parrot is Monkey So Conclusion I is TRUE.

In second statement we have Cow and Monkey so we will need to make relation between them. For this we need to take all 3 statements.



Now we have left with ((All + All) + NO) that is No Cow is Monkey. We don't have any rule to convert this statement into Possibility so second conclusion is FALSE

I think we are clear with above explanation now see about Vertical Cancellation

Example 2

Statements:

Some Mails are Messages

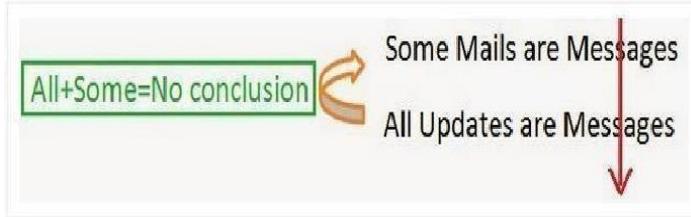
All Updates are Messages

Conclusion:

All Mails Being Update is a Possibility

No Update is Mail

Let's take Conclusion —All Mails Being Update is a Possibility|| that means we have to make relation between Mails and Updates



This is called Vertical cancellation. In this case direction of adding first phrase will be reversed i.e In Above example the conclusion will be All + Some = No Conclusion.

If we get No Conclusion in case of Possibility, then according to Rules in Possibility case will be definitely true. So Conclusion 1 follows and Conclusion 2nd Don't.

So far we have seen how to deal with All, Some, Some Not and No now let us see about Some Not in reversed condition.

What is Some Not (Reversed)?

To explain this let's take a Simple example

No A is B

All B is C

So the conclusion you get for this will be as follows (No + All) A is C = (Some Not Reversed) A is C Therefore Some C are Not A.

Practice questions- Set I

1. Statement:

All the phones are scales.

All the scales are calculators.

Conclusions:

I. All the calculators are scales.

II. All the phones are calculators.

III. All the scales are phones.

IV. Some calculators are phones.

A. Only conclusion I and IV follows

B. Only conclusion III and IV follows

C. Only conclusion II and IV follows

D. Only conclusion I and II follows

E. Only conclusion I and III follows

2. Statement:

All fishes are birds.

No Bird is an animal.

All animals are mammals.

Conclusions:

I. At least some birds are mammals.

II. All mammals being birds is a possibility.

A. Only conclusion II follows

B. Neither conclusion I or II follows

C. Both conclusion I and II follows

D. Either conclusion I or II follows

E. Only conclusion I follows

3. Statement:

Some triangle are circles.

All squares are circles.

Conclusions:

I. At least some squares are triangle

II. All circles are triangles

A. Only conclusion I follows

B. Either conclusion I or II follows

C. Both conclusion I and II follows

D. Only conclusion II follows

E. Neither conclusion I or II follows

4. Statement:

All books are scales

All scales are pencils

Some scales are pens

Conclusions:

I. No book is pen

II. All pencils are scales

A. Only conclusion I follows

B. Only conclusion II follows

C. Either conclusion I or II follows

D. Neither conclusion I or II follows

E. Both conclusion I and II follows

5. Statement:

Some actors are singers.

All the singers are dancers.

Conclusions:

I. Some actors are dancers.

II. No singer is actor.

A. Only conclusion I follows

B. Only conclusion II follows

C. Either I or II follows

D. Neither I nor II follows

E. Both I and II follows

6. Statement

All the harmoniums are instruments.

All the instruments are flutes.

Conclusions:

I. All the flutes are instruments.

II. All the harmoniums are flutes.

A. Only conclusion I follows

B. Only conclusion II follows

C. Either I or II follows

D. Neither I nor II follows

E. Both I and II follow

7. Statement:

Some mangoes are yellow.

Some tixo are mangoes.

Conclusions:

I. Some mangoes are green.

II. Tixo is a yellow.

A. Only conclusion I follows

B. Only conclusion II follows

C. Either I or II follows

D. Neither I nor II follows

E. Both I and II follow

8. Statement:

All the windows are doors.

No door is a wall.

Conclusions:

- I. Some windows are walls.
- II. No wall is a door.
- A. Only conclusion I follows
- B. Only conclusion II follows
- C. Either I or II follows
- D. Neither I nor II follows
- E. Both I and II follow

9. Statement:

Some cows are crows.

Some crows are elephants.

Conclusions:

- I. Some cows are elephants.
- II. All cows are elephants.
- A. Only conclusion I follows
- B. Only conclusion II follows
- C. Either I or II follows
- D. Neither I nor II follows
- E. Both I and II follows

10. Statement:

All green are blue.

All blue are white.

Conclusions:

- II. Some blue are green.
- III. Some white are green.
- IV. Some green are not white.
- V. All white are blue.
- A. Only conclusion I and II follows
- B. Only conclusion I and III follows
- C. Only conclusion I and IV follows
- D. Only conclusion II and IV follows

Practice questions- Set II

1. Statement:

All men are vertebrates.

Some mammals are vertebrates.

Conclusions:

- I. All men are mammals.
- II. All mammals are men.
- III. Some vertebrates are mammals.
- IV. All vertebrates are men.
- A. Only conclusion IV follows
- B. Only conclusion II follows
- C. Only conclusion III follows
- D. Only conclusion I follows
- E. Only conclusion I and III follows

2. Statement:

Some tables are T.V.

Some T.V. are radios.

Conclusions:

- I. Some tables are radios.
- II. Some radios are tables.
- III. All the radios are T.V.
- IV. All the T.V. are tables.
- A. Only conclusion II and IV follows
- B. Only conclusion I and III follows
- C. Only conclusion IV follows
- D. Only conclusion I and IV follows
- E. None of the four follows.

3. Statement:

All the research scholars are psychologists.

Some psychologists are scientists.

Conclusions:

- I. All the research scholars are scientists.
- II. Some research scholars are scientists.
- III. Some scientists are psychologists.
- IV. Some psychologists are research scholars.
- A. Only conclusion III and IV follows
- B. None of the four follows
- C. All the four follows
- D. Only conclusion III follows
- E. Only conclusion II and IV follows

4. Statement:

Some questions are answers.

Some answers are writers.

All the writers are poets.

Conclusions:

I. Some writers are answers.

II. Some poets are questions.

III. All the questions are poets.

IV. Some poets are answers.

A. Only conclusion I and II follows

B. Only conclusion I and IV follows

C. Only conclusion I and III follows

D. Only conclusion II and IV follows

5. Statement:

All the locks are keys.

All the keys are bats.

Some watches are bats.

Conclusions:

I. Some bats are locks.

II. Some watches are keys

III. All the keys are locks.

A. Only conclusion I and II follows

B. Only conclusion I follows

C. Only conclusion II follows

D. Only conclusion I and III follows

6. Statement:

All the papers are books.

All the bags are books.

Some purses are bags.

Conclusions:

I. Some papers are bags.

II. Some books are papers.

III. Some books are purses.

A. Only conclusion I follows

B. Only conclusion II and III follows

C. Only conclusion I and II follows

D. Only conclusion I and III follows

7. Statement:

Some boys are rude.

Some rude are human.

All human are good.

Conclusions:

I. All human being boys is a possibility.

II. At least some rude are good.

III. Some good are not boys.

A. Only conclusion I follows

B. Only conclusion II and III follows

C. All follows

D. Only conclusion I and III follows

E. None of these

8. Statement:

All metals are black.

Some pots are black.

No black is thread.

Conclusions:

I. At least some thread are pots.

II. All thread being pot is a possibility

III. Some metals are black.

A. Only conclusion I follows

B. Only conclusion II and III follows

C. All follows

D. None follows

E. Only conclusion I and III follows

9. Statement:

Some trains are metro.

No bus is a train.

All taxi is a metro.

Conclusions:

I. Some metro is not a bus.

II. At least some taxi is a bus.

III. All bus being metro is a possibility.

A. Only conclusion I follows

B. Only conclusion I and II follows

C. All follows

D. Only conclusion I and III follows

E. None follows

10. Statement:

Some blue are black.

Some black are grey.

All grey are red.

All red are pink.

Conclusions:

Some red are black.

Some pink are black.

Some pink are grey.

Some red are blue.

- A. Only conclusion I & II follow
- B. Only conclusion II & III follow
- C. Only conclusion I, II and III follow
- D. All follow
- E. None of these

NUMBER SERIES



A set of numbers arranged in a definite order according to some definite rule is called a sequence.

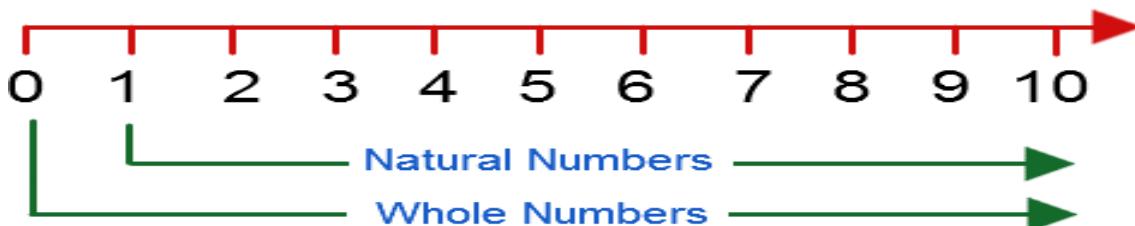
Different types of numbers

Natural numbers

Natural number is a number that occurs naturally. It is also called as counting numbers. It is denoted by n and it starts from 1, 2, 3 ...n

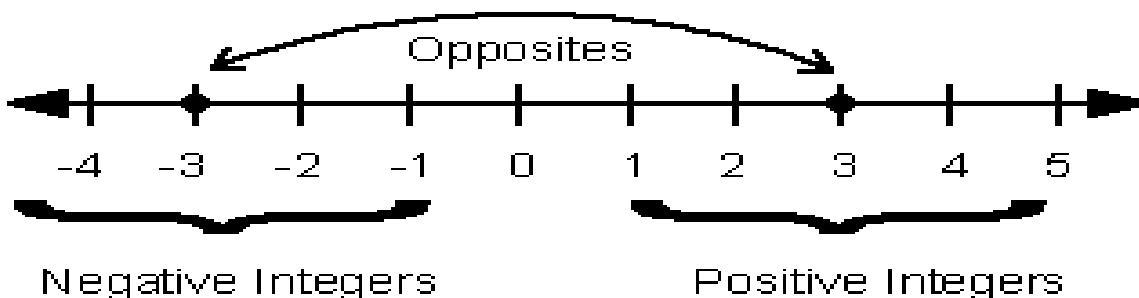
Whole numbers

Whole number is a number which does not contain fractional or decimal part or negative values. It is denoted by w and it starts from 0, 1, 2, 3 ...n



Integer numbers

Integer numbers can be a whole number but it contains both positive and negative numbers like -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5 and it is denoted by Z .



Rational numbers

It is denoted by q and it is expressed in the form of a/b . Where a and b are integers and b (denominator) cannot be 0.

Rational number (Q)

Its Form : $Q = p/q$

$p, q \in I$

$q \neq 0$

Irrational numbers

It is a number that cannot be expressed as a fraction for any integer and irrational have decimal expansions that neither terminate nor become periodic.

It is denoted by $_i_\$

Examples of Irrational Numbers

- $\pi = 3.14159 \dots$
- $e = 2.71828 \dots$
- $\sqrt{2} = 1.4142 \dots$
- $\sqrt{17} = 4.1231 \dots$

='

Complex numbers

It contains 2 parts i.e. Real and imaginary values. It is denoted by $_z_\$. Real part of complex numbers are denoted by $re(z)$ and imaginary part of complex numbers are denoted by $im(z)$. It is expressed as $a + ib$.

Complex Numbers

$$a + bi$$

↗ ↗
 Real Imaginary
 Part Part

Different types of series

1. Arithmetic series

A sequence in which the difference between two consecutive terms remains constant. Arithmetic series contains two rules, they are addition and subtraction.

Example

242, 244, 246, 248..... ?

250

252

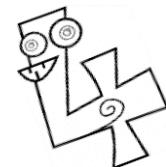
254
253

Solution:

Answer: Option a
The difference between the numbers is 2.

2. Geometric series

The number is obtained by multiplying or dividing the sequence with a fixed number.
Geometric series contains other two rules, they are multiplication and division.



Example

4.5, 9, 27, 54..... ?
78
102
148
162

Solution:

Answer: option d
The given sequence is multiple of alternative 2 and 3.

3. Alternative series

Alternative series is nothing but every alternate term forms a part of series.



Example

12, 10, 24, 20...?
36
48
58
None of these

Solution:

Answer: Option b
The given sequence is multiple of 12 in the 1st alternative and multiple of 10 in the 2nd alternative.

4. Arithmetic and geometric series

This series contains all the four rules i.e. Addition, subtraction, multiplication and division.
The sequence can be solved using any two rules.

Example

7, 16, 30, 62, 122.... ?
224

242
246
285

Solution:

Answer: Option c

The given sequence is solved using any two rules.

$$\begin{aligned}7 * 2 + 2 &= 16 \\16 * 2 - 2 &= 30 \\30 * 2 + 2 &= 62 \\62 * 2 - 2 &= 122 \\122 * 2 + 2 &= 246\end{aligned}$$

5. Perfect square and cube numbers

This series contains square and cube of a number may be in ascending or descending order.
In squares and cubes, we have three different rules.

N^2 $N^2 + 1$ $N^2 - 1$	n^3 $n^3 + 1$ $n^3 - 1$
---------------------------------	---------------------------------

Example 1

- 10, 14, 28, 32..... ?
44
54
64
74

Solution:

Answer: Option b

The given sequence is a perfect square number by adding or subtracting some numbers.

$$\begin{aligned}3^2 + 1 &= 10 \\4^2 - 2 &= 14 \\5^2 + 3 &= 28 \\6^2 - 4 &= 32 \\7^2 + 5 &= 54\end{aligned}$$

Example 2

- 28, 62, 128, 212...?
297
313
343
348

Solution:

Answer: Option d

The given sequence is a perfect cube number by adding or subtracting some numbers.

$$\begin{aligned}3^3 + 1 &= 28 \\4^3 - 2 &= 62 \\5^3 + 3 &= 128\end{aligned}$$

$$6^3 - 4 = 212$$

$$7^3 + 5 = 348$$

6. Special number series

Prime numbers: prime numbers are special number who is divisible by 1 and itself. Which means it is not possible to factorize the prime numbers.

Example

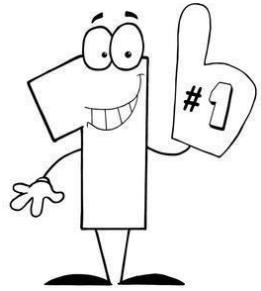
2, 3, 13, 17, 19.... ?

A. 21

B. 23

C. 25

D. 27



Solution:

Answer: Option b

The given sequence is a prime number which contains _ letter in that.

7. Fibonacci series:

Fibonacci series is a special series where current value is determined by adding previous two values.

Example

1, 1, 2, 3, 5, 8.....?

A. 11

B. 14

C. 12

D. 13

Solution:

Answer: Option d. 13

Here we need to add 5 and 8 in order to get 13.

Steps/methods to solve number series questions:

Find the difference between the two numbers.

Finding the different alternatives logic.

Using any two rules together to find the logic.

Finding the perfect square and cube numbers.

Checking the letters and vowels in the given sequence.

Activity:

How will you divide 1000 one rs. Coins in ten bags so that you can give any amount between 1-1000 by just giving the bags without changing the number of coins in each bag.

Practice questions Set-I

1. 7, 6, 8, 15, 44, ?
 A. 88
 B. 120
 C. 159
 D. 150
 E. 195
2. 16, 25, 42, 66, 96, ?
 A. 120
 B. 160
 C. 118
 D. 131
 E. 130
3. 4, 6, 9, 34, 68, ?
 A. 56
 B. 64
 C. 161
 D. 68
 E. 116
4. 7.6, 9.9, 14.5, 23.7, ?
 A. 42.2
 B. B.42.1
 C. C.42.3
 D. 42.4
 E. 42.5
5. 9, 5, 4, 7.5, 13, ?
 A. 35
 B. 26
 C. 34
 D. 48
 E. 62
6. 11, 13, 20, 48, 111, ?
 A. 237
 B. 125
 C. 273
 D. 255
 E. 555
7. 1. 6, 3.5, 4.5, 11, 48, ?
 A. 96
 B. 56
 C. 392
 D. 192
 E. 292
8. 8, 4.5, 5.5, 13, 56, ?
 A. 566
 B. 496
 C. 596
 D. 450
 E. 456
9. 11, 14, 23, 50, ?
 A. 111
 B. 121
 C. 151
 D. 131
 E. 141
10. 21, 35, 30, 44, 39, ?
 A. 59
 B. 53
 C. 55
 D. 45
 E. 46

Practice questions- Set II

1. 12, 6.5, 7.5, 17, 72, ?
 A. 356
 B. 82
 C. 185
 D. 292
 E. 584
2. 463, 220, 139, 112, ?, 100
 A. 101
 B. 100
 C. 104
 D. 103

- E. 102
3. 17, 30, 21, 34, 25, ?
A. 38
B. 30
C. 27
D. 36
E. 40
4. 31, 33, 40, 68, 131,?
A. 232
B. B.257
C. C.132
D. 250
E. 275
5. 10, 14, 28, 52, 134, ?
A. 302
B. 268
C. 300
D. 304
E. 208
6. 4500, 900, 90, 6, ?,
0.012.
A. 0.3
B. 3
C. 3.33
D. 0.33
E. 3.3
7. 24, 11, 10, 14, 27, ?
A. 66
B. 70.5
C. 68
D. 66.5
E. 68.5
8. 8, 7, 12, 33, 128, ?
A. 528
B. 365
C. 653
D. 825
E. 635
9. 18.3, ?, 16, 22.9, 13.7,
2.2, 11.4
A. 25.2
B. 18.3
C. 13.7
D. 22.9
E. 20.6
10. 9, 5, 6, 10.5, 23, 61, ?
A. 183
B. 10.5
C. 61
D. 5
E. 9

CALENDARS

Odd Days:

In a given period, the number of days more than the complete weeks are called odd days.



Leap Year:

- (i). Every year divisible by 4 is a leap year, if it is not a century.
- (ii). Every 4th century is a leap year and no other century is a leap year. Note: A leap year has 366 days.



Examples:

Each of the years 1948, 2004, 1676 etc. is a leap year.

Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

Ordinary Year:

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

Counting of Odd Days:

$$1 \text{ ordinary year} = 365 \text{ days} = (52 \text{ weeks} + 1 \text{ day.})$$

∴ 1 ordinary year has 1 odd day.

$$1 \text{ leap year} = 366 \text{ days} = (52 \text{ weeks} + 2 \text{ days})$$

∴ 1 leap year has 2 odd days.

$$100 \text{ years} = 76 \text{ ordinary years} + 24 \text{ leap years}$$

$$= (76 \times 1 + 24 \times 2) \text{ odd days} = 124 \text{ odd days.}$$

$$= (17 \text{ weeks} + \text{days}) \equiv 5 \text{ odd days.}$$

∴ Number of odd days in 100 years = 5.

$$\text{Number of odd days in } 200 \text{ years} = (5 \times 2) \equiv 3 \text{ odd days.}$$

$$\text{Number of odd days in } 300 \text{ years} = (5 \times 3) \equiv 1 \text{ odd day.}$$

$$\text{Number of odd days in } 400 \text{ years} = (5 \times 4 + 1) \equiv 0 \text{ odd day.}$$

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.





Concept and Calculations in Calendar

Concept of Calendar

We know that

Any non-leap year contains 365 days = 52 weeks + 1 day and leap-year contains 366 days = 52 weeks + 2 days

This 1-day and 2 days extra added to any year create changes in the calendar and this is the reason why calendar of Nth year will not be same as N+1th year. Before we proceed ahead, we should be very clear about two things:

i. Which years are leap years?

It takes the earth about 365.2422 days to go around the sun, but a normal calendar year is only 365 days. The extra fraction of a day added up four times makes four years (or, four revolution of earth around sun) 1460.9688 days, but four calendar years would only be 1460 days. That 0.9688 is almost a whole day, so every four years we add an extra day to our calendar, February 29. We call that year leap year. To make things easier, leap years are always divisible by four: 2004 and 2008 will both be leap years.

For hundreds of years, people used a calendar called the Julian calendar that followed this rule, adding a leap year every four years. However, because 0.9688 isn't exactly a whole day, the Julian calendar slowly began to disagree with the real seasons. In 1582, Pope Gregory fixed this problem by ordering everyone to use a new set of rules. These rules are named the Gregorian calendar, after him. They work like this:

The Gregorian Calendar Rule

Examples

Every fourth year is a leap year.

2004, 2008, and 2012 are leap years.

However, every centenary year is not a leap year.

1900 and 2100 are no leap years.

In case of centuries, every four hundred years, there's 2000 and 2400 are leap years. a leap year after all.

In layman terms, all the century years divisible by 400 will be leap years and all the non-century years divisible by 4 will be leap years. So, leap year next to 2096 AD is 2104 AD and not 2100 AD.

Because 2000, 4000, 6000, etc. are leap years and 1000, 3000, 5000, etc. are not, the number of leap days in each millennium alternates between 242 and 243, with the first, third, etc. millennia (i.e., 1-1000, 2001-3000, etc.) having 242 leap days, and the second, fourth, etc. (i.e., 1001-2000, 3001-4000, etc.) having 243 leap days.

ii. How the days of consecutive years change?

Due to any non-leap year, calendar of next year goes ahead by 1 day and due to any leap year, calendar of next year goes ahead by 2 days, but this change in calendar will be there only before 29th February.

It can be seen through the example given below:

	1991	1992	1993
1st January	Sunday	Monday	Wednesday
28th February	Tuesday	Wednesday	Friday
1st March	Wednesday	Friday	Saturday

In the above example, we have assumed that 1st January of 1991 is Sunday. 1991 and 1993 are non-leap years and 1992 is a leap year.

If now we try to find out the symmetric of calendars, we can see this in the following way:

I. For any leap-year

Let us see, for example, the case of 1972.

Year	1972	1973	1974	1975	1976
Excess days	2	1	1	1	2

Since no. of excess days are 7, so the days of the year 1972 and year 1977 will be same from 1st of January and 28th of February.

ii. For any (leap-year+1) year

Year	1973	1974	1975	1976	1977	1978
Excess days	1	1	1	2	1	1
Excess days	1	1	1	2	1	1

Since no. of excess days are 7, so calendar of year 1973 and 1979 will be same for whole year.

iii. For any (leap-year+2) year

Year	1974	1975	1976	1977	1978	1979
Excess days	1	1	2	1	1	1

Since excess days are 7, so calendar of year 1974 and 1980 will be same till 28th of February.

iv. For any (leap-year+3) year

Year	19 75	19 76	19 77	19 78	19 79	19 80	19 81	19 82	19 83	19 84	19 85
Excess days	1	2	1	1	1	2	1	1	1	2	1

Since no. of excess days are 14, so calendar of year 1975 and 1986 will be same for whole year.

This whole mechanism can be summed up in following way:

Nature of year	No. of years after which 1st January will be same
Leap year	5
Leap year + 1	6
Leap year + 2	6
Leap year + 3	11

So, if 1st January of 1972 and 1st January of 1977 will be on same day.

If 1st January of 1973 and 1st January of 1979 will be on same day and so on.

Exception – No century year, which is not a leap year, should be included in this calculation.

Example: 1. Sum of dates of last Monday of previous month and 1st Thursday of next month is 38. If both the dates are of the same year, then which month is the current month?

Solution-Sum of dates of last Monday of previous month and 1st Thursday of next month is 38 is possible only if last Monday is 31st and 1st Thursday is 7th. (Since if we take 30+8=8, then 30 can be last Monday of any month but 8th cannot be the 1st Thursday of any month)

So, 31st of last month is a Monday.

Hence 7th of current month - 14th of current month – 21st of current month -28th of current month will be a Monday.

Now, if current month is a month with 30 days, then 5th of next month will be a Monday, so 7th of next month cannot be a Thursday.

If current month is a month with 31 days, then 4th of next month will be a Monday, so 7th of next month will be a Thursday.

Finally, we can conclude that previous month and current month, both are having 31 days. Since both the dates are of the same year, so the current month is August.

Finding day of a date by using a reference date:

Let us see this with the help of an example:

If 9th Dec of 1972 is Sunday, then which day it will be on 14th December 1998?

Process: - There are several processes to do this calculation:

Year method,

Days method,

Actual calculation method

Year Method – We use the above given table to find out about any of the years. 9.12.1972 – Sunday

1.1.1973 – Tuesday (It is a Leap year + 1 year)

So, 1.1.1979 – Tuesday, (It is a Leap year + 3 year) So, 1.1.1990- Tuesday, (It is a Leap year + 2 year) So, 1.1.1996 – Tuesday

Now, we can find out all the next years one-by-one. 1.1.1997 – Thursday

1.1.1998 – Friday – 31.12.1998 – 24.12.1998 – 17.12.1998

So, 14.12.1998 – Tuesday

Days method –We use the no. of excess days every year to find out the no. of days calendar will move ahead by. 1.1.1973 –Tuesday

Due to 1973, calendar will go ahead by 1 day, similarly due to 1974 – 1 day, due to 1975 – 1 day and due to 1976- 2 days.

So, in four years, calendar will go ahead by 5 days.

Using unitary method, in four years, calendar will move ahead by 5 days.

So, in 24 years_ calendar will move ahead by 30 days. Hence calendar will move ahead by 2 days.

So, 1.1.1997 will be two days ahead of Tuesday i.e. Thursday. Now, it is calculation as given in Year Method.

Actual Calculation method-With the help of this method, we can find the actual day of any date of 20th century. To use this method effectively, we need to remember the Month Codes of all the months.

Let us learn this method by finding the date of 15th August 1947 –

At 1st, add the Date + Month code of August + Last two digits of year + (Where [x] represents the greatest integer value of x.)

So, corresponding to 15th August 1947 – 15 + 3 + 47 + 11 = 76 Now, divide this value by 7 to find out the remainder.

If the remainder is 0 then it is a Saturday

If the remainder is 1 then it is a Sunday

If the remainder is 2 then it is a Monday If the remainder is 3 then it is a Tuesday

If the remainder is 4 then it is a Wednesday

If the remainder is 5 then it is a Thursday

If the remainder is 6 then it is a Friday

Here, remainder is 6, so 15th August 1947 was a Friday. (It should have been 'Free_day) List of Month Code:

	J a n	F e b	M a r	A p r	M ay	J u n	J u l	A u g	S e p	O ct	N o v	D ec
Leap Year	0	3	4	0	2	5	0	3	6	1	4	6
Non-leap year	1	4	4	0	2	5	0	3	6	1	4	6

Let us observe the Examples

Example1: How many times does the 29th days of the month occur in 400 consecutive years

1) 97 times

- 2) 4400 times
- 3) 4497 times
- 4) none

Solution: In 400 consecutive years there are 97 leap years. Hence in 400 consecutive years, February has the 29th day 97 times, and the remaining 11 months have the 29th day 400×11 or 4400 times.

Therefore, 29th day of the month occurs $(4400 + 97)$ or 4497 times

Example 2:

Given that on 10th November 1981 is Tuesday, what was the day on 10th November 1581

- 1) Monday
- 2) Thursday
- 3) Sunday
- 4) Tuesday

Solution: After every 400 years, the same day comes.

Thus if 10th November 1981 was Tuesday, before 400 years i.e. on 10th November 1581, it has to be Tuesday.

Example 3: What was the day on 9 February 1979?

Solution: You know that in 1600 years, there will be 0 odd days. And in the next 300 years, there will be 1 odd day. From 1901 to 1978 we have 19 leap years and 59 non leap years. So, the total number of odd days up to 31st Dec. 1978 is $19 \times 2 + 59 = 97$. On dividing 97 by 7 we get 6 as the remainder, which is the total number of odd days in these years. So, till 31 Dec. 1978 we have $1 + 6 = 7$ odd days, which forms one complete week. Now, in 1979, we have 3 odd days in January, and 2 odd days in the month of February (up to 9 Feb). So, the total odd days are $3 + 2 = 5$. Hence, 9 February 1979 was a Friday.

Example 4: If May 10, 1997 was a Monday, what will be the day on Oct 10, 2001?

Solution: In this question the reference point is May 10, 1997 and you have to find the number of odd days from May 10, 1997 up to Oct 10, 2001. Now, from May 11, 1997

- May 10, 1998 = 1 odd day

May 11, 1998 - May 10, 1999 = 1 odd day

May 11, 1999 - May 10, 2000 = 2 odd days (2000 was leap year)

May 11, 2000 - May 10, 2001 = 1 odd day

Thus, the total number of odd days up to May 10, 2001 = 5.

Now, the remaining 21 days of May will give 0 odd days. In June, we have 2 odd days; in July, 3 odd days; in August, 3 odd days; in September, 2 odd days and up to 10 October, we have 3 odd days. Hence, total number of odd days = 18 i.e. 4 odd days. Since, May 10, 1997 was a Monday, then 4 days after Monday will be Friday. So, Oct 10, 2001 would be a Friday.

Example 5: If 11th April 1911 was a Tuesday, what would be the day on 17 September 1915?

Solution:

Firstly, in terms of years, the year 1911 to 1912 would give us 2 odd days and 1913, 1914, 1915 would give 1, 1 and 1 odd days respectively.

Now shift the focus on months. If you move one month ahead i.e. from 11th April to 11th May, the month ending in between is April, which gives you 2 days. Now after that the month of May, June, July, and August gives you 3, 2, 3, and 3 odd days respectively. With this you reach on 11th September 1915. After this there are 6 more September days (from 11th to 17th September).

The total number of odd days is $2 + 1 + 1 + 1 + 2 + 3 + 2 + 3 + 3 + 6 = 24$.

Subtracting 21 (3 full weeks) from this the odd number of days left is 3. Adding three days to the day given i.e. Tuesday, the answer becomes Friday.

Practice questions- Set I

1. How many years have 29 days in February from 2001 to 2100.
 - A. 26
 - B. 25
 - C. 23
 - D. 24
2. 2012 January 1st is Sunday, then which day is the Indian Independence day of the same year.
 - A. Saturday
 - B. Wednesday
 - C. Thursday
 - D. Friday
3. Which year has the same calendar as 1700?
 - A. 1705
 - B. 1706
 - C. 1707
 - D. 1708
4. If Arun_s birthday is on May 25 which is Monday and his sister_s birthday is on July 13. Which day of the week is his sister_s birthday?
 - A. Monday
 - B. Wednesday
 - C. Thursday
 - D. Friday
5. March 1st is Wednesday. Which month of the same year starts with the same day?
 - A. October
 - B. November
 - C. December
 - D. None Of These
6. What was the day on 15th august 1947?
 - A. Friday
 - B. Saturday
 - C. Sunday
 - D. Thursday
7. What was the day of the week on, 16th July, 1776?
 - A. Tuesday
 - B. Wednesday
 - C. Monday
 - D. Saturday

8. What will be the day of the week 15th August, 2010?
 - A. Sunday
 - B. Saturday
 - C. Wednesday
 - D. Monday
9. If 6th March, 2005 is Monday, what was the day of the week on 6th March, 2004?
 - A. Sunday
 - B. Saturday
 - C. Tuesday
 - D. Wednesday
10. On what dates of April, 2001 did Wednesday fall?
 - A. 2nd,9th,16th,23rd
 - B. 4th,11th,18th,25th
 - C. 12th,18th,27th,6th
 - D. 1st,8th,15th,22nd

Practice questions- Set II

1. How many leap years do 100th year have?
 - A. 25
 - B. 24
 - C. 4
 - D. 26
2. Find out the number of days from 26th January, 1996 to 15th May, 1996 (if both days are included)?
 - A. 110
 - B. 111
 - C. 112
 - D. 113
3. Find the day of the week on 25th December, 1995?
 - A. Monday
 - B. Tuesday
 - C. Wednesday
 - D. Thursday
4. Monday was a holiday for Republic Day and 16th of the next month is given a holiday for Pongal. What day it on the 16th?
 - A. Monday
 - B. Tuesday
 - C. Wednesday
 - D. None of these

5. February 1, 1984 was a Wednesday just like the February 29, 1984. When will the calendar show another February with a similar situation?

- A. 1996
- B. 2000
- C. 2004
- D. 2012

6. On what dates of April, 2001 did Wednesday fall?

- A. 1st, 8th, 15th, 22nd, 29th
- B. 2nd, 9th, 16th, 23rd, 30th
- C. 3rd, 10th, 17th, 24th
- D. 4th, 11th, 18th, 25th

7. The calendar for the year 1993 will be same for the year:

- A. 2004
- B. 1992
- C. 1998
- D. 2003

8. Prove that any date in March of a year is the same day of the week corresponding date in November that year.

- A. Same day
- B. Not same day
- C. Next day
- D. Previous day

9. What was the day of the week on 16th August, 1947?

- A. Sunday
- B. Monday
- C. Saturday
- D. Thursday

10. It was Tuesday on Jan 1, 2006. What was the day of the week Jan 1, 2010?

- A. Sunday
- B. Friday
- C. Monday
- D. Tuesday

CLOCKS



Introduction

A clock is a devise which shows time in consists of two hands the hour hand and the minute hand. The hour hand (short hand) indicates time in hours and the minute had or long hand indicates time in minutes. For simplicity we will not consider the third hand – the second hand which is also presenting many clocks and watches.

Usually clocks are circular in shape but we can see many other different shapes also. Irrespective of the shape of the dial of the clocks, the minute hand and hour hand always rotate in circular direction.

The circumference of a dial clock is divided into 12 equal parts called hour spaces.

Each hour space is further subdivided into 5 equal parts called minute spaces.

Thus the complete circumference of a dial of a clock is divided into $12 \times 5 = 60$ minute spaces.

In an hour the hour hand crosses 5 minute spaces. While the minute hand crosses 60 minute spaces therefore is obvious that in 60 minutes, the minute hand gains $(60-5=55)$ minute spaces or minutes over the hour hand. This is very important and useful conclusion and therefore should be remembered while solving the problems on clocks.



Some important facts:

In one hour the minute hand makes 3600

In 60 minutes the minute hand makes 3600

So in one minute the minute hand makes $360 / 60$ that is 60

Therefore, each minute_s space = 3600 i.e., minute spaces are 60 apart.

In one hour the hour hand makes $360 / 12$ i.e., 300

In 60 minutes the hour hand makes 300

So in one minute the hour hand makes $30 / 60$ that is $1/20$

Therefore, each hour space = 300 i.e., hour spaces are 300 apart.

The minute hand moves through 60 in each minute whereas the hour hand moves through $1/2$ 0 in each minute. Thus in one minute the minute hand gains $5 \frac{1}{2}$ 0 than the hour hand.

When the 2 hands are coincident, they are at 00 are 0 minutes apart.

In every hour the hour hand and minute hand coincide only once.

In twelve hours the 2 hands coincide 11 times. Because there is a common position 12_O clock between 11 and 1_O clock.

In 24 hours the 2 hands coincide 22 times

When the 2 hands are in opposite direction they are $180/6$ i.e., 30 minutes apart.

It happens only once in an hour.

In every hour the hour hand and minute hand are in opposite direction only once.

In 12 hours the 2 hands are in opposite direction 11 times. Because there is a common position 6_O clock between 5 and 11_O clock.

In 24 hours the 2 hands opposite direction 22 times.

When the 2 hands at right angle (90°) or 15 minutes spaces apart.

It happens 2 times in every hour.

In 12 hours the hands are at right angle only 22 times. It is so because there are 2 common positions in every 12 hours i.e., 3_O clock & 9_O Clock. iii) In 24 hours the 2 hands at right angle 44 times.

The minute and hour hands of a clock coincide for every 655 minutes.



Too Fast:

When the clock indicates time more than the correct time it is said to be running too fast by the difference between the correct time and the time indicated by the clock. For example, the clock indicates 9:30 am when the correct time is 9:15 am it is set to be 15 minutes fast

Too slow:

When the clock indicates time less than the correct time it is said to be running too slow by the difference between the correct time and the time indicated by the clock. For example, the clock indicates 9:30 am when the correct time is 9:45 am it is set to be 15 minutes slow

Note: The minute hand moves 12 times as fast as the hour hand.

Note: The clock has two hands—the hour hand and the minute. The hour hand (or short hand) indicates time in hours and the minute hand (or long hand) indicates time in minutes. In an hour, the hour hand covers 5 minute spaces while the minute hand covers 60 minute spaces. Thus, in one hour or 60 minutes, the minute hand gains 55 minute spaces over the hour hand.

Types:

Clock questions are categorized in to two ways.

Concept on angles

Concept on incorrect clocks

Concept on angles

Before we actually start solving problems on angles, we need to know couple of basic facts clear:

Speed of the hour hand = 0.5 degrees per minute
(dpm)

Speed of the minute hand = 6 dpm

At n o'clock, the angle of the hour hand from the vertical is $30n$.

Since there are 12 hours on a standard clock face we divided the total 360 degree measurement by 12 to get 30. $360/12 = 30$. This tells us that the space between any 2 hours is 30 degrees.

Example at 6:00 clock angle between the hour hand and the minute hand is $360*6 = 180$.
At, this point the hour hand and minute hand are opposite to each other.

Concept of Incorrect clocks:

Such sort of problems arises when a clock runs faster or slower than expected pace. When solving these problems, it is best to keep track of the correct clock.

Example: What is the angle between the hands of the clock at 7:20?

Sol: At 7_o_clock, the hour hand is at 210 degrees from the vertical.

In 20 minutes, hour hand = $210 + 20*(0.5) = 210 + 10 = 220$.

{The hour hand moves at 0.5 dpm}. Minute hand = $20*(6) = 120$.

{The minute hand moves at 6 dpm}. Difference or angle between the hands = $220 - 120 = 100$ degrees.

Example: Find the angle between the hands of a clock at 3:45

Sol:

At 3: 45

H= 3, M= 45

Then, the measure of angle between hands of a clock at 3:45 = $30(H+M/60) - 6M$

$30(3+45/60) - 6*45$

$30(3.75) - 270$

- 157.5

Note

If you get an angle more than 180° subtract from 360°

If you get a negative sign ignore it.

If hands of a clock (which do not show the correct time) coincide every m minutes, then

If $m > 65 \frac{5}{11}$, then the watch is going slow or losing time.

If $m < 65 \frac{5}{11}$, then the watch is going fast or gaining time.

Example: A watch gains 5 seconds in 3 minutes and was set right at 8 AM. What time will it show at 10 PM on the same day?

Sol: The watch gains 5 seconds in 3 minutes = 100 seconds in 1 hour.

From 8 AM to 10 PM on the same day, time passed is 14 hours

In 14 hours, the watch would have gained 1400 seconds or 23 minutes 20 seconds.

So, when the correct time is 10 PM, the watch would show 10: 23: 20 PM

Example: A clock which gains 1 Hr in every 24 Hours is set right on Sun 9 am, then what will be the true time when the clock indicates Tue 11 am?

Sol: Sun - 9 am - the clock was set right

50Hr {The gap of no. of hours between Sun – Tue}

Tue - 11 am – what will be true time?

WRONG TIME - RIGHT TIME

25 hrs - 24 hrs

50 hrs - ? (x)

$$25X = 50 * 24$$

$$X = (50 * 24)/25$$

$$X = 48 \text{ hrs}$$

SUN - 9 AM} 24 HRS

MON - 9 AM} 48 HRS

TUE - 9 AM

So, the True time is TUE 9 A.M.



Types of Questions Asked:

Finding the angle between minute and hour hand when makes some time, H:M hrs (given).

Finding the time when both minute and hour hand coincide (0°), perpendicular (90°), straight and opposite to each other (180°).

Problems on Incorrect clocks.

Problems on Mirror and Water images.

Clock problems based on direction.

Practice questions- Set I

1. How many times in a day, the hands of a clock are straight?
 - A. 22
 - B. 24
 - C. 44
 - D. 48
2. When will the clock_s hands be in a straight line but pointing in opposite directions between 2 am and 3 am?
 - A. $43(7/11)$ minutes past 2 am
 - B. $25(1/11)$ minutes before 3 am
 - C. $38(7/11)$ minutes before 3 am
 - D. $50(5/11)$ minutes past 2 am
3. In every 60 minutes, the minute hand gains _____ minutes on the hour hand.
 - A. 53
 - B. 54
 - C. 55
 - D. 56
4. How many times are the hands of a clock at right angle in a day?
 - A. 22
 - B. 24
 - C. 44
 - D. 48
5. How often between 11 _o_ clock and 12 _o_ clock are the hands of the clock together at an integral number value?
 - A. 55
 - B. 56
 - C. 4
 - D. 5
6. At what time between 2 and 3 o'clock will the hands of a clock be together?
 - A. $10 \times 10/11$
 - B. $10 \times 11/10$
 - C. $11 \times 10/11$
 - D. $12 \times 10/11$

7. At what angle the hands of a clock are inclined at 15 minutes past 5?
 - A. $117\frac{1}{2}^\circ$
 - B. 64°
 - C. $135\frac{1}{2}^\circ$
 - D. $145\frac{1}{2}^\circ$
8. At 3 : 40, the hour hand and the minute hand of a clock form an angle of
 - A. 120°
 - B. 125°
 - C. 130°
 - D. 135°
9. Find the angle between the hour and the minute hand of a clock when the time is 9:45.
 - A. $47\frac{1}{2}$
 - B. $49\frac{1}{2}$
 - C. $22\frac{1}{2}$
 - D. $57\frac{1}{2}$
10. A clock strikes every hour once at 1.00 twice at 2.00 and so on. the clock takes 6 seconds to strike 5.00 and 12seconds to strike 9.00 the time needed to strike 1.00 is negligible. how long does the clock need for all its striking in24 hours?
 - A. 168 Seconds
 - B. 178 Seconds
 - C. 188 Seconds
 - D. 198 Seconds

Practice questions- Set II

1. A watch which gains uniformly is 2 minutes slow at noon on Monday and is 4 min. 48 sec fast at 2 p.m. on the following Monday. When was it correct?
 - A. 2 p.m. on Tuesday
 - B. 2 p.m. on Wednesday
 - C. 3 p.m. on Thursday
 - D. 1 p.m. on Friday

2. A clock is set at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?
- A. 9 pm
 - B. 10 pm
 - C. 11 pm
 - D. 12 pm
3. The minute hand of a clock overtakes the hour hand at intervals of 65minutes of the correct time. How much a day does the clock gain or lose?
- A. $10 \frac{10}{140}$
 - B. $10 \frac{10}{150}$
 - C. $10 \frac{10}{143}$
 - D. $10 \frac{12}{140}$
4. The minute hand of a clock overtakes the hour hand at intervals of 50minutes of the correct time. How much a day does the clock gain or lose?
- A. $7 \frac{23}{55}$
 - B. $8 \frac{24}{58}$
 - C. $5 \frac{55}{60}$
 - D. $9 \frac{45}{50}$
5. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?
- A. 10 min_s past 12
 - B. 48 min_s past 12
 - C. 25 min_s past 15
 - D. 12 min_s past 45
6. A watch which gains uniformly is 6 min. slow at 8 o'clock in the morning Sunday and it is 6 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?
- A. Tuesday 7:20 pm
 - B. Wednesday 7:00 pm
 - C. Wednesday 7:20 pm
 - D. Tuesday 7:00 pm
7. At what time between 5 and 6 o'clock are the hands of a clock 3 minute apart?
- A. 24 min past 5
 - B. 20 min past 3
- C. 25 min past 6
- D. 24 min past 4
8. Find the MIRROR IMAGE for the following
- A. 7:20
 - B. 4:30
 - C. 8:45
 - D. 10:30
9. Find the WATER IMAGE for the following
- A. 7:30
 - B. 4:20
 - C. 8:30
 - D. 12:30
10. The time is 7:20 am, find the direction of minute hand if the direction of hour hand is in North direction?
- A. East
 - B. West
 - C. South-East
 - D. South-West

BLOOD RELATIONS

Names of Relationships

Mother's or Father's son – Brother

Mother's or Father's Daughter – Sister

Son's wife - Daughter in law

Daughter's husband - Son in law

Husband's father or Wife's father - Father in law

Husband's mother or Wife's mother - Mother in law

Mother's father or Father's father – Grandfather (Maternal or Paternal respectively)

Mother's mother or Father's mother – Grandmother (Maternal or Paternal respectively)

Mother's brother - Maternal Uncle

Father's Brother - Paternal uncle

Mother's sister or Father's sister – Aunt (Maternal or Paternal respectively)

Husband's brother or Wife's brother - Brother in law

Husband's sister or Wife's sister or brother's wife - Sister in law

Sister's husband - Brother in law

Brother's son or Sister's son – Nephew

Brother's daughter or Sister's daughter – Niece

Please note, always read carefully, whose relation is being asked. Some times in a hurry, we get the correct relation from the statement but we do not note which relation is being asked.

For example:

Pointing to a girl in photograph. Amir said, "Her mother's brother is the only son of my mother's father." How is the girl's mother related to Amir?

Here the actual relation is asked "How is the girl's mother related to Amir" but in hurry we can give the answer how the girl is related to Amir.

Consider the following:

If A's mother is B and B's only brother is C, then

A's relation to C is niece.

C's relation to A is maternal uncle.

Therefore, it is important to check whose relation with whom is asked.

Trick for solving Blood relations:

We can use the following symbols instead of using words. This will help in solving the questions faster.

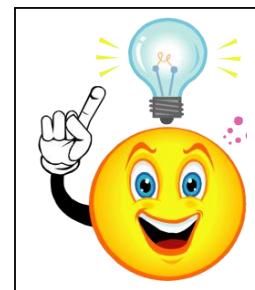
Male can be represented as 

Female can be represented as 

Relation between parents and children can be represented as 

Relation between siblings can be represented as 

Finally, the relation between husband and wife can be represented as =



For example,

i) Q and R are brothers.

ii) P is the wife of Q.

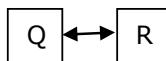
iii) T is the brother of S.

iv) S is the daughter of R.

How is P related to S?

Solution:

(i) can be represented as



Since Q and R are males, it is represented in squares.

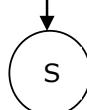
(ii) can be represented as  = 

Since, P is the wife and Q is the husband, they are represented in a circle and square respectively.

(iii) can be represented as  ↔ S

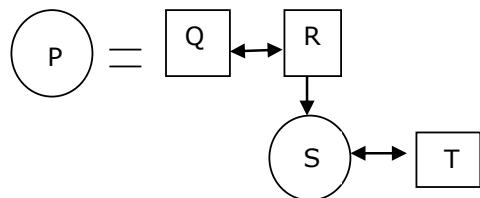
T is in a square since T is a male. The gender of S is not known. Therefore, no representation.

(iv) can be represented as R ↓



We know S is the daughter of R. Therefore S is in a circle.

Now, combining all the four statements we have,



From the above figure, we can get the relation between P and S.
 i.e., S is P's husband's brother's daughter. Therefore, P is S's aunt.

QUESTIONS:

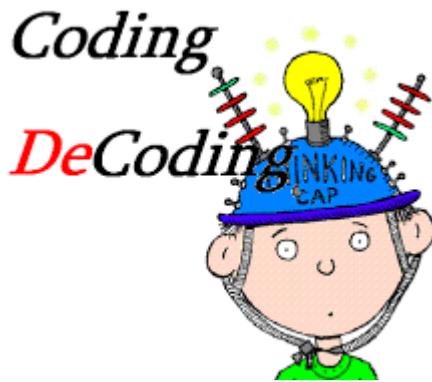
1. Pointing to a photograph, Vipul said, "She is the daughter of my paternal grandfather's only son." How is Vipul related to the girl in the photograph?
 - A. Father
 - B. Brother
 - C. Cousin
 - D. Uncle
 - E. Grandson
2. Looking at a portrait of a man, Harsh said, "His mother is the wife of my father's son. Brothers and sisters I have none." At whose portrait was Harsh looking?
 - A. His son
 - B. His cousin
 - C. His uncle
 - D. His nephew
 - E. None of these
3. If Neena says, "Anita's father Raman is the only son of my father-in-law Mahipal", then how is Bindu, who is the sister of Anita, related to Mahipal?
 - A. Niece
 - B. Daughter
 - C. Wife
 - D. Daughter-in-law
 - E. None of these
4. Pointing to a girl in the photograph, Amar said, "Her mother's brother is the only son of my mother's father." How is the girl's mother related to Amar?
 - A. Mother
 - B. Sister
 - C. Aunt
 - D. Grandmother
 - E. None of these
5. A is the son of B.
C, B's sister has a son D and a daughter E.
F is the maternal uncle of D.
How is E related to F?
6. (i) In a family of six persons A, B, C, D, E and F, there are two married couples.
(ii) D is grandmother of A and mother of B
(iii) C is wife of B and mother of F.
(iv) F is the granddaughter of E.
What is C to A?
 - A. Daughter
 - B. Grandmother
 - C. Mother
 - D. Cannot be determined
 - E. None of these
7. Pointing to a man on the stage, Rita said, "He is the brother of the daughter of the wife of my husband." How is the man on the stage related to Rita?
 - A. Son
 - B. Husband
 - C. Cousin
 - D. Nephew
 - E. Brother-in-law
8. Pointing to a photograph, a person tells his friend, "She is the granddaughter of the elder brother of my father." How is the girl in the photograph related to his man?
 - A. Niece
 - B. Sister
 - C. Aunt
 - D. Sister-in-law
 - E. Maternal aunt
9. P's father's sister's father is Q. How is Q related to P?
 - A. Father
 - B. Uncle
 - C. Grandfather
 - D. Son

- E. None of these
10. E is the son of A.
 D is the son of B.
 E is married to C.
 C is B's daughter.
 How is D related to E?
 A. Brother
 B. Uncle
 C. Father-in-law
 D. Brother-in-law
 E. None of these
11. A man pointing to a photograph says, "The lady in the photograph is my nephew's maternal grandmother." How is the lady in the photograph related to the man's sister who has no other sister?
 A. Mother
 B. Cousin
 C. Mother-in-law
 D. Sister-in-law
12. Pointing to a photograph, a lady tells Pramod, "I am the only daughter of this lady and her son is your maternal uncle." How is the speaker related to Pramod's father?
 A. Wife
 B. Sister-in-law
 C. Daughter
 D. Either (a) or (b)
13. Pointing to a gentleman, Deepak said, "His only brother is the father of my daughter's father." How is the gentleman related to Deepak?
 A. Brother-in-law
 B. Uncle
 C. Father
 D. Grandfather
14. Rita told Mani, "The girl I met yesterday at the beach was the youngest daughter of the brother-in-law of my friend's mother." How is the girl related to Rita's friend?
 A. Daughter
 B. Niece
- C. Friend
 D. Cousin
15. A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z. Which of the following statements is true?
 A. P and A are cousins
 B. P is the maternal uncle of A
 C. Q is the maternal grandfather of A.
 D. C and P are sisters
16. If $X + Y$ means X is the daughter of Y; $X - Y$ means X is the brother of Y; $X \% Y$ means X is the father of Y and $X \times Y$ means X is the sister of Y. Which of the following means I is the niece of J?
 A. $J - N \% C \times I$
 B. $I \times C - N \% J$
 C. $J + M \times C \% I$
 D. $I \times C + N - J$
17. Pointing to Gopi, Nalni says, "I am the daughter of the only son of his grandfather." How Nalni is related to Gopi?
 A. Niece
 B. Daughter
 C. Sister
 D. Cannot be determined
18. Pointing to a lady a person said, "The son of her only brother is the brother of my wife." How is the lady related to the person?
 A. Maternal aunt
 B. Grandmother
 C. Sister of father-in-law
 D. None of these
19. Pointing to Varman, Madhav said, "I am the only son of one of the sons of his father." How is Varman related to Madhav?
 A. Nephew
 B. Uncle
 C. Father or Uncle
 D. Father

20. Introducing Sonia, Aamir says, "She is the wife of only nephew of only brother of my mother." How Sonia is related to Aamir?

- A. Wife
- B. Sister
- C. Sister-in-law
- D. Data is inadequate

CODING AND DECODING



For solving coding and decoding questions, you have to learn a few basics.

Coding and decoding

1. Letter position (A=1, B=2, C=3, D=4.....Y=25, Z=26)
2. Opposite position of letters (A=26, B=25....Z=1)
3. Opposite of each letter (A is opposite to Z and B is opposite to Y and C is opposite to X.....learn all alphabets opposite)

TRICKS for solving Coding and Decoding.

Type-1- Letter: In a certain code, CODING is written as DPEJOH. How will DECODING be written in that code?

Answer: The simple formula or the shortcut trick to solve this type of question is to compare the first pair of words (CODING and DPEJOH). The number of steps that are used to increase each letter in first conversion, the same formula should be applied to the second set that has to be converted. Just like in CODING and DPEJOH, 1 point step is added (C+1=D, O+1=P as per alphabet).

Now compare each letter one to another like compare each letter of CODING and DPEJOH.

Comparing 1st letter of CODING to 1st of DPEJOH.

- C is converted to D.
- You know C's next letter in alphabets is D.
- O's next letter is P.
- D's next letter is E.
- I's next letter is J.
- N's next- O
- G's next- H

This was how CODING was converted to DPEJOH. Now the same way you have to convert DECODING.

So

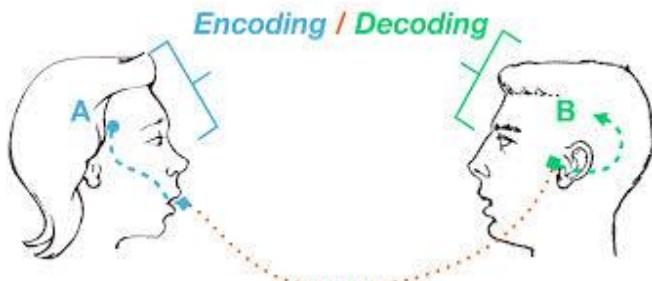
- D- Is converted to -E.
- E-F
- C-D
- O-P
- D-E
- I-J

- N-O
- G-H

So DECODING should be converted to EFDPEJOH.

Note: in the shown conversion of a question, there might be letters in which +2, +3, +4 etc. positions are added. In that case, add the same position to the asked word.

Type 2-Number: Position of letter is added or remains constant.



The Main idea to solve these questions is to apply the conversion that is shown like in above question "CODING is written as DPEJOH". You have to apply the same conversion to DECODING word. Same trick you have to apply to Number coding. For converting letters to numbers, you have to remember the conversion basic given above.

Like FEAR can be converted to 65118 as shown below

- F=6
- E=5
- A=1
- R=18

Or it can also be converted to the addition of these means $6+5+1+18=30$. The same way should be applied for the asked CODE.

Number-Symbol coding

In these types of questions, letters and numbers are made related to each other. These are of two types

When numerical/symbol codes are assigned to words

Example: If REQUEST is written as S2R52TU, then how will ACID be written?

Sol. In this question, alternate letters are moved to next letter in alphabet series and vowels are written as their position in their own series.

- R-S
- E-2 (A-1, E-2, I-3, O-4, U-5)
- Q-R
- U-5
- E-2
- S-T
- T-U

Similarly

- A-1
- C-D
- I-3

- D-E

Means ACID will be coded as 1D3E

When alphabetical codes are assigned to numbers

In this type of questions, different words are coded in a certain symbol. We have to encode them as per the conditions given in the questions. So read the conditions very carefully and then try to solve these types of questions.

Substitution based questions: In substitution based questions, three types of questions are basically asked.

In the first type of questions, the letters are directly coded in the same pattern as per the given examples and we have to form the letters as per those given letters changes.

Example: If EARTH is coded as QPMZS, then how would HEART be coded?

The main detection point of these questions is that the word that is asked to be coded will contain those same alphabets as given in the first sentence. So in this example, the letters of EARTH and HEART are same, so HEART will be coded as SQPMZ

In the second type of questions, each letter is moved certain steps forward to obtain the corresponding letter of the code.

Example: In a certain code, VICTORY is written as YLFWRUB, then how will SUCCESS be written in that language?

Solution: In this question, every letter is moved three step forward in their position in the alphabetical series to obtain the corresponding letter of the code.

In the third type of questions, the letters are arranged opposite to the position of them in the alphabetical series. The rule is as under

A-Z, B-Y, C-X, D-W, E-V, F-U, G-T, H-S, I-R, J-Q, K-P, L-O, M-N, N-M, O-L, P-K, Q-J, R-I, S-H, T-G, U-F, V-E, W-D, X-C, Y-B, Z-A

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
Z	Y	X	W	V	U	T	S	R	Q	P	O	N
26	25	24	23	22	21	20	19	18	17	16	15	14

Fig-1

Sum of each highlighted Alphabets
for e.g. > M+N = 13+14=27

Memorize this series on your fingertips for easy solution of the problems.

Example: In a certain language, MILK is written as NROP, how will DEAF be written

Solution: First you have to find out the question type. Now MILK is written as NROP. You can understand if you have memorized the series given above that N is opposite to M, R is opposite to I, O is opposite to L and P is opposite to K. Now as per the rule given above this question, D is opposite to W, E is opposite to V, A is opposite to Z and F is opposite to U. so DEAF will be coded as WVZU.

Deciphering number and symbol codes in a message

These types of questions are mostly asked in coding and decoding type of reasoning questions. In this type of question, a few groups of numbers or symbols which are codes are

given. The candidate is required to find the number or symbol code for each code or you have to formulate all the codes of the message. You can do that through the process of comparison of the given coded message taking at least two at a time.

For example, look at the given question below-

In a certain language, 'come now' is written as 'ha na', 'now and then' is written as 'pa do na' and 'go then' is written as 'sa pa'. how is 'and' written in that language.

Solution:

Now there are three statements given

1. Come now-ha na
2. Now and then- pa do na
3. Go then-sa pa

The shortcut trick for these questions is trying to find out the common words among the given statements. Try to find out the common word among the 1st and 2nd statements. The common word is 'now'. As you see, the corresponding coded conversion is 'na'. Similarly, try to find out the common word in 2nd and 3rd statement. Again you see the common word is 'then'. So the corresponding coded word (among 'pa do na' and 'sa pa') is 'pa'.

Now you have found out two conversions...

Now-na

Then-pa

So from the 2nd statement, 'pa' and 'na' correspond to 'then' and 'now' respectively. So from 2nd statement, the remaining word 'and' will correspond to 'do'.

So 'and' will be coded as 'do'.

QUESTIONS:

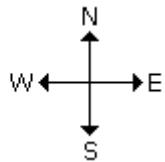
1. In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?
 - A. QDFHS
 - B. SDFHS
 - C. SHFDQ
 - D. UJHFS
 - E. None of these
2. In a certain code, COMPUTER is written as RFUVQNPC. How is MEDICINE written in the same code?
 - A. EOJDJEFM
 - B. EOJDEJFM
 - C. MFEJDJOE
 - D. MFEDJJOE
 - E. None of these
3. If GIVE is coded as 5137 and BAT is coded as 924, how is GATE coded?
 - A. 5427
 - B. 5724
 - C. 5247
 - D. 2547
4. In as a certain code, 15789 is written as EGKPT and 2346 is written ALUR. How is 23549 written in that code?
 - A. ALEUT
 - B. ALGTU
 - C. ALGUT
 - D. ALGRT
 - E. None of these
5. If white is called blue, blue is called red, red is called yellow, yellow is called green, green is called black, black is called violet and violet is called orange, what would be the colour of human blood?
 - A. Red
 - B. Green
 - C. Yellow
 - D. Violet
6. If the animals which can walk are called swimmers, animals who crawl are called flying, those living in water are called snakes and those which fly in the sky are called hunters, then what will a lizard be called?
 - A. Swimmers
 - B. Snakes
 - C. Flying
 - D. Hunters
 - E. None of these
7. In a certain code language,
 - (A) 'pit na som' means 'bring me water'
 - (B) 'na ja tod' means 'water is life'
 - (C) 'tub od pit' means 'give me toy'
 - (D) 'jo lin kot' means 'life and death'To find out the answer to the code 'bring me is life', which of the following statements can be dispensed with ?
 - A. A only
 - B. C only
 - C. B or C only
 - D. D only
 - E. None of these
8. In a certain code, '37' means 'which class' and '583' means 'caste and class'. What is the code for 'caste'?
 - A. 3
 - B. 7
 - C. 8
 - D. Either 5 or 3
 - E. Either 5 or 8
9. In a certain code, '289' means 'read from paper'; '276' means 'tea from field' and '85' means 'wall paper'. Which of the following is the code for 'tea'?
 - A. 2
 - B. 6

- C. Either 2 or 6
 D. Either 2 or 7
 E. Either 7 or 6
10. If FRAGRANCE is written as SBHSBODFG, how can IMPOSING be written?
 A. NQPTJHOJ
 B. NQPTJOHI
 C. NQTPJOHJ
 D. NQPTJOHJ
 E. None of these
11. If 'nso ptr kli chn' stands for 'Sharma gets marriage gift', 'ptr lnm wop chn' stands for 'wife gives marriage gift', 'tti wop nhi' stands for 'he gives nothing', what would mean 'gives'?
 A. chn
 B. nhi
 C. ptr
 D. wop
12. If 'tee see pee' means 'Drink fruit juice', 'see kee lee' means 'Juice is sweet' and 'lee ree mee' means 'He is intelligent', which word in that language means 'sweet'?
 A. see
 B. kee
 C. lee
 D. pee
 E. tee
13. In a certain code, SIKKIM is written as THLJJL.
 How is TRAINING written in that code?
 A. SQBHOHOH
 B. UQBHOHOF
 C. UQBJOHHO
 D. UQBJOHOOH
 E. None of these
14. If in a code, ALTERED is written as ZOGVIVW, then in the same code, RELATED would be written as
 A. IVOZGVW
 B. IVOZGWV
 C. IVOGZVW
 D. VIOZGVW
15. In a certain code, 'ile be pee' means 'roses are blue'; 'sik hee' means 'red flowers' and 'pee mit hee' means 'flowers are vegetables'. How is 'vegetables are red flowers' written in this code?
 A. pee sik mit hee
 B. sik pee hee be
 C. il sik mit hee
 D. Cannot be determined
 E. None of these
16. If in a certain language MYSTIFY is coded as NZTUJGZ, how is NEMESIS coded in that language?
 A. MDLHRDR
 B. OFNFTJT
 C. ODNHTDR
 D. PGOKUGU
17. In a certain code language, 743 means 'Mangoes are good', 657 means 'Eat good food' and 943 means 'Mangoes are ripe'. Which digit means 'ripe' in that language?
 A. 5
 B. 4
 C. 9
 D. 7
 E. None of these
18. In a certain code language,
 (A) 'mxy das zci' means 'good little frock';
 (B) 'jmx cos zci' means 'girl behaves good';
 (C) 'nvg drs cos' means 'girl makes mischief';
 (D) 'das ajp cos' means 'little girl fell'. Which of the given statements is superfluous to find the code for 'girl'?
 A. A

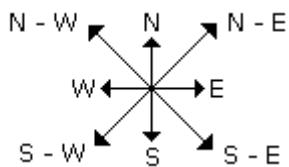
- B. B
C. C
D. D
E. None of these
19. In a certain code language, '3a, 2b, 7c' means 'Truth is Eternal';
'7c, 9a, 8b, 3a' means 'Enmity is not Eternal' and
'9a, 4d, 2b, 8b' means 'Truth does not perish'.
Which of the following means 'enmity' in that language?
A. 3a
B. 7c
C. 8b
- D. 9a
E. None of these
20. In a certain language, 36492 is written as SMILE and 058 is written as RUN.
How are the following figures coded in that language?
33980
A. SSLNR
B. SSLRN
C. SLSNR
D. Can't be determined
E. None of these

DIRECTIONS

There are four main directions - East, West, North and South as shown below:



There are four cardinal directions - North-East (N-E), North-West (N-W), South-East (S-E), and South-West (S-W) as shown below:



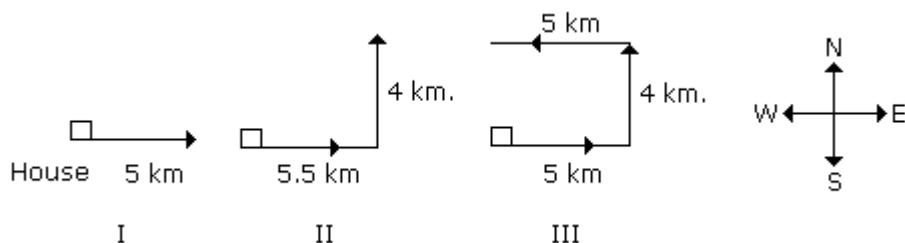
1. At the time of sunrise, if a man stands facing the east, his shadow will be towards west.
2. At the time of sunset the shadow of an object is always in the east.
3. If a man stands facing the North, at the time of sunrise his shadow will be towards his left and at the time of sunset it will be towards his right.
4. At 12:00 noon, the rays of the sun are vertically downward hence there will be no shadow.

Main types of questions are given below:

Type 1:

Siva starting from his house, goes 5 km in the East, then he turns to his left and goes 4 km. Finally he turns to his left and goes 5 km. Now how far is he from his house and in what direction?

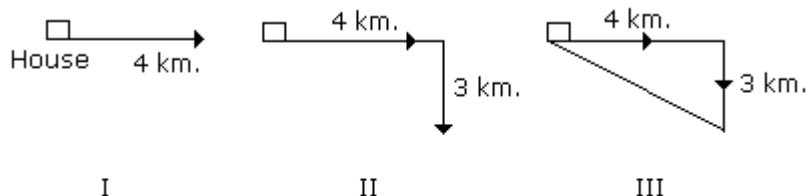
Solution:



From third position it is clear he is 4 km from his house and is in North direction.

Type 2:

Suresh starting from his house, goes 4 km in the East, then he turns to his right and goes 3 km. What minimum distance will be covered by him to come back to his house?

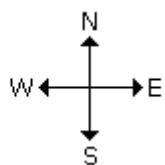
Solution:


$$\begin{aligned}
 \text{Minimum distance} &= \sqrt{(4)^2 + (3)^2} \\
 &= \sqrt{16 + 9} \\
 &= \sqrt{25} \\
 &= 5 \text{ km.}
 \end{aligned}$$

Type 3:

One morning after sunrise Juhi while going to school met Lalli at Boring road crossing. Lalli's shadow was exactly to the right of Juhi. If they were face to face, which direction was Juhi facing?

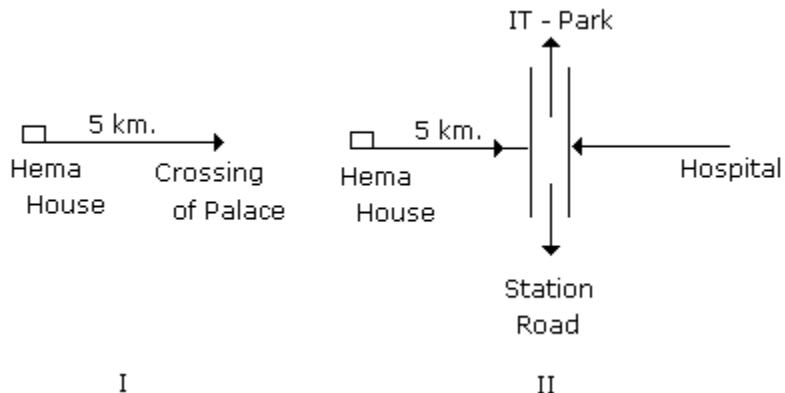
Solution: In the morning sunrises in the east.



So in morning the shadow falls towards the west. Now Lalli's shadow falls to the right of the Juhi. Hence Juhi is facing South.

Type 4:

Hema starting from her house walked 5 km to reach the crossing of Palace. In which direction she was going, a road opposite to this direction goes to Hospital. The road to the right goes to station. If the road which goes to station is just opposite to the road which goes to IT-Park, then in which direction to Hema is the road which goes to IT-Park?

Solution:

From II it is clear that the road which goes to IT-Park is left to Hema.

QUESTIONS:

1. From his house, Lokesh went 15 km to the North. Then he turned west and covered 10 km. Then he turned south and covered 5 km. Finally turning to the east, he covered 10 km. In which direction is he from his house?
 - A. East
 - B. West
 - C. North
 - D. South
2. A child went 90 m in the East to look for his father, and then he turned right and went 20 m. After this he turned right and after going 30 m he reached his uncle's house. His father was not there. From there he went 100 m to his north and met his father. How far did he meet his father from the starting point?
 - A. 80 m
 - B. 100 m
 - C. 140 m
 - D. 260 m
3. Shyam walks 5 km towards East and then turns left and walks 6 km. Again he turns right and walks 9 km. Finally he turns to his right and walks 6 km. How far is he from the starting point?
 - A. 26 km
 - B. 21 km
 - C. 14 km
 - D. 9 km
4. One morning after sunrise, Suresh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?
 - A. East
 - B. South
 - C. West
 - D. Data is inadequate
5. If the minute hand of a clock points towards west direction at 6 pm then in which direction does the hour hand points at 4.30 am?
 - A. North-East
 - B. South-West
6. Umesh directly went from P, to Q which is 9 feet distant. Then he turns to the right and walked 4 feet. After this he turned to the right and walked a distance which is equal from P to Q. Finally he turned to the right and walked 3 feet. How far is he now from P?
 - A. 6 feet
 - B. 5 feet
 - C. 1 feet
 - D. 0 feet
7. A man walks 2 km towards North. Then he turns to East and walks 10 km. After this he turns to North and walks 3 km. Again he turns towards East and walks 2 km. How far is he from the starting point?
 - A. 10 km
 - B. 13 km
 - C. 15 km
 - D. None of these
8. Some boys are sitting in three rows all facing North such that A is in the middle row. P is just to the right of A but in the same row. Q is just behind of P while R is in the North of A. In which direction of R is Q?
 - A. South
 - B. South-West
 - C. North-East
 - D. South-East
9. From his house, Lokesh went 15 kms to the North. Then he turned west and covered 10 kms. Then he turned South and covered 5 kms. Finally, turning to East, he covered 10 kms. In which direction is he from his house?
 - A. East
 - B. North
 - C. West
 - D. South

10. I am facing South. I turn right and walk 20 metre. Then I turn right again and walk 10 metre. Then I turn left and walk 10 metre and then turning right walk 20 metre. Then I turn right again and walk 60 metre. In which direction am I from the starting point?
- North-East
 - North-West
 - North
 - West
11. Meeru lost her way to home and was standing 25 meters away from her house in the south-west direction. She walks 20 metres north and reaches point A. How far and in which direction would she have to walk to reach her house?
- 20 metres, East
 - 15 metres, East
 - 15 metres, West
 - 20 metres, West
12. A person is walking from particular point A. he walks south 100km from point a and turns right and walk 40 km. Then he turns right and walk 70 km. how far that person from starting point?
- 25
 - 45
 - 50
 - 70
13. Starting from a point P, Sachin walked 20 metres towards South. He turned left and walked 30 metres. He then turned left and walked 20 metres. He again turned left and walked 40 metres and reached a point Q. How far and in which direction is the point Q from the point P?
- 30 metres, West
 - 10 metres, West
 - 30 metres, North
 - 10 metres, North
14. Rohit walked 25 metres towards South. Then he turned to his left and walked 20 metres. He then turned to his left and walked 25 metres. He again turned to his right and walked 15 metres. At what distance is he from the starting point and in which direction?
- 35 metre, North
 - 30 metre, South
 - 35 metre, East
 - 30 metre, North
15. A boy rode his bicycle Northward, then turned left and rode 1 km and again turned left and rode 2 km. He found himself 1 km west of his starting point. How far did he ride northward initially?
- 1 km
 - 2 km
 - 3 km
 - 5 km
16. A dog runs 20 metre towards East and turns Right, runs 10 metre and turns to right, runs 9 metre and again turns to left, runs 5 metre and then turns to left, runs 12 metre and finally turns to left and runs 6 metre. Now which direction dog is facing?
- East
 - North
 - West
 - South
17. A man is facing north. He turns 45 degree in the clockwise direction and then another 180 degree in the same direction and then 45 degree in the anticlockwise direction. Find which direction he is facing now?
- North
 - East
 - West
 - South
18. Ashok started walking towards South. After walking 50 metre, he took a right turn and walked 30 metre. He then took a right turn and walked 100 metre. He again took a right turn and walked 30 metre and stopped. How far and in which direction was he from starting point?
- 50 metre South
 - 150 metre North
 - 180 metre East
 - 50 metre North

19. Gaurav walks 20 metres towards North. He then turns left and walks 40 metres. He again turns left and walks 20 metres. Further, he moves 20 metres after turning to the right. How far is he from his original position?
- A. 40 metres
 - B. 50 metres
 - C. 60 metres
 - D. 70 metres
20. Kunal walks 10 km towards North. From there he walks 6 Km towards South. Then, he walks 3 Km towards east. How far and in which direction is he with reference to his starting point?
- A. 5 Km North
 - B. 5 Km South
 - C. 5 Km East
 - D. 5 Km North-East

SEATING ARRANGEMENTS

The process of making a group of people to sit as per a prefixed manner is called "seating arrangement". In these questions, some conditions are given, on the basis of which students are required to arrange objects, either in a row or in a circular order. While making arrangements, it should be noted that all the conditions given are complied with.

In order to solve seating arrangement questions, first of all, the correct diagram should be made. By doing so, questions are easily and quickly solved.

Here are a few guidelines to be followed to solve seating arrangement problems:

- 1) First of all take a quick glance at the given information. After performing this step, you would get an idea of the situation of people or objects.
- 2) Next, determine the usefulness of the given information and classify them accordingly into 'definite information', 'comparative information' and 'negative information'.
- 3) When the place of any object or person is definitely mentioned, then we can say that it is **definite information**. E.g.) A is sitting on the right end of the bench.
- 4) When the place of any person or object is not mentioned definitely but mentioned only in comparison of another person or object, then we can say it is **comparative information**. E.g.) A is sitting second to the right of E. This type of information can be helpful when we get the definite information about E.
- 5) A part of definite information may consist of **negative information**. Negative information does not tell us anything definitely but it gives an idea to eliminate a possibility. E.g.) C is not sitting on the immediate left of A.

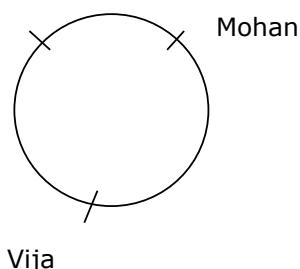
Example 1:

1. 6 Boys are sitting in a circle and facing towards the centre of the circle.
2. Rajeev is sitting to the right of Mohan but he is not just at the left of Vijay.
3. Suresh is between Babu and Vijay.
4. Ajay is sitting to the left of Vijay. Who is sitting to the left of Mohan?

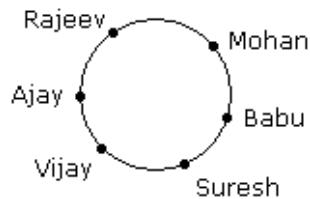
Solution:

Here people are sitting in a circle facing towards the centre, so we arrange the boys in such a way that when we move from the person's left to right side, we will be in the anti-clockwise direction. If the boys were arranged facing away from the centre, then when we move from the person's left to right side, we will be in the clockwise direction.

From the given statement (2) above, we can fix the position of Mohan. Rajeev is somewhere to the right of Mohan. It is given that Rajeev is not just at the left of Vijay but somewhere to the left.



From statements (3) and (4), we have Suresh sitting between Babu and Vijay and Ajay sitting to the left of Vijay.



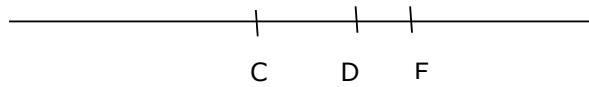
Hence, Babu is sitting to the left of Mohan.

Example 2:

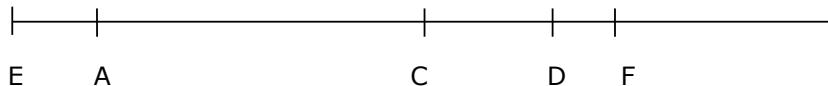
1. Eleven students A, B, C, D, E, F, G, H, I, J and K are sitting in first line facing to the teacher.
 2. D, who is just to the left of F, is to the right of C at second place.
 3. A is second to the right of E who is at one end.
 4. J is the nearest neighbor of A and B and is third to the left of G.
 5. H is next to D to its left and is at the third place to the right of I.
- Who is just in the middle?

Solution:

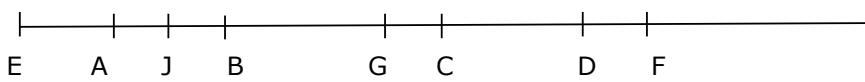
We suppose the students are facing the teacher in the North direction. From statement (2), we have,



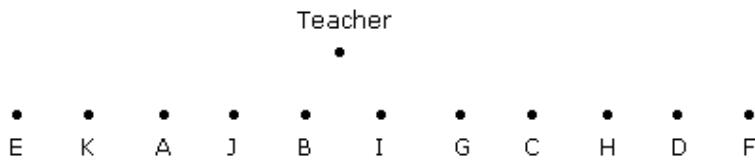
Since A is to the right of E and E is at one of the ends, clearly E cannot be at the right end.



From statement (4), we have,



From statement (5), we get the positions of the remaining boys.

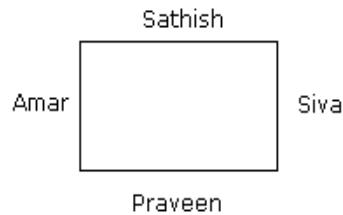


Hence, I is just in the middle.

Example 3:

Siva, Sathish, Amar and Praveen are playing cards. Amar is to the right of Sathish, who is to the right of Siva. Who is to the right of Amar?

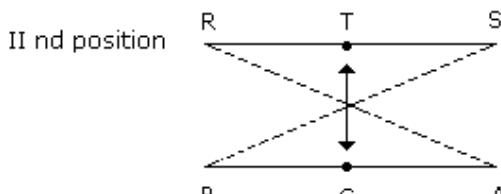
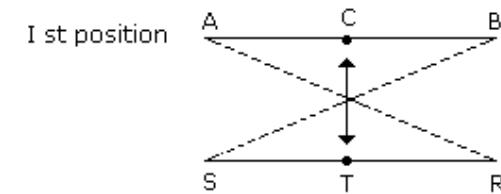
Solution:



Hence Praveen is to the right of Amar.

Example 4:

1. A, B and C are three boys while R, S and T are three girls. They are sitting such that the boys are facing the girls.
 2. A and R are diagonally opposite to each other.
 3. C is not sitting at any of the ends.
 4. T is left to R but opposite to C.
- (A) Who is sitting opposite to B?
 (B) Who is sitting diagonally opposite to B?



Solution:

- (A) Hence, R is sitting opposite to B.
 (B) Hence, S is sitting diagonally opposite to B.

QUESTIONS

SAMPLE: Six girls are sitting in a circle facing to the centre of the circle. They are P, Q, R, S, T and V. T is not between Q and S but some other one. P is next to the left of V. R is 4th to the right of P. Which of the following statement is not true?

- A. S is just next to the right to R
 - B. T is just next to the right of V
 - C. R is second to the left of T
 - D. P is second to the right of R
1. If P and R interchange their positions then which of the following pair will sit together?
 - A. RT
 - B. PV
 - C. VT
 - D. QV
 2. What is the position of T?
 - A. Just next to the right of Q
 - B. Second to the left of P
 - C. Between Q and R
 - D. To the immediate right of V
 3. Which one is sitting just right to the V?
 - A. P
 - B. T
 - C. R
 - D. S/Q
- A, B, C, D, E, F and G are sitting in a row facing North:
 F is to the immediate right of E.
 E is 4th to the right of G.
 C is the neighbour of B and D.
 Person who is third to the left of D is at one of ends.
4. Who are to the left of C?
 - A. Only B
 - B. G, B and D
 - C. G and B
 - D. D, E, F and A
 5. Which of the following statement is not true?

- A. E is to the immediate left of D
- B. A is at one of the ends
- C. G is to the immediate left of B
- D. F is second to the right of D

6. Who are the neighbours of B?
 - A. C and D
 - B. C and G
 - C. G and F
 - D. C and E

7. What is the position of A?
 - A. Between E and D
 - B. Extreme left
 - C. Centre
 - D. Extreme right

In an Exhibition, seven cars of different companies - Cadillac, Ambassador, Fiat, Maruti, Mercedes, Bedford and Fargo are standing facing to east in the following order:

- i) Cadillac is next to right of Fargo.
- ii) Fargo is fourth to the right of Fiat.
- iii) Maruti car is between Ambassador and Bedford.
- iv) Fiat which is third to the left of Ambassador is at one end.

8. Which of the cars are on both the sides of Cadillac car?
 - A. Ambassador and Maruti
 - B. Maruti and Fiat
 - C. Fargo and Mercedes
 - D. Ambassador and Fargo
9. Which of the following statement is correct?
 - A. Maruti is next left of Ambassador.
 - B. Bedford is next left of Fiat.
 - C. Bedford is at one end.
 - D. Fiat is next second to the right of Maruti.
10. Which one of the following statements is correct?

- A. Fargo car is in between Ambassador and Fiat.
 B. Cadillac is next left to Mercedes car.
 C. Fargo is next right of Cadillac.
 D. Maruti is fourth right of Mercedes.
11. Which of the following groups of cars is to the right of Ambassador?
 A. Cadillac, Fargo and Maruti
 B. Mercedes, Cadillac and Fargo
 C. Maruti, Bedford and Fiat
 D. Bedford, Cadillac and Fargo
12. Which one of the following is the correct position of Mercedes?
 A. Next to the left of Cadillac
 B. Next to the left of Bedford
 C. Between Bedford and Fargo
 D. Fourth to the right of Maruti.
13. There are five different houses, A to E in a row. A is to the right of B and E is to the left of C and right of A. B is to the right of D.
 Which of the houses is in the middle?
 A. A
 B. B
 C. D
 D. E
14. Four girls are sitting on a bench to be photographed. Shikha is to the left of Reena. Manju is to the right of Reena. Rita is between Reena and Manju.
 Who would be second from the left in the photograph?
 A. Reena
 B. Shikha
 C. Manju
 D. Rita
15. Five children are sitting in a row. S is sitting next to P but not T. K is sitting next to R who is sitting on extreme left and T is not sitting next to K. Who are sitting adjacent to S?
- A. K and P
 B. R and P
 C. only C
 D. P and T
 E. Insufficient
- A, B, C, D, E, F and G are sitting on a wall and all of them are facing east.
 • C is on the immediate right to D.
 • B is at an extreme end and has E as his neighbour.
 • G is between E and F.
 • D is sitting third from the south end.
16. Who is sitting to the right of E?
 A. A
 B. C
 C. D
 D. F
 E. None of these
17. Which of the following pairs of people are sitting at the two extreme ends?
 A. AB
 B. AE
 C. CB
 D. FB
 E. Cannot be determined
18. Name the person who is sitting third from the North end?
 A. E
 B. F
 C. C
 D. G
19. Which among the following pairs have the persons sitting adjacent to each other?
 A. AC
 B. AF
 C. CE
 D. CF
 E. None of these

DATA SUFFICIENCY

Data Sufficiency questions test your knowledge of basic math facts and skills coupled with reasoning, analytical and problem solving abilities. Each Data Sufficiency item presents you with a question where you need to decide whether or not the information presented along with the question would be sufficient to answer the question.

What makes Data Sufficiency questions different?

The challenge in DS questions, as they are popularly called, is not question solving but rather establishing whether the question has a solution or not. A special array of five answer choices is provided, each of which categorizes the relationship between the question and the information provided in a different way. You must select the answer choice that describes this relationship accurately.

Let's have a cursory look at these answer options which generally feature in this question type

Give answer (A) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

Give answer (B) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.

Give answer (C) if the data either in Statement I or in Statement II alone are sufficient to answer the question.

Give answer (D) if the data even in both Statements I and II together are not sufficient to answer the question.

Give answer (E) if the data in both Statements I and II together are necessary to answer the question.

Note:

Students often confuse the different answer options, and end up marking the incorrect choice. Always double check whether you are marking the correct option, and do not assume that the examiner would always present the options in a default order. Go through the answer options to check whether the order of statements is as expected.

Example 1:

Is the product of two numbers greater than 100?

The sum of the two numbers is greater than 50.

Each of the numbers is greater than 10.

Solution:

Let us analyze the individual statements first and then take them together.

Statement A alone is not sufficient to answer the question and this can be proved by examples. If the two numbers are 30 and 31, their sum is greater than 50 and their product is greater than 100; but if the two numbers are 50 and 1, though their sum is greater than 50, their product is only 50, and less than 100.

Statement B is sufficient. If both of the numbers are greater than 10, then their product must be greater than 10×10 , or greater than 100.
Hence only second statement is sufficient to solve the question. Option B is the answer.

Example 2:

Is x a prime number? $91 < x < 97$
 x is a factor of 121

Solution:

Here the first statement is sufficient to answer the question as we see that there is no prime number between $91 < x < 97$. Hence ' x ' is not a prime number.

In second statement, the factors of 121 are 1, 11 and 121. Here 1 and 121 are not prime numbers whereas 11 is a prime number. Hence in this case ' x ' may or may not be a prime number.

Hence, only the first statement is sufficient to solve the question. Option A is the answer.

Note: Remember, even if a question has an answer as 'no', even then it is a valid answer.

Example 3:

Is $x = -5$? $x^2 = 36$ x is a natural number.

Solution:

Here the question directly asks whether x is equal to -5 or not.

From statement A, we have $x = 6$ or -6 . In both the cases x is not equal to -5. Hence first statement is sufficient to get the answer.

Statement B says that x is a natural number. Since x is a natural number, it cannot be negative. Hence it is not equal to -5. So, the second statement is also sufficient to solve the question.

Hence, both statements are independently sufficient to answer the question.

Option C is the answer.

How ready are you in Data Sufficiency now? Take this test and find the answer.

Note:

To conclude, it is very important to read the question carefully in the case of Data Sufficiency questions.

One major mistake committed by a number of students is that when the answer has to be yes/no and normally whenever you get the answer as no, you mark the answer as insufficient.

Remember: 'NO' is also an answer for Data Sufficiency questions.

Practice Questions -Set I

1. How is Sulekha related to Nandini?

Statements:

- I. Sulekha's husband is the only son of Nandini's mother.
- II. Sulekha's brother and Nandini's husband are cousins.
- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

2. What is Gagan's age?

Statements:

- I. Gagan, Vimal and Kunal are all of the same age.
- II. Total age of Vimal, Kunal and Anil is 32 years and Anil is as old as Vimal and Kunal together.
- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

3. In which year was Rahul born?

Statements:

- I. Rahul at present is 25 years younger to his mother.
- II. Rahul's brother, who was born in 1964, is 35 years younger to his mother.
- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

4. What will be the total weight of 10 poles, each of the same weight?

Statements:

- I. One-fourth of the weight of each pole is 5 kg.

II. The total weight of three poles is 20 kilograms more than the total weight of two poles.

I alone is sufficient while II alone is not sufficient

- A. II alone is sufficient while I alone is not sufficient
- B. Either I or II is sufficient
- C. Neither I nor II is sufficient
- D. Both I and II are sufficient

5. How many children does M have?

Statements:

- I. H is the only daughter of X who is wife of M.
- II. K and J are brothers of M.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

6. How much was the total sale of the company?

Statements:

- I. The company sold 8000 units of product Each costing Rs. 25.

II. This company has no other product line.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

7. The last Sunday of March, 2006 fell on which date?

Statements:

- I. The first Sunday of that month fell on 5th.

II. The last day of that month was Friday.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

8. What is the code for 'sky' in the code language?

Statements:

I. In the code language, 'sky is clear' is written as 'de ra fa'.

II. In the same code language, 'make it clear' is written as 'de ga jo'.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

9. How many children are there between P and Q in a row of children?

Statements:

I. P is fifteenth from the left in the row.

II. Q is exactly in the middle and there are ten children towards his right.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

10. How is T related to K?

Statements:

I. R's sister J has married Ts brother L, who is the only son of his parents.

II.K is the only daughter of L and J.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

Practice Questions -Set II

1. In a certain code, '13' means 'stop smoking' and '59' means 'injurious habit'. What do '9' and '5' mean respectively in that code?

Statements:

I. '157' means 'stop bad habit'.

II. '839' means 'smoking is injurious'.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

2. Who is to the immediate right of P among five persons P, Q, R, S and T facing North? Statements:

I. R is third to the left of Q and P is second to the right of R.

II. Q is to the immediate left of T who is second to the right of P.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

3. On which date of the month was Anjali born in February 2004?

Statements:

I. Anjali was born on an even date of the month.

II. Anjali's birth date was a prime number.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

4. How is X related to Y? Statements:

I. Y says, "I have only one brother".

II. X says, "I have only one sister".

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

5. How is F related to P? Statements:

I. P has two sister's M and N.

II. F's mother is sister of M's father.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

6. Among Monika, Anita, Sonal, Ratna and Tanvy, who came last for the program? Statements:

I. Monika came after Anita but not after Tanvy.

II. Ratna came after Tanvy but not after Sonal.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

7. Who among P, Q, R, S and T is the lightest?

Statements:

I. R is heavier than Q and T but lighter than S.

II. S is not the heaviest.

- A. I alone is sufficient while II alone is not sufficient
- B. II alone is sufficient while I alone is not sufficient
- C. Either I or II is sufficient
- D. Neither I nor II is sufficient
- E. Both I and II are sufficient

8. How is T related to K? Statements:

- I. K has two sons; one of the sons is A.
- II The mother of T has only two sons-A and B.
 - A. I alone is sufficient while II alone is not sufficient
 - B. II alone is sufficient while I alone is not sufficient
 - C. Either I or II is sufficient
 - D. Neither I nor II is sufficient
 - E. Both I and II are sufficient

9. What is the shortest distance between Devipur and Durgapur?

Statements:

- I. Durgapur is 20 kms away from Rampur.
- II. Devipur is 15 kms away from Rampur.
 - A. I alone is sufficient while II alone is not sufficient
 - B. II alone is sufficient while I alone is not sufficient
 - C. Either I or II is sufficient
 - D. Neither I nor II is sufficient
 - E. Both I and II are sufficient

10. Which word in the code language means 'flower'?

Statements:

- I. 'de fu la pane' means 'rose flower is beautiful' and 'la quiz' means 'beautiful tree'.
- II. 'de la chin' means 'red rose flower' and 'pa chin' means 'red tea'.
 - A. I alone is sufficient while II alone is not sufficient
 - B. II alone is sufficient while I alone is not sufficient
 - C. Either I or II is sufficient
 - D. Neither I nor II is sufficient
 - E. Both I and II are sufficient

ONE WORD SUBSTITUTES

Important points to remember while answering the One Word Substitutes questions

The only way out for this section is by building up your vocabulary because the section asks you to replace group of words or a full sentence effectively without creating any kind of ambiguity in the meaning of the sentences. Like the word "autobiography" can be used in the place of the sentence "The life story of a man/woman written by himself/herself"

Here are some tips and techniques:

How to learn a new word

There are 3 stages of understanding when you learn a new word.

First, you learn the meaning of the word and how it is used. Repeated exposure to the word develops familiarity with the word.

Then you can differentiate this word from similar words. Usually words may seem similar but there is a difference in the context in which it is employed.

Finally, you have accumulated a word in your vocabulary when you can recollect the word and use it in your speech or writing.

Why is it important to learn a word thoroughly?

Let's look at 3 words and how they differ and why it is important to know more than just the meaning.

Wisdom is the knowledge of an individual learnt from life or learnt collectively by a group. For example, the wisdom of native American communities, or the wisdom of the Sufi sect.

Erudition is academic knowledge. For example, Prof. Shetty's erudition in Economics makes him a front runner for the Nobel Prize.

Acumen is knowledge gained through practice of a profession. For example - a person with a strong business, political or financial acumen.

Now imagine solving Fill in the Blanks/One Word Substitutes armed with such knowledge and confidence.

How to build a vocabulary

1. Read

Regular reading increases exposure to words. We recommend you read editorials, essays and opinion pages of prominent new papers and magazines, many of which are easily available online of which are easily available online.

2. Use the words

Creating sentences of your own reinforces your understanding.

3. Enjoy

Though you may be developing your vocabulary for company tests or competitive exams, remember that a good vocabulary is a permanent asset.

If you are not regularly using or revising the new words you've learned, you will forget them. There is no substitute for regular revision. Create flash cards for yourself as you learn or download wordlists on your mobile phone and learn on the move.

QUESTIONS

1. Study of earth.
 a) Seismology c) Geology
 b) Earthquake d) Histology
2. One who pretends to be what he is not.
 a) Robber c) Fraud
 b) Hypocrite d) Fallible
3. Worship of idols.
 a) Idolatry c) Pessimist
 b) Optimist d) Topiary
4. A person who is made to bear the blame due to others
 a) Omnipresent c) Anonymous
 b) Scapegoat d) Epilogue
5. That which cannot be conquered.
 a) Indestructible c) Invincible
 b) Indefensible d) Ambiguous
6. A person who speaks less.
 a) Garrulous c) Dandy
 b) Blabbermouth d) Reticent
7. The act or habit of talking in one's sleep.
 a) Somniloquy c) Somnambulism
 b) Stammer d) Monogamy
8. A man who is always ready to help others.
 a) Civilized c) Kind
 b) Samaritan d) Belligerent
9. A person who is too stubborn to admit.
 a) oblique c) obdurate
 b) determined d) fascinated
10. A person who is generous and has an exalted soul
 a) magnanimous c) miser
 b) stingy d) frugal
11. Be the embodiment or perfect example of
 a) Characterize c) Idol
12. A person interested in reading books and nothing else
 a) Book-keeper c) Scholar
 b) Book-worm d) Student
13. Something that relates to everyone in the world
 a) General c) Common
 b) Usual d) Universal
14. A style full of words
 a) Verbose c) Pedantic
 b) Rhetorical d) Abundant
15. To issue a thunderous verbal attack
 a) Languish c) Animate
 b) Fulminate d) Invigorate
16. Look at with fixed eyes.
 a) Stare c) Stardust
 b) Starch d) Stiff
17. Temporary loss of strength and energy resulting from hard physical or mental work.
 a) Devolve c) Degenerate
 b) Fade d) Fatigue
18. Characterized by tolerance and mercy
 a) Lenient c) Honest
 b) Obedient d) Confident
19. Convert ordinary language into code
 a) Encrypt c) Encrust
 b) Beset d) Harass
20. A component of a mixture or compound
 a) Mixture c) Alloy
 b) Ingredient d) Solvent
21. Long spell of dry weather
 a) Stare c) Survive
 b) Drought d) Stale
22. Express approval by clapping
 a) Applaud c) Cripple
 b) Repeatedly d) Century

23. Put up with something or somebody unpleasant
a) Allow c) Endure
b) Resist d) Defy
24. Person who is insane or very foolish
a) Crawl c) Lunatic
b) Invincible d) Habitual
25. Unmistakably clear; visibly clear; in an evident manner
a) Noticeable c) Detected
b) Perceived d) Obviously
26. Keep oneself away from
a) Notorious c) Avoid
b) Youth d) Pedestrian
27. Able to bend easily without breaking
a) Breakable c) Discover
b) Shout d) Flexible
28. Way out
a) Exit c) Entry
b) Abnormal d) Portable

29. Send goods to another country for sales
a) Export c) Abort
b) Assort d) Travel
30. One who looks on the bright side of things
a) Pessimist c) Optimist
b) Atheist d) Believer
31. The act of making a strong public expression of disagreement and disapproval
a) Aver c) Protest
b) Assert d) Insist
32. Google's operating system for mobile platform
a) iOS c) Window8
b) MAC d) Android
33. Period of 10 years
a) Century c) SilverJubilee
b) Decade d) Stellar
34. Succeed in doing
a) Accomplish c) Scream
b) Haggle d) Reveal

READING COMPREHENSION

What is Reading comprehension?

Reading comprehension is the ability to read text, process it and understand its meaning. An individual's ability to comprehend text is influenced by their traits and skills, one of which is the ability to make inferences.

Why Reading comprehension is important?

RC is very important as far as placement tests are concerned because, how well the candidate understands the language is measured. Without comprehension, reading is simply following words on a page from left to right while sounding them out. The words on the page have no meaning. And while people read for many different reasons, the chief goal is to derive some understanding of what the writer is trying to convey and make use of that information – whether for fact gathering, learning a new skill, or for pleasure. That's why reading comprehension skills are so important

Different type of passages

Average length of a passage is around 300-400 words and most probably passages are unfamiliar and unfriendly to the test taker. Passages might deal with events, issues, ideas, person, place etc. Passages may be about the physical or biological sciences, social science, the humanities (history, art, archaeology), or business topics, such as economic models, marketing strategies, or human resource theories.

Types of Questions

Countable questions are asked along with passages. There are two types of questions referential and inferential questions. As the name suggests referential means answer is taken directly from passage. Inferential queries are skeptical because answer is found out with the help of clues. Proper understanding of the entire passage will give way to inferential questions.

Strategies

1. Never read aloud, it decreases speed reading capability.
2. Avoid using supports like pen or pencil while reading.
3. Avoid recursive reading.
4. Follow balanced approach to understand which portion of the passage speaks what.
5. Never worry about the meaning of complicated words. Understand the meaning based on the context of the sentence.
6. Give suitable main title and subtitle to every paragraph to understand the flow of the passage.

7. Understand the key points and summarize them.

Concept of Eye Span

Eye span plays vital role in reading."Eye-span" is referred to the amount of text someone takes in with the eyes for each stopping, or "fixation" of the eyes or the number of words a reader can grasp in a single glance. News publishers know the concept of eye span; hence all readers can quickly read the articles on the newspaper. All news is published in rectangular shape boxes with minimum width and length. Read newspapers for better eye span.

Sample passage

Speech is great blessings but it can also be great curse, for while it helps us to make our intentions and desires known to our fellows, it can also if we use it carelessly, make our attitude completely misunderstood. A slip of the tongue, the use of unusual word, or of an ambiguous word, and so on, may create an enemy where we had hoped to win a friend. Again, different classes of people use different vocabularies, and the ordinary speech of an educated may strike an uneducated listener as pompous. Unwittingly, we may use a word which bears a different meaning to our listener from what it does to men of our own class. Thus speech is not a gift to use lightly without thought, but one which demands careful handling. Only a fool will express himself alike to all kinds and conditions to men.

What is the theme of the passage? (Inferential question)

Give suitable title for the passage? (Inferential question)

What is the author's opinion about a fool person? (Referential question)

What is the author's tone? (Inferential question)

What is the conclusion of the passage? (Referential Question)

Key points:

- Speech is a blessing as well as a great curse.
- Consequences of careless speech.
- Careful handling is required for speech.

Whenever a passage is read, make sure key points are noted. All the key points will lead to the flow of the passage. No subtitles for this passage since there is only one paragraph.

Complicated words from the above passage: ambiguous, pompous, unwittingly
Most probably meanings of complicated words are not required for understanding the whole passage.

Actual meanings of words:

Unwittingly: unintentional or accidental

Ambiguous: open to more than one interpretation, not clear

Pompous: Self important

There is an unusual scenario where one of the questions would be to select the suitable meaning of any word from the passage.

What is the closest meaning of the word "ambiguous"?

Options:

- a. Intentions
- b. Carelessness
- c. Unusual

Solution:

A slip of the tongue, the use of unusual word, or of an ambiguous word..... (3rd line of this passage)

Here ambiguous word is connected to the use of unusual word. So it's easy to infer the meaning is unusual. (Note: This is not the exact meaning, meaning inferred from the passage)

What is the theme of the passage? (Inferential question)

Theme should be one common point which flows throughout the passage.

Options:

- A. Speech is great blessings but it can also be great curse.
- B. Different classes of people use different vocabularies.
- C. Careful handling is required for speech.

Solution: Option C is the most appropriate answer.

Here comes the importance of Key points. The flow of the passage is about the consequences of careless speech.

Options A and B are just parts of the passage and hence the answer.

Give suitable title for the passage? (Inferential question)

Title and theme of a passage are almost same. A title should be one common point which is discussed throughout the passage.

What is the author's opinion about a fool person? (Referential question)

The answer is in last line of the passage. "A fool will express himself alike to all kinds and conditions to men".

What is the author's tone? (Inferential question)

This is the most commonly asked question. Understand the tone of author throughout the paragraph.

Options:

1. Assertive
2. Ambiguous
3. Skeptical

Solution: Option 1.

Author gives clear explanations about careless speech and its consequences. So definitely author is not ambiguous and confused about the content.

What is the conclusion of the passage? (Referential Question)

Look for sign post clue words like thus, therefore, at last, finally and these words lead to conclusion.

Solution: Second last line of the paragraph "Thus speech is not a gift to use lightly without thought, but one which demands careful handling."

QUESTIONS

Directions: In the given passages below, Read through the passage and choose ONE answer for each question

PASSAGE 1:

Male lions are rather reticent about expanding their energy in hunting more than three quarters of kills are made by lionesses are in front, tensely scanning ahead, the cubs lag playfully behind and the males bring up the rear, walking slowly, their massive heads nodding with each step as if they were bored with the whole matter. But slothfulness may have survival value. With lionesses busy hunting, the males function as guard for the cubs, protecting them particularly from hyenas.

Questions

1. According to the passage male lions generally do not go for hunting because

a. They don't like it.

b. they want lioness to get training

c. they wish to save their vigor for other things

d. they are very lazy

2.

Male lions protect their cubs

a. from the members of their own species

b. from hyenas only

c. from hyenas as much as from other enemies

d. more from hyenas than from other animals

3. Lioness go for hunting

a) All alone.

b) With their male partners only.

c) With their cubs and male partners

d) With their cubs only

4. When the lionesses go in search for their prey, they are very

a) Cautious.

b) Playful.

c) Sluggish.

d) Serious

PASSAGE 2:

In 1995 the criminologist James Alan Fox wrote a report for the U.S. attorney general that grimly detailed the coming spike in murders by teenagers. Fox proposed optimistic and pessimistic scenarios. In the optimistic scenario, he believed, the rate of teen homicides

would rise another 15 percent over the next decade; in the pessimistic scenario, it would more than double. "The next crime wave will get so bad," he said, "that it will make 1995 look like the good old days."

Other criminologists, political scientists, and similarly learned forecasters laid out the same horrible future, as did President Clinton. "We know we've got about six years to turn this juvenile crime thing around," Clinton said, "or our country is going to be living with chaos. And my successors will not be giving speeches about the wonderful opportunities of the global economy; they'll be trying to keep body and soul together for people on the streets of these cities." The smart money was plainly on the criminals.

Questions

5. What is the writer's main purpose in writing this text?
 - a) To know how other criminologists react to this situation
 - b) To write a report to U.S attorney.
 - c) To make U.S. attorney understand about the pessimistic scenario of teen homicides.
6. Who is James Alan Fox?
 - a) One of the members of U.S. attorney association.
 - b) Generalist.
 - c) Criminologist.
7. "The next crime wave will get so bad"
What does Fox mean?
 - a) Acknowledges about teen homicide.
 - b) The situation of U.S. over the past decade.
 - c) Ignorance of homicide rate leads to horrible future.
8. What is the primary response of the President?
 - a) Assertive
 - b) Confused about living condition of people
 - c) He gave solutions.
9. What was the solution given by the President?
 - a) To stop the juvenile thing around.
 - b) His inheritors work hard for the common people.
 - c) Spending smart money for Criminals.

PASSAGE 3:

Proactivity is based on the unique human endowment of self-awareness. The two additional unique human endowments that enable us to expand our proactivity and to exercise personal leadership in our lives are imagination and conscience.

Through imagination, we can visualize the uncreated worlds of potential that lie within us.

Through conscience, we can come in contact with universal laws or principles with our own singular talents and avenues of contribution, and with the

personal guidelines within which we can most effectively develop them. Combined with self-awareness, these two endowments empower us to write our own script. Because we already live with many scripts that have been handed to us, the process of writing our own script is actually more a process of "re-scripting," or Paradigm Shifting -- of changing some of paradigms that we already have. As we recognize the ineffective scripts, the incorrect or incomplete paradigms within us, we can proactively begin to re-script ourselves.

I think one of the most inspiring accounts of the re-scripting process comes from the autobiography of Anwar Sadat, past president of Egypt. Sadat had been reared, nurtured, and deeply scripted in a hatred for Israel. He would make the statement on national television, "I will never shake the hand of an Israeli as long as they occupy one inch of Arab soil. Huge crowds all around the country would chant, "Never, never, never!" He marshaled the energy and unified the will of the whole country in that script.

Questions

10. Give suitable title for the passage?
 - a) Importance of Re-Scripting.
 - b) Power of imagination and conscience.
 - c) Proactivity.

11. Why did author mention about autobiography of Anwar Sadat?
 - a) To show Sadat's attitude towards Israeli
 - b) To mention re-scripting process
 - c) To point out the response of huge crowd.

12. Infer the meaning of Proactivity
 - a) Human endowment.
 - b) Personal Leadership
 - c) Self initiated behavior
 - d) Re-scripting.

14. What does one do to write script?
 - a) Change existing paradigms.
 - b) Using universal laws and personal guidelines.
 - c) Using self awareness, imagination and conscience.

PASSAGE 4:

Organizations are institutions in which members compete for status and power. They compete for the resources of the organization, for example, finances to expand their own departments, for career advancement and for power to control the activities of others. In pursuit of these aims, groups are formed and sectional interests emerge. As a result, policy decisions may serve the ends of career and political systems rather than those of organization.

In this way, the goals of the organization may give way to favor sectional interests and individual ambitions. These preoccupations sometimes prevent the emergence of organic

systems. Many of the electronic firms in the study had recently created research and development departments employ highly qualified and well paid scientists and technicians. Their high pay and expert knowledge were sometimes seen as a threat to the established order of rank, power and privilege. Many senior managers had little knowledge of technicality and possibilities of new developments and electronics. Some felt that close cooperation with the experts in an organic system would reveal their ignorance and show their experience was now redundant.

Questions

15. The theme of the passage?

- a) Groupism in organizations
- b) Individual ambitions in organizations.
- c) Frustration of senior managers.
- d) Short comings of established order of rank, power and privilege

16. Members in an organization contend with each other for all the following reasons except:

- a) Authority to regulate the functions of other members in the organization.
- b) Progress in the careers of the members.
- c) Gaining expert knowledge of technicality and possibilities of new development.
- d) Capital for expansion of departments

17. Policy decisions in organization may involve

- a) Cooperation at all levels.
- b) Assisting the objectives of political and career structures in the organization.
- c) Keeping in view the larger objectives of the organizations

18. Author makes out a case for

- a) Organic systems
- b) Research and development.
- c) An understanding between senior and middle level executives.
- d) A refresher course for senior staff.

19. The author tends to look at the senior manager as

- a. ignorant and incompetent
- b. a little out of step with their work environment
- c. jealous of their younger colleagues

SENTENCE CORRECTION

Important points to remember while answering the Sentence Correction section

The Sentence Correction section of any test can be intimidating, especially for test-takers who grew up speaking a language other than English. Luckily there are some tricks to help you out. There are specific words and phrases that you can use to eliminate options, and you can learn how different constructions must fit together in order to form a "correct" sentence.

The directions for these questions look like this:

Directions: The following questions consist of sentences that are either partly or entirely underlined. Below each sentence are five versions of the underlined portion of the sentence. Choice (A) is a copy of the original version. The four other answer choices change the underlined portion of the sentence. Read the sentence and the five choices carefully and select the best version.

Choose answers according to the norms of **standard written English** for grammar, word choice, and sentence construction. Your selected answer should express the intended meaning of the original sentence as **clearly and precisely** as possible, while avoiding ambiguous, awkward, or unnecessarily wordy constructions.

Sentence Correction Tips

1. Sentence Correction adheres to the rules of "Standard Written English"

"Standard Written English" refers to the grammar rules that you find in grammar books and in formal writing. Since proper written English often differs from spoken English, the best answer will not always be the one that sounds the best. You cannot rely on your ear alone; you must become familiar with the grammar rules of written English.

2. You will be tested on a limited number of grammar rules.

English grammar contains hundreds of very specific rules. Only a few of these will be tested, so devote your energies to mastering the rules that most frequently come up.

3. Grammar is key - but style is important, too

The best answer must be clear, without unnecessary redundancy, and with proper punctuation. Idioms must be used correctly. Style is a secondary concern. Look for grammar errors first, and then check for errors in style.

4. Don't change the meaning of the sentence

In the sentence correction section, you'll sometimes find two answer choices that are equally correct in terms of grammar and style conventions. When this happens, choose the

answer that best maintains the meaning of the original sentence. The correct answer will never significantly alter the original meaning.

5. Incorrect answer choices are incorrect

Sentence Correction answer choices are variations on the correct answer. Incorrect answers will almost always be identifiable as such. Even if an answer choice sounds funny, if you can't find a definite error, then don't rush to eliminate it.

Sentence Correction Three-Step Method

A Sentence Correction question looks like this:

1. When Roopa goes to the park, she likes to run, swim, and to play basketball.

- A. *she likes to run, swim, and to play basketball*
- B. *she likes to run, swim, and play basketball*
- C. *she likes running, to swim, and to play basketball*
- D. *she likes running, swimming, and to play basketball*
- E. *she likes all of the following, to run, swim, and to play basketball*

You are given *a sentence with one section underlined*, and five **answer choices**. The underlined portion is reproduced five different ways in the answer choices. Your task is to find the answer choice which is most grammatically correct according to the rules of Standard Written English. Sometimes more than one answer choice will appear to be free of grammatical errors. This is not a mistake. Style conventions must be taken into consideration as well. When this occurs, you must look for the answer that is clearly expressed and concise.

Three-Step Method to Sentence Correction questions:

1. READ

Read the entire sentence. Do not simply read the underlined part of the sentence, because context may be important in determining the correct answer. Don't worry about spelling, capitalization, or punctuation; they are not covered in Sentence Correction questions. If you do find an error in the underlined portion, or if you're not sure, proceed to step two.

2. DISSECT

only a limited number of grammar error types are tested. **After you've read the sentence, look for clues indicating which grammar rule the question is testing.** These grammar rules, and the clues to look for, will be covered in more detail in the next section.

Keep an eye out for:

Agreement Issues: Look for pronouns, verbs, and nouns: do they agree?

Modifiers: Look for introductory phrases set off by a comma: is the modifier used

correctly?

Parallels: Look for commas separating words in a list, as well as expressions such as *not only...but also, both...and, either...or, neither...nor*: is everything parallel?

3. COMPARE

after you've dissected the question, compare answer choices and note the differences. **Look for the answer choice that preserves the meaning of the original sentence and fixes its errors without creating any new ones.** Eliminate answer choices with grammar errors.

Types of Errors

Remember, more often than not, a sentence correction question will present to you a combination of multiple errors simultaneously. All the errors should be corrected and no other error should be introduced during the correction process. Following sections deal with different errors separately, but the examples taken may present you multiple errors. So, be careful!

Here are the most common errors in sentences.

1. Using the wrong word.
2. Singular-Plural pronouns.
3. Modifier errors
4. Parallelism errors
5. Idiomatic errors.
6. Diction errors

Let us have a detailed look at them now.

1. Using the wrong word

- Practice vs. Practise
- Affect vs. Effect
- Lay vs. Lie

Practice vs. Practise

These words sound alike and are spelled similarly – they differ only by one alphabet in spelling. They have a variety of meanings (to practise an instrument, a profession; a doctor's practice etc.) but there is one golden rule:

The word practise with an S is a **verb**, whereas the word practice with a C is a **noun**.

For example: I practise the piano (verb), but I did my piano practice (noun). The doctor practised for twenty years (verb), but his brother, the solicitor, had a practice that lasted over thirty years.

2. Singular-Plural Errors

Illustrative Sentences

- A picture of the All-Star Team, composed of players from different leagues, were given to each member. (This is incorrect)

Explanation

The trick to catch these errors is to isolate the true subject of a sentence. Remember to use the bracket technique to isolate the distracting phrases so that you can focus on the important elements of each sentence.

In the above example, the subject—**picture—is singular**, but the verb—**were—is plural**.

Therefore, this is not a correct sentence. The correct statement would be:

A picture of the All-Star Team, composed of players from different leagues, was given to each member.

3. Modifier Errors:

Modifiers should be close to what they modify. **This is the golden rule used for finding modifier errors.**

Example: The man saw the house on the hill with the telescope.

So, why is this wrong?

We have two modifiers here, which are phrases that give additional information: on the hill and with the telescope. It isn't clear from the way in which the sentence has been corrected to what these modifiers refer. We can reasonably assume that the seeing was done with the telescope, since that is what telescopes are for. Probably, the house was on a distant hill, so it seems the scenario on the left is the most likely one. However, it would have been better to put the modifier with the telescope next to the seeing, and the on the hill next to the house:

The man saw with the telescope the house on the hill.

... or better still ...

Using the telescope, the man saw the house on the hill.

This makes it abundantly clear that the seeing was done with the telescope, and it is a quite legitimate change to make.

4. Parallelism Errors

Parallelism is comparing or listing of two or more phrases or clauses which should both/all take the same form. Here is an example of two parallel items being compared:

Seeing is believing.

In this case, seeing is being listed alongside believing. They both take the same form, i.e. a verb ending in "-ing" which is being used as a noun (termed a gerund in grammatical text books). The three-word proverb above does not contain any parallelism errors. We could also rewrite the proverb as follows:

To see is to believe.

This time, both verbs are listed as infinitives, "to ..." Again, since they are both in the same form, there is no parallelism errors. However, if we wrote the following, it would be wrong:

To see is believing.

Here a gerund is being compared to an infinitive. This is grammatically wrong. The same applies to the following:

Seeing is to believe.

5. Idiom Errors

Idiom errors arise due to incorrect usage of idioms. It is not easy to spot these errors if one hasn't heard of them before. Go through the various idioms booklets for practice.

- Many teenagers feel a great deal of pressure to conform with the values, attitudes, and behavior of their peers.(Incorrect idiomatic usage).

The correct expression is "conform to"; the preposition "with" is incorrect.

6. Diction Errors

A diction error refers to using the wrong word for the meaning intended. You have to be very careful to spot this error because the word given in the sentence is spelled almost exactly like the word that should have been used.

Example:

- The space launch will take place next month, providing that the weather is good.
- The word **providing** in the example should have been **provided**. A diction error is not a spelling error, but rather the wrong word.

QUESTIONS

Directions: Look at the underlined part of each sentence. Below each sentence are given three possible substitutions for the underlined part. Choose the one which is better than the underlined part. If none of the substitutions improves the sentence, indicate (d) as your response. Thus a 'No improvement' response will be signified by the option (d).

1. The tension between us has eased a little.
 a) Eased off
 b) Easied
 c) Eased Over
 d) No Improvement
2. Whenever my students come across new words, I ask them to look for them in the dictionary.
 a) To look it up
 b) To look them up
 c) To look at them
 d) No Improvement
3. The company goes to great lengths to ensure that employees can be comfortable in their work environment.
 a) Should enjoy comfort
 b) Will be comfortable
 c) Are comfortable
 d) No Improvement
4. If you are living near a market you should be ready to bear the disturbances caused by traffic.
 a) To bear upon
 b) To bear with
 c) To bear in
 d) No Improvement
5. He insisted on he was innocent.
 a) Insistent on that
 b) Insisted that
 c) Insisted with
 d) No Improvement
6. The child shouted that he might attract attention.
 a) To attract attention
 b) In attracting attention
7. Each furniture on display in this hall is on sale.
 a) Each of the furniture
 b) Each piece of furniture
 c) Each one of the furniture's
 d) No Improvement
8. You must atone with your sins before you expect any pardon.
 a) Atone at
 b) Atone to
 c) Atone for
 d) No Improvement
9. Gangs of youth went at the rampage in the city yesterday.
 a) With the rampage
 b) On the rampage
 c) Over the rampage
 d) No Improvement
10. The whole family rallied round when mother was ill.
 a) Rallied over
 b) Rallied at
 c) Rallied with
 d) No Improvement
11. The building is a primer example of 1960s architecture.
 a) Prime number
 b) Paramount
 c) Primitive
 d) No Improvement
12. Bogus social workers have been preying over old people living alone.
 a) Preying with
 b) Preying at
 c) Preying on
 d) No Improvement
13. The negotiations will call for considerable dexterity.
 a) Will call considerable
 b) Will call at considerable
 c) Will call with considerable
 d) No Improvement

14. Looked over from that point of view, his decision is easier to understand.

- a) Looked with
- b) Looked at
- c) Looked for
- d) No Improvement

15. He showed total disregard for anyone else's feelings.

- a) Disregard at
- b) Disregard over
- c) Disregard with
- d) No Improvement

16. He does not like I coming so late.

- a) My coming so late
- b) I coming so late
- c) Me come so late
- d) No Improvement

17. I have to cut down my expenses due to my failing income.

- a) I have to cut off
- b) I have to cut out
- c) I have to cut of
- d) No Improvement

18. Of the two candidates, I think he is the best suited.

- a) He is suited best
- b) He is the better suited
- c) He is best suited
- d) No Improvement

19. The climate of Delhi is somewhat like Jaipur.

- a) Like Jaipur's
- b) As like Jaipur's
- c) As Jaipur
- d) No Improvement

20. His brother never has and never will be dependable.

- a) Never had
- b) Never has been
- c) Was never being
- d) No Improvement

21. If you would have remembered to bring the map, we would not have lost our way.

- a) Had remembered
- b) Were remembering
- c) Remembered

d) No Improvement

QUESTIONS: SET 2

1. The thieves distributed the loot **within themselves**.
 - a) Among themselves
 - b) With themselves
 - c) Amongst themselves
 - d) No Improvement
2. The **preservation of peace** is necessary.
 - a) Maintenance of peace
 - b) Persuasion of peace
 - c) Establishment of peace
 - d) No Improvement
3. The spirit of democracy had **sped into** our way of thinking.
 - a) Leaked into
 - b) Soaked into
 - c) Permeated
 - d) No Improvement
4. The **descent** from the mountain peak was slow and painful.
 - a) Descent
 - b) Decrease
 - c) Decline
 - d) No Improvement
5. The building **will redecorate** during the summer season.
 - a) Will be redecorated
 - b) Will be redecorate
 - c) Will be redecorating
 - d) No Improvement
6. Our armed forces are **superior to** those of any other country in the world.
 - a) Superior
 - b) Superior over
 - c) Superior from
 - d) No Improvement
7. He secured the first position in **hundred meters race**.
 - a) A hundred meters race
 - b) One hundred meters race
 - c) Hundred meters race
 - d) No Improvement
8. Working in the slums **brought her in** against the realities of poverty.
 - a) Brought her forward
 - b) Brought her on
 - c) Brought her up
9. I hope it's not imposing on your hospitality, but could I **stay to** for dinner.
 - a) Stay over
 - b) Stay at
 - c) Stay up to
 - d) No Improvement
10. I have dreamt all my life **for owning** a beautiful maroon - coloured car.
 - a) Of owning
 - b) At owning
 - c) To owning
 - d) No Improvement
11. The war has had a negative impact **over the economy** of the country.
 - a) In the economy
 - b) In economy
 - c) On the economy
 - d) No Improvement
12. They claimed to bring the best products and services **on the doorsteps** of their consumers.
 - a) Up to the doorsteps
 - b) At the doorsteps
 - c) To the doorsteps
 - d) No Improvement
13. My company has decided to go it alone rather than **set up** a joint venture.
 - a) Put up
 - b) Go along with
 - c) Deal with
 - d) No Improvement
14. The last exercise was **fair** easier than I thought it would be.
 - a) More
 - b) Comparatively
 - c) Fairer
 - d) No Improvement
15. The Chief Manager asked me to **carry on** his orders immediately.
 - a) Carry forward
 - b) Carry with
 - c) Carry out
 - d) No Improvement

16. The profits will be **shared amongst**

the investors.

- a) Shared all
- b) shared amongs
- c) Shared with
- d) No Improvement

17. Your advice is **no difference from**

the other friend.

- a) Not different from
- b) Not different from the
- c) No different from that of the
- d) No Improvement

18. It was **she, not me** who put forth

the attractive propositions.

- a) She, not I
- b) Her, not I
- c) Her, not me
- d) No Improvement

19. His love of language **bent him**

towards a career as a translator.

- a) Inclined him
- b) Directed him
- c) Twisted him
- d) No Improvement

20. His suggestion was **greeted with**

hoots of laughter.

- a) Greeted in
- b) Greeted at
- c) Greeted over
- d) No Improvement

21. He has been absent **since three**

days.

- a) From three days
- b) For three days
- c) During three days
- d) No Improvement

22. He bought a new house but

disposed it off immediately.

- a) Disposed immediately
- b) Disposed of it immediately
- c) Disposed off it immediately
- d) No Improvement

23. He **could not be able** to think

logically because of his illness.

- a) Cannot be able
- b) Can be able
- c) Was not able
- d) No Improvement

Definition of Articles:

An **article** is a word used to modify a noun, which is a person, place, object, or idea. Technically, an article is an **adjective**, which is any word that modifies a noun. Usually adjectives modify nouns through description, but articles are used instead to point out or refer to nouns.

There are two different types of articles that we use in writing and conversation to point out or refer to a noun or group of nouns: **definite and indefinite articles**.

Definite Article:

This article is the word '**the**', and it refers directly to a specific noun or groups of nouns.

For example:

*the freckles on my face
the alligator in the pond
the breakfast burrito on my plate*

Each noun or group of nouns being referred to - in these cases freckles, alligator, and breakfast burrito - is direct and specific.

Indefinite articles:

These are the words '**a**' and '**an**'. Each of these articles is used to refer to a noun, but the noun being referred to is not a specific person, place, object, or idea. It can be any noun from a group of nouns.

For example:

*a Mercedes from the car lot
an event in history*

In each case, the noun is not specific. The Mercedes could be any Mercedes car available for purchase, and the event could be any event in the history of the world.

Some exceptions in choosing articles:

Choosing A/An:

The word '**honour**' is a consonant. Here even though it starts with letter '**H**' it sounds like a vowel. So, we use '**An**'.

Example:

My mother is an honest woman.

Similarly, When the first letter of a word is a vowel but is pronounced with a consonant sound, we use '**A**'

Example:

She is a United States senator.

Here **United** has a **Y** sound in the beginning.

Note:

This holds true with acronyms & Initialisms also.

Example:

An LCD display.

A U.K based company.

An HR Department.

A URL.

Indefinite articles with uncountable nouns:

As the name itself suggests it is either difficult or impossible to count.

Example: Information, air.

Things those are too large to count.

Example: sand, wood etc.

Note: When these things can't be counted, you should never use a/an.

Example: Please give me a water. (Wrong usage)

Please give me some water. (Right usage)

However, if we describe water in terms of countable units (like bottles) then we can use A/an.

Example: Please give me a bottle of water.

Please give me an ice cube.

Using Articles with pronouns:

If we use pronouns and an article at the same time, it causes confusion.

Example: Why are you reading the my book?

Omission of articles:

Occasionally articles are omitted before certain nouns. In these cases, the article is implied but not actually present. This implied article is sometimes called a **Zero article**.

Example:

Let's go out for a dinner tonight. (wrong usage)

Let's go out for dinner tonight. (Right usage)

Note:

Many languages and nationalities are not preceded by an article.

Sports and academic do not require articles.

Practice Questions Set I:

1. I have uncle who lives in home for elderly. He is.....honest man. He used to be FBI agent. He once savedone-year-old boy from fire. He has many interesting stories.
2. He told me that he once met.....alien from.....space. This alien didn't need oxygen to live; it didn't have.....nose. That's one-year-old boy fromfire. He has many interesting stories.
3.stress can make..... life unpleasant. In.....day, I work at.....office people I work with are busy, and work we do isn't easy.
4. When I drive to.....work, usually.....highways are really busy. If there's accident during..... rush hour, it can be.....chaos on the roads.
5. I don't watch....TV. I get.....information and.....news, etc., from... .. Internet. I don't often go to cinema, either.
6. I'm interested in.....finance. I heard.....Euro is losing value, compared to.....US dollar.

7. I like Japan.crime is infrequent there. When I fly to Japan, I usually fly to....Narita Airport. The last time I was in Japan, I climbed....Mount Fuji. It was fun.

8. I am tall Japanese are generally shorter than I am.

9. I have two brothers. One, Greg, is still incollege, and.....other, Mike, has already graduated. Mike is.....kind of guy that is very serious. I don't remember last time I saw him. It may have been in.....August. He was wearing.....red sweater. It matched his... . red hair.

10. After.....dinner, I usually wash dishes. My wife hates doing it. I waste a lot of.....water when I do it. That's bad for.....environment, I think.

Practice Questions Set II:

1. The difference between _____ successful person and _____ others is not _____ lack of strength, not _____ lack of knowledge, but rather _____ lack in will.

- a) A, the, a, a, a
- b) A, no article, the, the, the
- c) An, no article, a, a, a
- d) A, no article, a, the, a

2. _____ best and most beautiful things in. _____ world cannot be seen or even touched - they must be felt with. _____ heart.

- a) The, the, the
- b) The, no article, the
- c) The, the, no article
- d) No article, the, the

3. Pashupatinath temple is _____ busy centre. There entrance is only for _____. Hindus. Along with _____ people is _____ menace of animals like dogs and monkeys. As it is visited by _____ large number of people often problems could be created there.

- a) A, no article, the, a, a
- b) A, no article, the, the, a
- c) A, no article, the, no article, a
- d) A, the, the, the, a

4. Consider example from another field. At ordinary temperatures, electricity flows only so long as driving force from battery or generator is imposed upon circuit. At temperatures near absolute zero, metals exhibit superconductivity: unique property that causes electric current to flow for months after voltage is cut off.

- a) An, no article, a, a, the, a, an, the
- b) A, no article, a, a, the, a, a, the
- c) An, no article, a, the, the, a, an, the
- d) An, no article, a, a, no article, a, an, the

5. stroke is similar to heart attack, except it attacks brain. Strokes are among biggest killers in the USA. Many are result of unhealthy eating. A diet that has large amount of fast food and other junk food increases the risk of having a stroke.

- a) A, a, the, a, the, a
- b) A, the, the, a, the, the
- c) A, a, the, the, the, a
- d) A, a, the, a, the, the

6. Joan of Arc was French peasant girl, who was believed to be voiced by God to lead French army to success in Hundred Years War. She was young, active, skilled and determined. She was person who believed in what she thought as right.

- a) A, the, a, no article, a
- b) A, the, the, no article, the
- c) A, no article, the, no article, a
- d) A, the, the, no article, a

7. computer lovers claim that they can learn everything with help of a computer. But there will be lot of difference between a test conducted by teacher and a computer. Thus, despite many advantages of a computer, wisdom lies in judiciously using it as a tool.

- a) No article, the, a, the, the
- b) No article, the, a, a, the
- c) The, the, a, a, the
- d) No article, the, a, a, no article

8. If you really want to do something, you will find way. If you don't, you'll find excuse.

- a) The, an
- b) A, the
- c) A, an
- d) The, the

9. Technology is just tool. In terms of getting the kids working together and motivating them, teacher is most importa]]nt.

- a) A, an, the
- b) A, the, a
- c) The, the, the
- d) A, the, the

10. Joan of Arc was French peasant girl, who was believed to be voiced by God to lead French army to success in Hundred Years War. She was young, active, skilled and determined. She was person who believed in what she thought as right.

- a) A, the, a, no article, a
- b) A, the, the, no article, the
- c) A, no article, the, no article, a
- d) A, the, the, no article, a

Direct Speech and Indirect Speech

When we report something using the exact words of the speaker, we use **direct speech**. In direct speech, we convey the message of the speaker in his own actual words without any change to another person

When we report something in our own words, we use **indirect speech**. In indirect speech, we convey the message of the speaker in our own words to another person

Direct speech uses quotation marks.

Indirect speech is also known as reported speech, It doesn't use quotation marks. The verb tenses do not change since we just copy what the person had said.

Direct speech: The boy said, 'I'm happy with my results.'

Indirect speech: The boy said that he **was happy** with his results.

With regards to direct speech, there is no interpretation or annotation; the words are taken directly from one source and repeated to another. In other words, we take the words directly from the speaker and repeat them exactly as they were originally stated.

When talking about indirect speech, we use words that refer to something that has already happened. To do so, we are speaking in the past tense and are summarizing, modifying, or synthesizing what has already been said.

DIRECT SPEECH

Direct speech repeats, or quotes, the exact words spoken. When we use direct speech in writing, we place the words spoken between quotation marks (" ") and there is no change in these words. We may be reporting something that's being said NOW (for example a telephone conversation), or telling someone later about a previous conversation.

EXAMPLES

She says, "What time will you be home?"

She said, "What time will you be home?" and I said, "I don't know!" "There's a fly in my soup!" screamed Simone.

John said, "There's an elephant outside the window."

INDIRECT SPEECH

Reported or indirect speech is usually used to talk about the past, so we normally change the tense of the words spoken. We use reporting verbs like 'say', 'tell', 'ask', and we may use the word 'that' to introduce the reported words. Inverted commas are not used.

She said, "I saw him." (direct speech) = She said that she had seen him. (indirect speech)

'That' may be omitted:

She told him that she was happy. = She told him she was happy.

'SAY' AND 'TELL'

Use 'say' when there is no indirect object:

He said that he was tired.

Always use 'tell' when you say who was being spoken to (i.e. with an indirect object):

He told me that he was tired.

'TALK' AND 'SPEAK'

Use these verbs to describe the action of communicating:

He talked to us.

She was speaking on the telephone.

Use these verbs with 'about' to refer to what was said:

He talked (to us) about his parents.

DIRECT SPEECH ILLUSTRATION:

She said, "I am twelve years old."

"I am twelve years old," she said.

INDIRECT SPEECH ILLUSTRATION:

Mom can't attend my graduation.

Simon said (that) his mom couldn't attend his graduation.

I am going to the bank

Mary said (that) she was going to the bank.

My teacher told me to study harder

Her teacher had told her to study harder.

DIRECT SPEECH



Direct speech reports what someone has said or written by quoting their exact words in quotation marks.

"Amazing Man is here to save the day!" declared Amazing Man.

INDIRECT SPEECH



Indirect speech reports what someone said, although it's not word for word and it doesn't use quotation marks.

Amazing Man said he's here to defeat the bad guys.

General rules for changing direct speech into indirect speech

Omit all inverted commas or quotation marks. End the sentence with a full stop.

If the verb inside the inverted commas/quotation marks is in the present tense, change it into the corresponding past tense. If it is in the simple past tense, change it into the past perfect tense.

Direct speech: The girl said, I like singing.'

Indirect speech: The girl said that she liked singing.

Direct speech: Rahul said, I will have to reach home by 8.30.'

Indirect speech: Rahul said that he **would have** to reach home by 8.30.

Direct speech: Alina said, I met James yesterday.'

Indirect speech: Alina said that she **had met** James yesterday.

When the verb inside the quotation marks expresses a universal truth, we do not normally change it into the past tense.

He said, All people have equal rights.'

He said that all people **have** equal rights. (More natural than He said that all people had equal rights.)

Use pronouns appropriately. Study

the examples given below.

Direct speech: The boy told the girl, I told you that we were not going on a holiday.' Indirect speech: The boy told the girl that **he had told her that they** were not going on a holiday.

Notes:

When the reporting verb is in a present or future tense, we do not change the tense of the verb inside the quotation marks.

Direct speech: She says, I will come.'

Indirect speech: She **says that she will come.**

Special cases in direct speech & indirect speech

Reported Speech or Direct & Indirect Speech is one of the trickiest topics in Grammar. Not only must you have a good understanding of the Rules of Converting Direct and Indirect Speech, but you must also be aware of the Exceptions in Direct and Indirect Speech.

Case 1:

He said, —I saw the sun rising in the east.||

He said that he had seen the sun rising in the east.

Case 2:

He said, —Sun rises in the East.||

He said that Sun rises in the East.

In case 1 and 2, the Reporting Verb is in the past tense (said) but still the tense of only Case 1 is changed. There are certain exceptions that follow the change of tense.

Exception #1: The tense of the verb doesn't change even if the Reporting verb is in past tense for the following cases.

Universal/ Scientific Truths:

E.g. She said, —Water boils at 100 degree Celsius.||

She said that water boils at 100 degree Celsius.

Historical Facts:

E.g. The teacher said, —Kalidas is the Shakespeare of India.|| The teacher said that Kalidas is the Shakespeare of India.

Proverbs:

E.g. Mohan said, —Where there is a will there is a way.|| Mohan said that where there is a will there is a way.

Habitual Facts:

Example:

She said, —I snooze my alarm everyday.||

She said that she snoozes her alarm every day.

Exception #2: The connector isn't always _that' for the following cases.

Interrogative Sentences:

Type 1 – Yes/No type question:

_If/whether' is used instead of _that'.

Example:

John said to Joanna, —Are you supporting Clinton?

John asked Joanna whether she was supporting Clinton.

Direct Speech to Indirect Speech Illustration using
table
Present tense:

Direct Speech		Indirect Speech	
Present Simple	—I eat pizza, Lisa said.	Past Simple	Lisa said she ate Pizza.
Present continuous	—I am eating pizza, Lisa said.	Past continuous	Lisa said she was eating Pizza.
Present Perfect	—I have eaten pizza, Lisa said.	Past Perfect	Lisa said she had eaten Pizza.
Present Perfect continuous	—I have been eating pizza, Lisa said.	Past Perfect continuous	Lisa said she had been eating Pizza.

Direct Speech		Indirect Speech	
Past Simple	—I ate pizza, Lisa said.	Past Perfect	Lisa said she had Eaten Pizza.
Past continuous	—I was eating pizza, Lisa said.	Past Perfect continuous	Lisa said she had been eating Pizza.
Past Perfect	—I had eaten pizza , Lisa said.	Past Perfect	Lisa said she had eaten Pizza. (no change)
Past Perfect continuous	—I had been eating pizza, Lisa said.	Past Perfect continuous	Lisa said she had been eating Pizza. (no change)

Note:

If something is still true or still happening up to the time of reporting, then we may use the same tense.

Example:

1. —My father is a businessman,|| he said
-He said that his father was a businessman.
-He said that his father is a businessman.
2. The doctor said, —You need enough sleep.||
-The doctor said I needed enough sleep.
-The doctor said I needed enough sleep.

Modals

	Direct Speech		Indirect Speech
Will	—We will play football, he said	would	He said he would play football
Can	Tony said, I can play piano well.	Could	Tony said he could play piano well
May	—It may rain, the weatherman Said	Might	The weatherman said it might rain
Shall	—We shall go, Alex said.	should	Alex said we should go.
Must	Sandra said, I must study.	Had to	Sandra said she had to study hard

There are no changes to the following modals:

Could	Should
Would	Might

—**I could** he said.
cook,|| He said he **could** cook.

—Ms. James **would** like to talk to you,|| Harry said.
Harry said Ms. James **would** like to me.

Pronoun Change:

When making a reported speech, remember that the pronoun also changes since the person doing the reporting is not the original speaker.

Direct Speech	Indirect Speech	Pronoun
— I am sad, Maddy said.	Maddy said that he was said	I-He
— You have to be there, he said.	He said that I had to be there	You-I
— I saw your sister, she said.	She said that she had seen my sister	I-She Your-My

Practice Questions:

1. The government has announced, "Taxes will be raised":
A. The government has announced that taxes would be raised.
B. The government has announced that taxes would raised.
C. The government has announced that taxes will be raised.
D. The government has announced taxes will be raised.

2. The poor examine said, "O God, take pity on me."
A. The poor examine said, "O God, take pity on me."
B. The poor examine, involving God, implored him to take pity on him.
C. The poor examine exclaimed that God take pity on him.
D. The poor examine asked God to take pity on him.

3. Suresh asked, —How long will it take to travel from Germany to South Africa ?"
A. Suresh asked how long it will take to travel from Germany to South Africa.
B. Suresh asked how long would it take to travel from Germany to South Africa.
C. Suresh asked how long it would take to travel from Germany to South Africa.
D. Suresh was asking how long must it take to travel from Germany to South Africa.

4. Manna asked Rohan, "Have you sat in a trolley bus before?!"
A. Manna asked Rohan had he sat in a trolley bus earlier.
B. Manna asked Rohan had he sat in a trolley bus before.
C. Manna asked Rohan if he sat on a trolley bus before.
D. Manna asked Rohan if he has ever sat in a trolley bus.

5. "You can't bathe in this sea," he said to me, "it's very rough."
A. He said that I can't bathe in this sea because it's very rough.
B. He said that you couldn't bathe in the sea if it was very rough.
C. He said that I couldn't bathe in that sea as it was very rough.
D. He said that you can't bathe in this sea since it was very rough.

6. Vissu said, "We passed by a beautiful lake when we went on a trip to Goa."
A. Vissu said that they passed by a beautiful lake when they had gone on a trip to Goa.
B. Vissu said that they has passed by a beautiful lake when they went on a trip to Goa.
C. Vissu said that they had passed by a beautiful lake when they had gone on a trip to Goa.
D. Vissu said they passed by a beautiful lake when they went on a trip to Goa.

7. He said. "Be quite and listen to my words.
A. He urged them to be quite and listen to his words.
B. He urged them and said be quite and listen to his words.
C. He urged they should be quite and listen to his words.
D. He said you should be quite and listen to his words

8. He said to me, I have often told you not to play with fire.
A. He said that he has often been telling me not to play with fire.
B. He told me that he had often told me not to play with fire.
C. He reminded me that he often said to me not to play with fire.
D. He said to me that he often told me not to play with fire.

9. The Captain said to his men, "Stand at ease."
A. The Captain urged his men to stand at ease.
B. The Captain wanted his men to stand at ease.
C. The Captain told his men that they should stand at ease.
D. The Captain commanded his men to stand at ease.

10. Pawan said to me, "If I hear any news, I'll phone you."
A. Pawan told me that if he heard any news, he will phone me.
B. Pawan told me that if he will hear my news, he will phone me
C. Pawan told me that if he had heard any news, he would phone me.
D. Pawan told me that if he heard any news, he would phone me.

11. He says, "They will hurt you".
A. He said to me that they will hurt me.
B. He said to me that he would be hurt me.
C. He said to me that they would hurt me.
D. He said to me that they would hurt you.

12. My father said to me, "You must work hard".
A. My father said to me that I must work hard. B.
My father said to me that you must work hard.
C. My father said to me that you have to work hard.
D. My father said to me that I had to work hard.

13. She said, "My parents are going to Karachi".
A. She said that my parents going to Karachi.
B. She said that my parents gone to Karachi.
C. She said that her parents were going to Karachi.
D. She said that her parents are going to Karachi.

14. Mother said, "Where are you going?" A. Mother said me where I am going.
B. Mother said to me where are you going.
C. Mother asked where I was going.
D. Mother asked me where I am going.

15. Did you come from France"? Said Sara to the boy. A. Sara asked the boy if he had come from France.
B. Sara asked the boy if he come from France. C. Sara asked the boy if he came from France. D. Sara said the boy did he come from France.

16. You said, "You have brought books for me".
A. You said that you brought books for me.
B. You said that you had brought books for me.
C. You said that you have brought books for me.
D. You said that you have been brought books for me.

17. My uncle said to me, "How is your studies going on now?"
. A. My uncle asked me how was my studies going then.
B. My uncle asked me how is your studies going on now.
C. My uncle asked me how my studies was going on
then. D. My uncle asked me how my studies going on
then.

18. You said, "He was there".
A. You said that he had been here.
B. You said the he was there.
C. You said that he had there.
D. You said that he is there.

19. Mother said, "Where are you
going?" A. Mother said me where I am
going.
B. Mother said to me where are you going.
C. Mother asked where I was going.
D. Mother asked me where I am going.

20. She said to him, "Where are you
living"? A. She said to him where he lived.
B. She asked him where he was
living. C. She asked me where I am
living. D. She asked him where he
was lived.

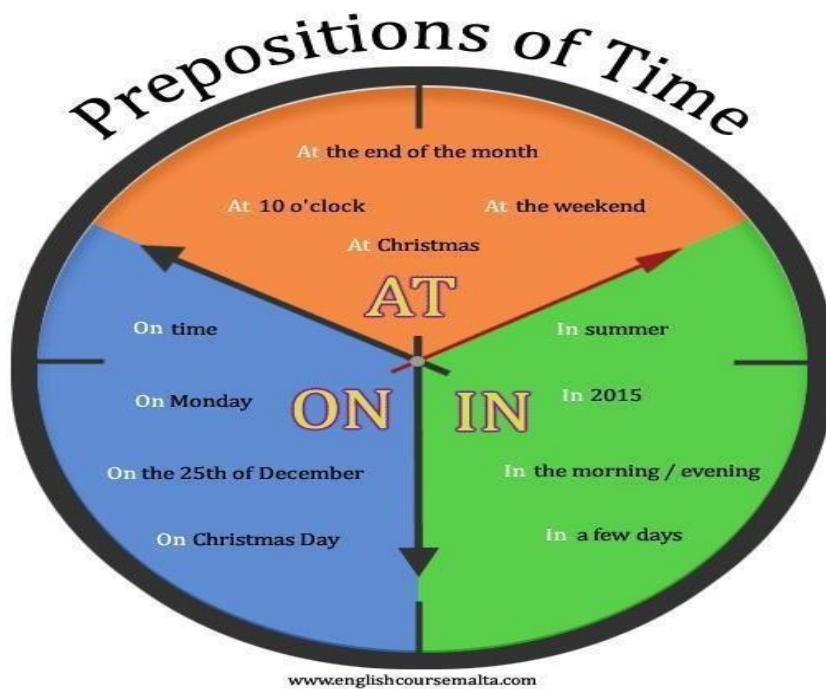
Prepositions

A **preposition** is a word such as after, in, to, on, and with. Prepositions are generally used in front of **nouns** or pronouns and it shows the **relationship** between the noun or pronoun and other words in a sentence. They are also words which begin with prepositional phrases, it means a group of words containing a preposition, a noun or pronoun object of the preposition and any modifiers of the object. We commonly use prepositions to show a relationship in space or time or a logical relationship between two or more people, places or things.

PREPOSITION OF TIME

Time preposition are those such as before, after, during and until. It allows you to discuss a specific time period such as a date, day or the actual time something takes place. Preposition of time are the same words as prepositions of place, however they are used in a different way.

- At – This preposition of time is used to discuss clock times, holidays and festivals, and other very specific time frames including exceptions, such as –at morning.||
- In – This preposition of time is used to discuss months, seasons, years, centuries, general times of day, and longer periods of time such as –in the past.||
- On – This preposition of time is used to discuss certain days of the week or portions of days of the week, specific dates, and special days such as –on Birthday.||



Examples:

Preposition _in'

I am going to London in July. (in is the preposition and July is the noun)

My birthday is in March.

Jane often goes skiing in winter.

Preposition _on'

I have a meeting with my manager on next Friday.

The bank is closed on Monday.

I received the letter on September 2nd

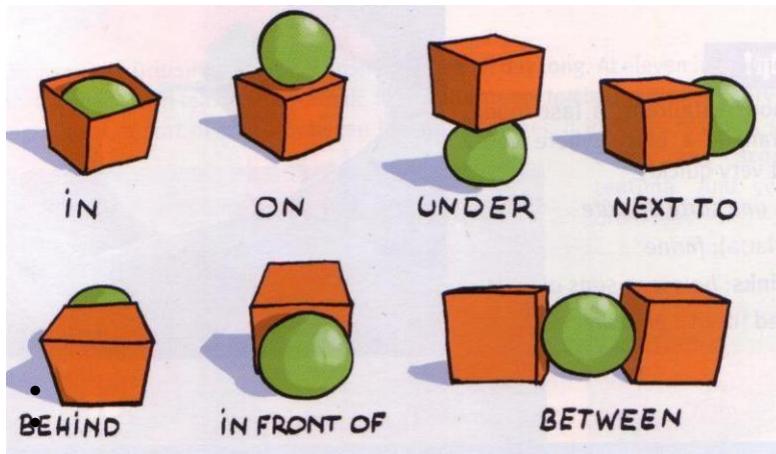
Preposition _at'

We have a meeting at 4'o clock.

I enjoy walking in Paris at night.

They sing carols at church.

PREPOSITION OF PLACE



Generally we use at, in and on when we talk about the location of things. It is a preposition which is used to refer to a place where something or someone is located.

- In - It tells us the noun is in an enclosed space (surround or closed off on all sides). Basically, when something is inside something.
- On – in this the noun is located on a surface. Use on when one thing is attached to or touching something.

- At – it says that the following noun is located at a specific point or location. It shows an exact position.

Examples:

Preposition _in'

He is playing in the garden.

The vehicle is in the parking area.

They are in London to visit Tower Bridge.

I read an article in the newspaper.

Preposition _on'

There is a photograph on the wall.

It is written on the first page.

She is wearing a ring on her finger.

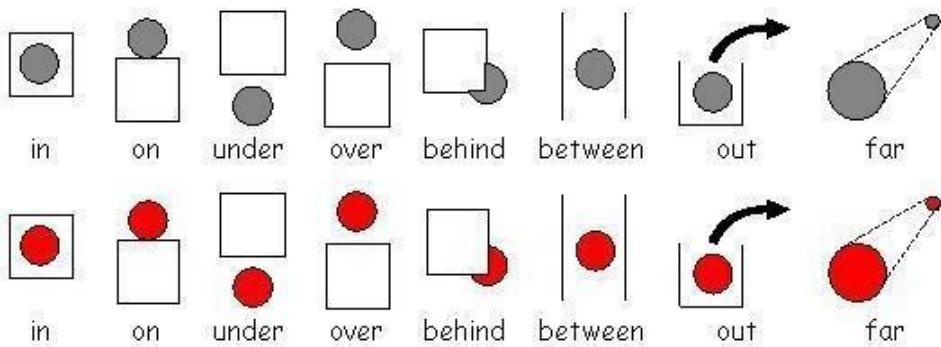
Preposition _at'

Somebody is at the door.

He lives at 2nd phase, Wilson layout.

I have a meeting at 9.

PREPOSITION OF DIRECTIONS/ MOVEMENTS



Prepositions of movement or direction are used to show movement from one place to another. These prepositions are most often used with verbs of motion and are found after the verb.

We use —toll to show movement in a specific direction.

—Across is used to show movement from one side to the opposite side.

—Along is used as a point in the length of; from one point to another.

Some other prepositions of directions are: around, along, down, into, off onto over etc.

NOTE: There are also sentences where prepositions are not necessary such as with the words home, downtown, uptown, inside, upstairs.

Examples:

I go **to** college by bus

You walk **to** railway station every day.

Stephy flew **to** Canada.

The boat will take you **across** the river.

The street runs **along** the seafront.

I prefer to ride my bike **down** the hills.

The cat jumped **over** the dog.

Who is the girl running **towards** the shop?

Questions

1. When we get ready for dinner, I have to take my books _____ the table.
A. Off
B. From
C. Out
D. Of
2. Every weekend, we put the trash can _____ for garbage collection.
A. Up
B. At
C. into
D. Out
3. You have to leave your shoes _____ the door when you enter the house.
A. In
B. Over
C. To
D. By
4. It was a long walk, so he began moving slowly _____ the town.
A. For
B. Towards
C. Until
D. At
5. David cleaned his room by stuffing everything _____ his bed.
A. In
B. On
C. Under
D. Along
6. We often go fishing _____ the river bank.
A. Towards
B. Inside
C. Along
D. Around
7. The lecture will be held right _____ the tutorial.
A. At
B. Before
C. Over
D. Beyond
8. He was caught by the teacher for cheating _____ the exam.
A. During
B. After
C. Near
D. Outside
9. John needs to submit the report _____ his boss before 5 pm.
A. At
B. To
C. Of
D. On
10. James' wife accused him _____ cheating.
A. Up
B. To
C. With
D. Of
11. Kathleen apologized _____ her brother's poor behaviour.
A. For
B. In
C. Off
D. At
12. There is no doubt that Adam is very good _____ telling jokes.
A. At
B. Over
C. Of
D. With
13. These dresses were _____ sale last week.
A. In
B. Of
C. On
D. Up
14. There is a bridge _____ the river.
A. Near
B. Across
C. On
D. Along

15. Amanda hasn't seen her older sister _____ last April.
- A. On
B. In
C. Around
D. Since
16. The dispute was because _____ jealousy.
- A. On
B. In
C. Over
D. Of
17. Julia came all the way _____ Russia to see her boyfriend.
- A. From
B. Since
C. Before
D. In
18. The detective chased the killer _____ the streets.
- A. Since
B. Through
C. During
D. Beyond
19. Ai Ling is worried _____ the test because she thinks that she will fail the test.
- A. Of
B. On
C. To
D. About
20. Jeanne was texting her boyfriend _____ the teacher was teaching.
- A. Until
B. Of
C. While
D. For
21. The father was anxious the safety of his _____ daughter.
- A. for
B. at
C. about
D. with
22. I was advised to abstain _____ all alcoholic drinks.
- A. at
B. by
C. from
D. in
23. You are supposed to take this medicine and you will get rid of the _____ bad cold.
- A. from
B. over
C. at
D. of
24. He had to repent _____ what he had done
- A. for
B. of
C. at
D. over
25. My voice reverberated _____ the walls of the castle.
- A. in
B. for
C. on
D. from
26. It is not difficult to sympathise an _____ unfortunate man.
- A. with
B. at
C. by
D. for
27. Finding himself short money, he wrote _____ his uncle _____ help.
- A. of, to, for
B. in, to, to
C. with, to, for
D. to, to, about
28. It is very dangerous to intrude _____ the enemy's camp
- A. into
B. on
C. through
D. for
29. The person refused to grovel the feet of his master
- A. by
B. on

C. about
D. at

30. These slums are disgrace the civic authorities.

- A. for
- B. to
- C. towards
- D. on

Active and Passive voice

In grammar, the **voice** of a verb describes the relationship between the action (or state) that the verb expresses and the participants identified by its arguments (subject, object, etc.). When the subject is the agent or doer of the action, the verb is in the active voice. When the subject is the patient, target or undergoer of the action, the verb is said to be in the passive voice. Voice is sometimes called **diathesis**.

For example, in the sentence:

The cat ate the mouse.

The verb "ate" is in the active voice. However, in the sentence:

The mouse was eaten by the cat.

The verbal phrase "was eaten" is passive.

Active voice:

The active voice is the most commonly used in many languages and represents the "normal" case, in which the subject of the verb is the agent.

In the active voice, the subject of the sentence performs the action or causes the happening denoted by the verb.

Example: **Antonio Manteno ate the potatoes.**

The verb *ate* indicates the active voice.

Passive voice:

The passive voice is employed in a clause whose subject expresses the theme or patient of the verb. That is, it undergoes an action or has its state changed.^[5]

In the passive voice the grammatical subject of the verb is the recipient (not the doer) of the action denoted by the verb.

Consider the following sentence which is in passive voice:

The potatoes were eaten by Antonio Manteno.

The words *were eaten* indicate the passive voice.

The passive voice shows that something has been acted upon by someone or something else.

In the Active Voice, the Subject (Noun) of a sentence **performs an Action** (Verb) on an Object. In the Passive Voice, the Subject (Noun) is **Acted Upon** (Verb) by an Object. Use of active or passive voice depends on what emphasis you want in your writing. Active voice emphasizes the doer of the action; passive voice emphasizes the recipient of the action.

Examples: To switch between passive and active voice, the noun in the object position must change places with the noun in the subject position. Edit the sentence accordingly.

		Subject	Verb	Object
1.	Active	The student	wrote	the essay.

Why: The subject is doing the action, and the object is receiving the action. The subject (student) performed the action (wrote) on the object (essay).

1.	Passive	The essay	was written	by the student.
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Why: The roles of the subject and the object are reversed. The subject (essay) was acted upon (written) by the object (student).

2.	Passive	The essay	was written.	
----	----------------	-----------	--------------	--

Why: The subject is still being acted upon, but the thing doing the action is unknown. This sentence is still passive. The reader just doesn't know who wrote the essay.

2.	Active	The student	wrote.	
----	---------------	-------------	--------	--

Why: The subject (student) is doing the action (wrote), but what the student wrote (object) is unknown. If a sentence has no recipient of the action, the sentence cannot be written in passive voice since nothing is being acted upon.

Notes: It is always good practice to write in the active voice unless your instructor says otherwise. However, please keep in mind that passive voice is NOT WRONG!

Passive voice is used to draw attention to the action rather than the doer of the action. For example, many scientific reports are written in the passive voice to emphasize the results of the experiment rather than the experiment itself.

How to tell if a Sentence is in Passive Voice

- When both the helping verb and the past-tense verb are used, you are using passive voice. Look for **helping verbs** such as "is," "was," "were," and "are." Helping verbs can "help" past-tense verbs such as "written," "graded," and "conducted" create the passive voice.

	Subject	Verb	Object
Active	The teacher	graded	the tests

Why: The subject (teacher) performed the action (graded) on the object (tests).

Passive	The tests	were graded	by the teacher.
----------------	-----------	-------------	-----------------

Why: The subject (tests) **was** acted upon (graded) by the object (teacher).

Active	The lab assistant	conducts	the experiment.
---------------	-------------------	----------	-----------------

Why: The subject (lab assistant) performs the action (conducts) on the object (experiment).

Passive	The experiment	is conducted	by the lab assistant.
----------------	----------------	--------------	-----------------------

Why: The subject (experiment) **is** acted upon (conducted) by the object (lab assistant).

- Sometimes no noun exists in the object position. In these situations, try adding "by zombies" to the end of the sentence. If the sentence makes sense with zombies performing the action, it's probably passive.

	Subject	Verb	"by zombies"
Passive	The tests	were graded	[by zombies].

Why: The preposition "by zombies" tells us that zombies could perform the action of the sentences, which means this is a passive sentence.

Active	The student	wrote	[by zombies].
---------------	-------------	-------	---------------

Why: The preposition "by zombies" sounds like the student wrote near the zombies instead of the zombies doing the writing, so this is an active sentence.

Some Easy Ways to identify to differentiate between active and passive voice:

Scenario: 1

One of my college English professors said the easiest way to see if a sentence is in passive voice is to see whether it still makes sense if you add the phrase —by zombies.||

Example 1:

I went to the store (by zombies). (doesn_t makes sense; active) Example 2:

I was dropped off at school (by zombies). (does make sense; passive)

Scenario: 2

In active voice-subject is doer

Example-Ram beats Mohan.

Here ram is subject as well as doer.

In passive voice-subject is not doer.

Example- Mohan is beaten by Ram. Here Mohan is not subject.

Convert the following statement in to passive forms

1. He would carry the bag.
2. Will the interviewer test our GK?
3. The technicians test the generator.
4. Who gave you these pencils?
5. She must fill in the form.
6. Mary is selling coffee.
7. The fire damaged the roof
8. They have stopped collecting donations.
9. He said that cakes are delicious
10. Paul solved these problems.

Re-write the sentences in Active Voice:

1. The thieves were caught by the police.
2. The principal was pleased with the student's result.
3. My niece is frightened by loud thunderstorms.
4. The ball is hit straight into the spectators by Gayle.
5. He was praised by the police
6. I was told the answer by Annie.
7. The passenger was shown his seat.
8. Her idea was not accepted by anyone.
9. The work was done by him is known to me.
10. Let the door be closed.

Tenses

Grammar **tenses refer to the state of the verb**. The state, or tense, of the verb **explains the time of the action**.

There are three major tenses in English. These include past, present, and future. Each of these tenses can explain an event that occurred in the past, an event that occurs in the present, or an event that will occur in the future.

What is Past Tense?

What does past tense mean? The past simple tense of verbs expresses events or actions that already occurred. These actions are finite in that they have both a starting and a stopping point.

Examples of Past Tense Verbs

For regular verbs, the past tense is formed by adding --ed to the end of the infinitive (base).

Example:

Verb: to listen

Past: listened

What is Present Tense?

What does present tense mean? The present simple tense expresses events that happen in the present but are not actions happening now. The present tense is used for facts, habits (habitual actions), general truths, and states of being.

This may seem confusing, so an example will clarify.

Examples of Present Tense Verbs

Examples:

He works at Target.

While he works at Target, he may not be working at Target at this very moment.

This statement expresses a fact.

What is Future Tense?

What does future tense mean? The future simple tense expresses actions that have not yet occurred but that will occur at a future date.

The future tense is formed in two ways:

Adding —will between the subject and the infinitive (base) of the verb.

I will go to the store later today.

She will answer the question shortly.

Adding —going between the subject + the infinitive of the verb.

I am going to remind you tomorrow.

We are going to sleep

Full List of English Verb Tenses

Here are all of the major types of tense in English.

Four Past Tenses:

Simple Past Tense > He smiled

Past Progressive Tense > He was smiling

Past Perfect Tense > He had smiled

Past Perfect Progressive Tense > He had been smiling

Four Present Tenses:

Simple Present Tense > He smiles

Present Progressive Tense > He is smiling

Present Perfect Tense > He has smiled

Present Perfect Progressive Tense > He has been smiling

Four Future Tenses:

Simple Future Tense > He will smile

Future Progressive Tense > He will be smiling
Future Perfect Tense > He will have smiled
Future Perfect Progressive Tense > He will have been smiling

1. Simple Past: (I did, she did)

An action completed at a specific time in the past.

Example:

She wrote that novel many years ago.

2. Past continuous: (I was doing, we were doing)

Here, action is in progress in the past, either:

- a. when a shorter action took place
- b. at a very precise moment

Example:

She was writing that novel when she became ill.

3. Past Perfect: (I had done, we had done)

Action that was completed in the past, either:

- a. Before another action took place
- b. Before a very precise moment

She had written that novel before she became ill.

4. Past perfect continuous: (I had been doing, she had been doing)

Action that started, continued for sometime & was still in the progress in the past,

Either:

- a. Before another action
- b. Before a very precise moment

She had been writing that novel for a few months, when she became ill.

5. Simple present :(I do, he does)

It is a fact, habit/regular action
Fact: Anna is an author

She writes novels

Habit: She always writes in the morning

Regular action: She writes two books a year

6. Present Continuous: (I'm doing, she's doing, and we're doing)

A temporary action in progress at present

Example:

She is currently writing a new book

7. Present perfect continuous: (I've been doing, she has been doing,)

An action that is under process or yet to be completed.

Example:

She has been writing books for over 20 years.

8. Present perfect: (I have done, she has done)

a. With an action verb:

A completed action without a specified time in the past.

Words used here are:

So far, already, not yet, recently, until now, etc.

Example:

She has already written 20 books.

b. With state verbs:

Words used here are:

to be, to have, etc.

Example:

She has been a writer for 15 years.
I have known her since 2010.

9. Future simple: (I will do, she will do and they will do)**a. A decision at the time of speaking** (used mainly in dialogues & in contracted form)

- Dan called earlier. He wants you to call him back. Ok. I will call him as soon as I can.

b. A possibility in future.

- I will probably call him this afternoon.

10. Future Continuous: (I will be doing, we will be doing) a. Action in progress at a specific time in the future. Example:

Will you be home at 9'0 clock tonight?

b. Going to' / _be**doing' Example:**

Are you going to pay for the meals?

We will be going to the Mexican restaurant.

11. Future perfect: (I will have done, we will have done) Action that will be completed in the future, either:

a. Before a specific moment

b. Before another action takes place.

We will have left the restaurant by midnight.

12. Future Perfect Continuous: (I will have been doing, We will have been

doing) Action that will have started, continued for sometime & will be in progress in the future, Either:

a. Before a specific moment

b. Before another action takes place.

We will have been drinking for few hours by the time we leave.

Tenses – Questions

Complete the following sentences with an appropriate tense form.

1. This time tomorrow, I on the beach.

A.....amlying

B. will lie

C. will be lyin

2. She said that she

help

me.

A. will

B. shall

C. would

3. Phone me when you time.

A.

have

B. would

have C. will

have

4. I will follow you wherever you

A. go B.

goes C.

will go

5. You an accident if you

go on driving like that.

A. are having

B. will have

C. would have

6. When you phoned, I in the garage.

A. worked

B. am working

C. was working

7. I all my childhood in

South India.

A. was spending

B. spend

C. spent

8. The phone while I was having a bath.

A. rings

B. rang

C. was ringing

9. I tried a little of the soup to see how it

A. tastes

B. is

tasting C.

tasted

10. When we were children we our own toys.

A. were making

B. made

C. had made

11. I can't dance because I my leg.

A. have broken

B. broke

C. had broken

12. I the bell six times but no one opened the door.

A. was ringing

B. have rung

C. rang

Chose the correct form of sentence for the following questions:

1. Suganya is typing

- A. simple past
- B. simple present
- C. present continuous
- D. past perfect

C. present continuous

D. past continuous

7. Prasanna would have reached

Bangalore by this time.

- A. present perfect
- B. past perfect
- C. future perfect
- D. future simple

2. Shakespeare has written dramas appealing to all people of all ages.

- A. simple present
- B. present continuous
- C. past perfect
- D. present perfect

8. He goes to office by train daily.

- A. simple present
- B. simple past
- C. past perfect
- D. past continuous

3. The Earth moves round the Sun.

- A. simple past
- B. past perfect
- C. simple present
- D. past future perfect

C. simple present

4. I will have finished this work by the end of this month.

- A. future perfect
- B. present perfect
- C. simple past
- D. simple present

A. future perfect

5. She could not marry the person of her choice.

- A. simple past
- B. simple future
- C. past continuous
- D. past perfect

B. simple future

6. Are you playing tennis?

- A. simple present
- B. simple past

A. simple present

Subject Verb Agreement



INTRODUCTION

One of the basic ideas behind the sentence agreement is that all the parts of the sentence should match. Verb need to agree with their subjects in number and in person.

WHAT IS SUBJECT VERB AGREEMENT

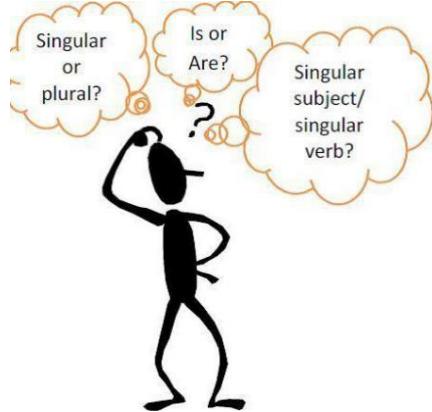
Subject verb agreement refers that the subject and the verb in the sentence must agree. Subject verb agreement is mainly done to find

- What is the verb
- The subject
- Subject matches the verb

Example : Godse killed Gandhi.

In the above example _ Godse is the subject, _killed is the verb and _Gandhi comes as the object.

In subject verb agreement, the subject becomes the —Doer of the action, the verb is the —Action words and the object is the —Receiver of the action.



Subject-Verb Agreement

General Rules



Basic rule of subject verb agreement is that

Rule 1: Singular subject takes a singular verb whereas plural subject takes plural verb.

Examples:

- The list of items is on the desk.
- Teacher teaches the student.
- My dog always growls t the postal carrier.

Rule 2: When two subjects are connected by or, either/or, or neither/nor require a singular verb.

Examples:

- My mom or my dad is arriving by bus today.
- Neither Anna nor Abel is available.
- Either Maya or Manu is helping today with stage decoration.

Rule 3: In general rule, use plural verb with two or more subjects when they are connected by and.

Examples:

- A car and bike are my means of transportation.
- Gelatin and honey are needed for the recipe.
- Pepperoni and cheese are great on pizza.

Rule 4: In case of verb, words ending with _s is singular and words not ending with s is plural and vice-versa in case of noun.

Examples:

- In case of verb, kill is plural and kills is singular.
- In case of noun, bench is singular and benches is plural.



Rule 5: Use singular verb with distance, periods of time, sums of money etc... when considered as a unit.

Examples:

- Four miles is too far to walk.
- Twelve dollars is a high price to pay.
- PG degree is for two years.

Rule 6: In case of collective noun such as family, jury, audience, population, the verb might be singular or plural according to the writer's intent.

Examples:

- All of my friends has arrived OR have arrived.
- Most of the jury is here OR are here.

Rule 7: Words that indicate portion eg- a lot, a majority, and we are guided by noun after of. If the noun after of is singular, use a singular verb. If it is plural, use plural verb.

Examples:

- A lot of **pie** has disappeared.
- A lot of **pies** have disappeared.
- A third of the **city** is unemployed.
- A third of the **people** are unemployed.

Rule 8: When not only..... but also is used to combine two subjects, the verb must agree with the subjects closest to it. (Rule of proximity)

Examples:

- Not only the students but also the teacher **wants** to take a day off.
- Not only the parliament but also the State Legislatures **are** required to pass a major constitutional amendment.

Rule 9: The words there and here are never subjects. The real subjects come after the verbs.

Examples:

- There **are** two reasons.
- There **is** no reason for this.
- Here **are** two apples.
- Here **is** the key.

-Rule 10: For all indefinite pronouns we always use singular verb.

Everybody	Anybody	Somebody	Nobody	Each
Everyone	Anyone	Someone	No one	Either
Everything	Anything	Something	Nothing	Neither

Examples:

- Everyone has done his or her homework.
- Somebody has left her purse.
- Someone has my book.

QUESTIONS

- 1) My dog always (growl/growls) at the postman.
- 2) Basketballs (roll/rolls) across the floor.
- 3) I don't (understand/understands) the homework.
- 4) These clothes (is/are) too big for her.
- 5) George doesn't (like/likes) vegetables.
- 6) Corn powder and egg (is/are) needed for the recipe.
- 7) Neither my uncle nor my father (know/knows) how to skip.
- 8) Tomato sauce and cheese (is/are) great on pizza.
- 9) Corned beef and cabbage (is/are) a traditional meal in Ireland.
- 10)The director and producer (are/is) arriving soon.
- 11)Each (get/gets) a trophy for studying.

PARTS OF SPEECH

A **part of speech** is a group of words that are used in a certain way. In other words, all words in the English language are divided into different categories. Each category has a different role or function in the sentence.

In grammar the words are classified based on eight parts of speech they are:-

1. The Noun
2. The Pronoun
3. The Verb
4. The Adverb
5. The Adjective
6. The Preposition
7. The Conjunction
8. The Interjection

Part of speech explains how the word is used and not how the word is. Sometimes the same word can be used as a noun in one sentence and verb or adjective in other sentence.

For example:

In the sentence "He would like a drink" the word "drink" is a noun.

However, in the sentence "We drink too much" the word "drink" is a verb.

So it all depends on the word's role in the sentence.

The Noun:-

A **noun** is a word used to name a person, animal, place, thing, and abstract idea. Nouns are usually the first words which small children learn.

The **underlined** words in the following sentences are all nouns:

Last year my neighbours bought a car.

Sonu Nigam is an Indian singer.

The inspector looked at all the travellers Licence.

According to Rajeev, the museum at Raighad is very old.

Geography gives the outline of a country.

A noun can function in a sentence as a subject, a direct object, an indirect object, a subject complement, an object complement, an appositive, an adjective or an adverb.

Types Of Nouns:-

Proper Nouns

You always write a **proper noun** with a capital letter, since the noun represents the name of a specific person, place, or thing.

Many people dread Monday mornings.

John appears in India.

Common Nouns

This is a noun referring to a person, place, or thing but in general usually, you should write it with a capital letter only when it begins a sentence.

According to this sign, that town is 20 miles away.

People often go on a walk.

Concrete Nouns

This a noun which names anything (or anyone) that you can perceive through your physical senses: touch, sight, taste, hearing, or smell.

The judge handed the files to the clerk.

I spend hours chasing waves.

Abstract Nouns

An **abstract noun** is a noun which names anything which you can *not* perceive through your five physical senses, and is the opposite of a concrete noun.

Childhood memories are the best.

Justice is often hard.

Countable Nouns

This is a noun with both a singular and a plural form, and it names anything (or anyone) that you can count. In each of the following sentences, the **highlighted** words are countable nouns:

We painted the table.

This tree lost three branches in the hurricane.

Non-Countable Nouns

This is a noun which does not have a plural form, and which refers to something that you could not usually count. A non-countable noun always takes a singular verb in a sentence.

Oxygen is essential to human life.

She gave a good advice.

Collective Nouns

This is a noun naming a group of things, animals, or persons. You could count the individual members of the group, but you usually think of the group as a whole is generally as one unit.

The flock of geese.

The jury is dining on.

The Pronoun:-

A pronoun is a word used instead of a noun. It designates a person, place, or thing without naming it.

Nouns and pronouns are called **substantives** .

The substantive to which a pronoun refers is called its **antecedent** .

You are surely my best friend.

I have never met her.

He stole my pen.

The Verb:-

A verb is a word which can assert an action concerning a person, place or thing.

Some verbs express state or condition rather than action.

It takes a good deal of dedication to complete a degree

She studied hard for the test.

The Adverb:-

An adverb is a word which modifies a verb, an adjective or another adverb.

A word or group of words that changes or modifies the meaning of another word is called a modifier.

Adjectives and adverbs are both **modifiers** .

The boldly spoken words would return to haunt the rebel.

The midwives waited patiently through a long labour.

The Adjective:-

An adjective is a word which describes or limits a substantive.

An adjective which describes is called a descriptive adjective; one which points out or designates is called a definitive adjective.

The small boat foundered on the wine dark sea.

The coal mines are dark and dank.

The Prepositions:-

A preposition usually indicates the temporal, spatial or logical relationship of its object to the rest of the sentence

The book is on the table.

The book is under the table.

The book is leaning in the box.

The most common prepositions are "about," "above," "across," "after," "against," "along," "among," "around," "at," "before," "behind," "below," "beneath," "beside," "between," "beyond," "but," "by," "despite," "down," "during," "except," "for," "from," "in," "inside," "into," "like,"

"near," "of," "off," "on," "onto," "out," "outside," "over," "past," "since," "through," "throughout," "till," "to," "toward," "under," "underneath," "until," "up," "upon," "with," "within," and "without."

The Conjunction:-

Conjunction to link words, phrases, and clauses.

I ate the bread and jam.

Call me when you are ready.

"and," "but," "or," "nor," "for," "so," or "yet", "after," "although," "as," "because," "before," "how," "if," "once," "since," "than," "that," "though," "till," "until," "when," "where," "whether," and "while."

The Interjection:-

An interjection is a word added to a sentence to convey emotion.

Ouch, coffee is hot!

Oh no, I lost my money.

Hey! Don't touch that!

Decide which parts of speech are the bold words

1. This is a **beautiful** flower.
 - a. Preposition
 - b. Adjective
 - c. Noun
2. **She** asked me to write this letter.
 - a. Conjunction
 - b. Preposition
 - c. Pronoun
3. She sat **under** the bed to hide from her sister.
 - a. Preposition
 - b. Conjunction
 - c. Pronoun
4. I want to finish this work **quickly** and get some rest.
 - a. Adverb
 - b. Conjunction
 - c. Interjection
5. I love to **work** on Mondays.
 - a. Verb
 - b. Preposition
 - c. Adverb
6. **University** of London is my dream place to study literature.
 - a. Adjective
 - b. Preposition
 - c. Noun
7. I have **met** this person before.
 - a. Verb
 - b. Preposition
 - c. Interjection
8. **Well**, she is too late, I don't think we can wait.
 - a. Interjection
 - b. Preposition
 - c. Noun
9. Rahul was eating pizza **but** his sister stole it from him.
 - a. Adverb
 - b. Adjective
 - c. Conjunction
10. Shall we go for coffee **after** lunch?
 - a. Pronoun
 - b. Preposition
 - c. Verb
11. You should always have faith in **yourself**.
 - a. Adverb
 - b. Pronoun
 - c. Adjective
12. Rakul **left** from her apartment yesterday.
 - a. Adverb
 - b. Noun
 - c. Verb
13. I won't leave from here **until** I meet John.
 - a. Preposition
 - b. Conjunction
 - c. Interjection
14. I am very hungry, would you mind joining me for **breakfast**?
 - a. Pronoun

- b. Adjective
 - c. Noun
15. I don't know what to name that object but it was **heavy**.
- a. Adverb
 - b. Adjective
 - c. Noun
16. Is **it** the same painting which Shima gifted you?
- a. Pronoun
 - b. Noun
 - c. Adverb
17. She is **very** difficult to understand.
- a. Conjunction
 - b. Interjection
 - c. Adverb
18. "Oh! He was so strong, I knew he would win this.
- a. Conjunction
 - b. Interjection
 - c. Adjective
19. **There** is a pen on her table, but it is too costly.
- a. Noun
 - b. Pronoun
 - c. Adverb
20. We got back home late at night **but** we didn't go to sleep immediately.
- a. Pronoun
 - b. Adverb
 - c. Adjective

AGES



This chapter deals with determining the ages of people from different generations. Ages problems are algebra-based word problems, in which we have to find the relation between the **past, present and future** ages of people.

Age is defined as a duration of time that a person has lived, or a thing has existed. It is measured in months, years, decades and so on.

Problems based on ages generally consists of data of ages of two or more persons and a relation between their ages in present or future or past. Using the data, it is asked to calculate the ages of one or more persons in present or future or past.

Important formula for solving Age problems

1. If the present age is X, then n times the age is Xn
2. If the present age is M, then n years later the age is M+n
3. If the present age is M, then n years ago the age is M-n
4. Age in ratio A/B is AX and BX respectively.
5. If present age is A then 1/n times the age is A/n.

Examples based on ages for years ago

1. Seven years ago, the ratio of the ages of Karan and Sunil was 6:5. Five years hence, the ratio of their ages will be 11:10. What is Sunil's age at present?

Sol: $K-7/S-7 = 6/5$. $K+5/S+5 = 11/10$.

$$5(K-7) = 6(S-7).$$

$$5K-35 = 6S-42 \quad \dots(1)$$

$$11(S+5) = 10(K+5)$$

$$11S+55 = 10K+50 \quad \dots(2)$$

Solving 1 and 2 we get

$$S=19 \text{ years.}$$

2. The present age of the father is 6 times the age of his son. Four years ago, the age of the Father was eight times the age of his son at that time. How old is the father 2 years ago?

Sol: Let F and S be the present age of father and son respectively $F = 6S$

$$F-4 = 8*(S-4)$$

$$6S-4 = 8S-32$$

$$2S=28$$

$$S = 14$$

$$F = 6*14 = 84$$

Two years ago, Father's age is $84 - 2 = 82$ years.

Examples based on present age

1. Ravi's age after 10 years will be 4 times his age 4 years back. What is the present age of Ravi?

Sol: Let R be Ravi's present age, $R+10 =$

$$4(R-4) \quad 3R = 26;$$

$$R = 8.66 \text{ years.}$$

2. The age of mother, one-decade age was thrice the age of her daughter. 5 years hence, mother's age will be two times that of her daughter. The proposition of their current ages is:

Sol: Let M and D be the present age of mother and daughter

$$\text{respectively } M-10 = 3(D-10)$$

$$M+5=2(D+5)$$

$$M-3D = -20$$

$$M-2D = 5$$

$$-D=-25; \quad D=25 \text{ years}$$

$$M = 5+50=55 \text{ years}$$

$$D/M = 25/55 = 5/11.$$

Problems based on ages for years hence

1. The ratio between the present ages of P and Q is 5: 6 respectively. If the difference between Q's present age and P's age after 6 years is 3, what is the total of P's and Q's present ages?

Sol: $P/Q = 5/6$

$$Q-(P+6) = 3$$

$$Q-P=9, \quad x=9$$

$$11x= 11*9=99 \text{ years.}$$

Practice Questions Set I

1. Rishabh is 10 years elder than Ram. If 5 years ago, Rishabh was 4 times as old as Ram was, then find Rishabh's present age?
 - A. 18.33 years
 - B. 19.2 years
 - C. 18.7 years
 - D. 19.4 years
2. Apoorva is 40 years old and Rohan is 60 years old. How many years ago was the ratio of their ages 3: 5?
 - A. 10 years
 - B. 15 years
 - C. 20 years
 - D. 25 years
3. The age of mother is thrice that of her daughter. After 12 years, the age of the mother will be twice that of her daughter. What is the present age of the daughter and mother?
 - A. 10, 30
 - B. 5, 15
 - C. 20, 60
 - D. 12, 36
4. My brother is 4 years elder to me. My father was 26 years of age when my sister was born while my mother was 24 years of age when I was born. If my sister was 6 years of age when my brother was born, then what was the age of my father when my brother was born?
 - A. 30
 - B. 32
 - C. 34
 - D. 36
5. The present ages of A, B and C are in proportions 4:6:11. Eight years ago, the sum of their ages was 18. What are their present ages (in years)?
 - A. 8:12:23
 - B. 8:12:22
 - C. 7:12:21
 - D. 4:6:10
6. Leela is as much younger than Manish as he is older than Naga. If the sum of the ages of Manish and Naga is 40 years, what is the difference between Manish and Leela's age?
 - A. 3 yrs.
 - B. 2 yrs.
 - C. 5 yrs.
 - D. Cannot be determined
7. Mr. X has three sons namely P, Q and R. P is the eldest son of Mr. X while R is the youngest one. The present ages of all three of them are square numbers the sum of their ages after 5 years is 44. What is the age of P after three years?
 - A. 15 years
 - B. 13 years
 - C. 19 years
 - D. 17 years
8. The present age of a son is 50% of his father age. And the age of his mother is 200% of his age. The average age of three members is 45. Find the present age of mother.
 - A. 54 years
 - B. 22 years
 - C. 20 years
 - D. 44 years
9. Fifteen years ago, a father was thrice as old as his son. Now the father is only twice as old as his son. Then the present ages of the son and the father is:
 - A. 30, 60
 - B. 20, 40
 - C. 40, 80
 - D. None of these
10. Two years ago, Priya was three times as old as her daughter Sona. 5 years hence, Priya's age will exceed her daughter's age by 8 years. The ratio of the present ages of Priya and her daughter is?
 - A. 7/4
 - B. 8/3
 - C. 7/3
 - D. 8/5

Practice Questions Set II

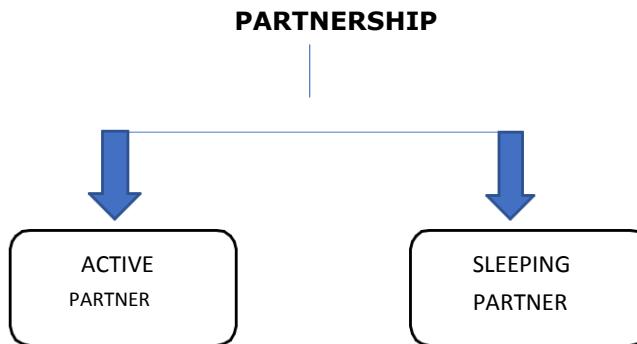
1. One year ago, ratio of Sriram and Sairam's ages was 3: 4 respectively. After 5 years, this ratio becomes 4: 5. How old is Sairam?
 - A. 22 years
 - B. 21 years
 - C. 25 years
 - D. 16 years
2. What is Jawad's age after 40 years, if after 20 years his age will be 10 times his age 10 years back.
 - A. 52.6 years
 - B. 53.3 years
 - C. 63.4 years
 - D. 42.4 years
3. Age of mother 5 years ago was 5 times the age of her son. After 15 years, mother's age will be thrice that of her son. Find the ratio of their present ages.
 - A. 33: 2
 - B. 31: 4
 - C. 11: 2
 - D. 21: 5
4. Sharu's age is 1/5th of her father age. Sharu's father's age will be thrice of Chirag's age after 10 years. If Chirag's 8th birthday was celebrated 2 years ago. Then what is the present age of Sharu?
 - A. 5
 - B. 10
 - C. 15
 - D. 20
5. The ratio of Sushma's age 3 years ago and Rahul's age after 3 years is 1: 1. If at present, the ratio of their ages is 7: 3, then find the ratio between Sushma's age 4 years hence and Rahul's age 4 years ago?
 - A. 28: 3
 - B. 27: 5
 - C. 29: 1
 - D. 30: 7
6. 15 years ago, my Uncle was 6 times older than me. After 10 years from today, my uncle will be twice as old as I will be at 20 years hence. What is the ratio of my age and my uncle age?
 - A. 1: 13
 - B. 1: 12
 - C. 3: 14
 - D. 2: 13
7. Ratio of the ages of A and B is 6:x. A is 20 years younger to C. After 8 years C will be 50 years old. If the difference between the ages of B and A is same as the age of C, what is the value of x?
 - A. 13.5
 - B. 12.35
 - C. 14.5
 - D. 17.45
8. Sitara's present age is $1\frac{3}{5}$ times of her age at the time of marriage. She married 15 yrs. ago. Now she has a son whose age is 1 more than $\frac{2}{5}$ th of her age at the time of marriage. Find the age of son?
 - A. 7 years
 - B. 9 years
 - C. 11 years
 - D. 13 years
9. Rohit's present age is four times his son's present age and he is three-fifth of his father's present age. The average of the present ages of all of them is 45 years. What is the difference between the Rohit's son's present age and Rohit's father's present age?
 - A. 66.56
 - B. 65.57
 - C. 67.54
 - D. 68.1
10. Two years ago, Priya was three times as old as her daughter Sona. 5 years hence, Priya's age will exceed her daughter's age by 8 years. The ratio of the present ages of Priya and her daughter is?
 - A. 7:4
 - B. 8:3
 - C. 7:3
 - D. 8:5

PARTNERSHIP

Partnership: Two or more people can get together to do business by pooling resources. The deal is known as partnership. All the people who have invested money in the partnership are called partners.

Partner: The persons who are involved in partnership are called partners.

Types of Partners:



Active Partner: A person who invests in the business and manages the business is called an **Active Partner** or **Working Partner**.

Sleeping Partner: A person who invests for the business but not managing the business is called a **Sleeping Partner**.

Example 1

In a certain business A is a working and B is a sleeping partner. A invests Rs. 3000 and B puts in Rs.8000. A receives 11.11 % of the profit for managing the business and the rest is divided in proportion to their capital. What does each get out of total profit of 880?

Solution:

$$11.11 \% \text{ of } 990 = 110$$

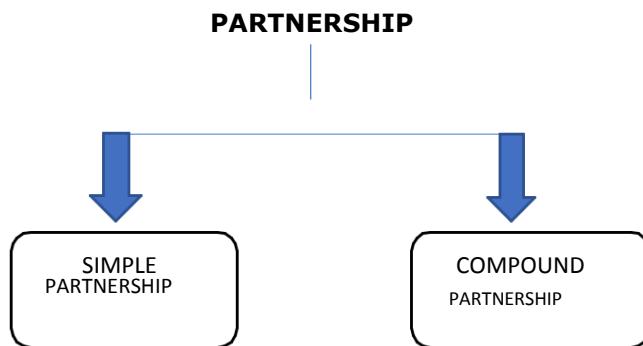
$$\text{Total profit} - 11.11 \% \text{ of profit} = 880$$

$$\text{Ratio of their investment} = 3:8$$

$$\text{A's share of profit} = 880 * 3/11 = 240$$

$$\text{B's share of profit} = 880 * 8/11 = 640$$

Types of Partnership:



Simple Partnership:

If all the partners invest amount in a partnership for same period, then it is called as simple partnership.

If investments of all the partners are for the same time period, the gain or loss is distributed among the partners in the ratio of their investments.

Assume P and Q invest Rs. a and Rs. b respectively for a year in a partnership, then at the

end of the year, (P's share of profit): (Q's share of profit) = a: b

Example 2

A and B started a business by investing Rs.12000/- and Rs.15000/- respectively. Find the A's share out of a total profit of Rs.1800.

Solution:

Investment of A = 12000

Investment of B = 15000

Ratio of their investment = 12000: 15000

4: 5

$$18000 * \frac{4}{9} = 8000$$

Compound Partnership:

If all the partners invest amount in partnership for different period, then it is called as Compound Partnership.

If investments are for different time periods, then equivalent capitals are calculated for a unit of time.

Assume A invests Rs. a for x months and B invests Rs. b for y months then,
(A's share of profit): (B's share of profit) = ax: by.

Example 3

X, Y and Z enter into a partnership, X invests Rs 6000 for 4 months, Y invests Rs 9000 for 5 months and Z invests Rs 12000 for 6 months. At the end of one year they make a profit of Rs 4700. Calculate the share of profit.

Solution:

Investment of x = 8000 for 4 months

Investment of Y = 9000 for 5 months

Investment of Z = 12000 for 6 months

Here, the investment of each person is for different time period,

So, ratio of their investment = $6000 * 4 : 9000 * 5 : 12000 * 6$

$$= 8 : 15 : 24$$

Profit of A = $4700 * 8 / 47$ = Rs. 800

Y's share of profit is = $15/47 * 4700$ = Rs.1500

Z's share of profit is = $24/47 * 4700$ = Rs.2400

Practice Questions Set I

1. Anu, Sonu and Meenu started a business by investing Rs. 1500, Rs.4500 and Rs.7500 respectively. Total profit is Rs.9900. Find the Meenu 's share?
 - A. Rs. 900
 - B. Rs. 1800
 - C. Rs. 4700
 - D. Rs. 5500
2. Sita, Gita started a business in which Sita invested Rs.40000/- for 1year, Gita invested Rs.80000/- for 2 years. At the end of the two years got a profit of Rs.25000/-. What is Sita"s share?
 - A. Rs. 4000
 - B. Rs. 2000
 - C. Rs. 5000
 - D. Rs. 1000
3. Kajal started a software business by investing Rs. 60,000. After 6 months, Radha joined her with a capital of Rs. 96,000. After 3 years, they earned a profit of Rs. 24,500. What was Kajal"s share in the profit?
 - A. Rs. 10110
 - B. Rs. 10500
 - C. Rs. 12000
 - D. Rs. 13000
4. P, Q, R enter into a partnership & their share are in the ratio 1/4: 1/3: 1/2, after two months, R withdraws half of the capitals & after 10 months, a profit of Rs.336 is divided among them. What is R's share?
 - A. Rs. 112
 - B. Rs. 154
 - C. Rs. 164
 - D. None of these
5. Mahesh started a business investing Rs. 60,000. After 3 months, Praveen joined him with a capital of Rs.75,000. After another 6 months, Ankur joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 1,60,000. What would be Ankur"s share?
 - A. Rs. 30000
 - B. Rs. 45000
 - C. Rs. 60000
 - D. Rs. 66000
6. In a business, A and C invested amounts in the ratio 4: 7, whereas the ratio between amounts invested by A and B was 6: 5. If Rs 301000 was their profit, how much amount did B receive?
 - A. Rs. 48600
 - B. Rs. 70000
 - C. Rs. 48500
 - D. Rs. 44800
7. Arun invested Rs 84000 in a business. After few months, Arjun joined him with Rs 72000. The total profit was divided between them in the ratio 2: 1 at the end of the year. After how many months did Arjun join?
 - A. Rs. 48600
 - B. Rs. 70000
 - C. Rs. 48500
 - D. Rs. 44800
8. A, B and C enter into a partnership. A initially invests Rs. 35 lakhs and adds another Rs. 10 lakhs after 1 year. B initially invests Rs. 45 lakhs and withdraws Rs.10 lakhs after 2 years and C invests Rs. 40 lakhs. In what ratio should the profits be divided at the end of 3 years?
 - A. 20: 19: 18
 - B. 20: 20: 19
 - C. 10: 10: 9
 - D. 25: 25: 24
9. Anitha started a business by investing Rs.5000/-. After 4 months Bindhu joined by investing Rs.4000/- After 2 months, Chandu joined by investing Rs.3000/-. At the end of the year, they got a profit of Rs.5500/-. What is A"s share?
 - A. Rs. 900
 - B. Rs. 1600
 - C. Rs. 3600
 - D. Rs. 3000

10. Pranith and Prakash invested in a business. The profit earned was divided in the ratio 2: 3. If Pranith invested Rs.60000, the amount invested by Prakash is
- A. Rs. 40000
 - B. Rs. 90000
 - C. Rs. 50000
 - D. Rs. 70000
4. Ashok started a business investing Rs.80000. After 3 months, Sameer joined him with a capital of 120000. If at the end of a year, the total profit made by them was Rs.105000, what will be the difference between their shares?
- A. Rs. 20000
 - B. Rs. 21000
 - C. Rs. 24000
 - D. None of these

Practice Questions Set II

1. A, B and C subscribe Rs. 60,000 for a business, A subscribes Rs. 5000 more than B and B subscribes Rs.3000 more than C. Out of total profit of Rs. 61000, A receives.
- A. Rs. 24000
 - B. Rs. 15000
 - C. Rs. 12000
 - D. Rs. 13500
2. A, B, C started a partnership business by investing Rs.24000, Rs.64000, Rs.72000 respectively. At the end of the year, the profit was distributed among them. If C's share of profit is Rs.54000. What is the total profit?
- A. Rs. 120000
 - B. Rs. 90000
 - C. Rs. 70000
 - D. Rs. 110000
3. Sindhu and Roopa started a partnership business investing capital in the ratio of 8 :7. C joined in the partnership after six months with an amount equal to that of B. At the end of one year, the profit should be distributed among A, B and C in proportion.
- A. 16: 14: 7
 - B. 10: 5: 4
 - C. 3:4:5
 - D. 9:5:4
5. A, B started a business by investing Rs. 20000 and Rs. 30000 respectively. After one- year B withdrew Rs.5000 and A withdrew Rs.10000 more. At the end of 2 years they got total Profit Rs.8500. What is A's share?
- A. Rs. 3500
 - B. Rs. 2800
 - C. Rs. 2000
 - D. Rs. 2500
6. A and B enter into a partnership and A invests Rs. 30,000 in the partnership. At the end of 4 months he withdraws another Rs. 5000. At the end of another 5 months, he withdraws Rs.5000. If B invests a certain sum in the partnership at the beginning of year and leaves it intact and receives Rs.15600 as his share of total profit of Rs.46100 for the year, how much did B invest in the company?
- A. Rs. 12000
 - B. Rs. 96000
 - C. Rs. 13000
 - D. Rs. 6000
7. A started a business with Rs 28000 and is joined afterwards by B with Rs 56000. After how many months did B join if the profits at the end of the year are divided equally?
- A. 6 months
 - B. 3 months
 - C. 4 months
 - D. 8 months

8. A, B, C rent a pasture. If A puts 12 oxen for 7 months, B puts 9 oxen for 4 months and C puts 8 oxen for 3 months for grazing and the rent of the pasture is Rs. 192, then how much amount should C pay as his share of rent?

- A. Rs. 44
 - B. Rs. 47
 - C. Rs. 46
 - D. Rs. 32
9. A and B invest in a business in the ratio 5: 4. If 10 % of the total profit goes to charity and A's share is Rs.1000, the profit is

- A. Rs. 500
 - B. Rs. 2000
 - C. Rs. 1000
 - D. Rs. 1500
10. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Rs. 10000 for 6 months, B, Rs.7500 for 5 months and C, Rs.12,000 for 3 months. A want to be the working member for which, he was to receive 5% of the profits. The profit earned was Rs.9181. Calculate the share of B in the profit.
- A. Rs. 1900
 - B. Rs. 2450
 - C. Rs. 2800
 - D. Rs. 2840