$$1 + nc + c + nc^2$$
$$= 1 + (n+1)c + nc^2 \geqslant 1 + (n+1)c$$

HW1

0.3.6 (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$

$\therefore x \in A$ and $x \in B$ or $x \in C$

if $x \in B$ then $x \in A \cap B$

$\therefore$ for C $\therefore x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Let $x \in (A \cap B) \cup (A \cap C)$

if $A \cap B$ then $x \in A$ and $x \in B$

~~so~~ x is in $B \cup C$ also $\therefore x \in A \cap (B \cup C)$

similarly for C $\therefore x \in A \cap (C \cup B)$

$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$ $\begin{cases} A & A \cup B \text{ and } A \cup C \\ B \cap C & B \cup A \text{ and } C \cup A \end{cases}$

$x \in A$ or $x \in B \cap C$ $\therefore x \in B$ and $x \in C$

$\therefore x \in A$ or B and x A or C $\therefore$ LHS $\subseteq$ RHS

RHS :- $x \in A$ or $\in A$ $A \cup B$ $A \cup C$ if $\notin A$ $\in B$, $\in C$ $\therefore$ RHS $\subseteq$ LHS

0.3.11 $n < 2^n \quad \forall n \in \mathbb{N}$ $\quad 2^n$

Base: ① $1 < 2$

Induction ② $n+1 < 2^n + 1 < 2^n + 2^n = 2^{n+1}$

0.3.12 set A $\quad |A| = n \quad P(A) = 2^n$

0 to n $\quad \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$

$1 + n + \dots + 1$

for: Base $n=1$ $\quad P(A) = \{\phi, A\} \quad \#P(A) = 2$

Induction +1 $\quad$ __ __ __ __ ... n times

yes no

1 0

$2 \cdot 2 \cdot 2 \cdot 2 \dots \quad \therefore 2^n$

induction: $n+1$

n has 2^n

+1 can be in each of 2^n and it will be unique

$\therefore 2^n$ also

$\therefore 2^n + 2^n = 2^{n+1}$ $\quad \square$

0.3.15. $n^3 + 5n \quad \text{mod } 6 = 0$

Base: $1 = 6 \text{ mod } 6 = 0$

Induction: $(n+1)^3 + 5(n+1)$

$= n^3 + 3n^2 + 3n + 1 + 5n + 5$

$= n^3 + 5n + \cancel{n^2+3}$

$3n^2 + 3n + 6$

$3\cancel{(n^2+n+2)}$

$3\underbrace{n(n+1)}_{\text{even} \therefore 2m} + 6$

$n-1$

$n^2 + 2n + 1$

$n^3 + 2n^2 + n$

$= n^2 + 2n + 1$

$n^3 + 5n \quad + \quad 3 \cdot 2 \cdot m \quad + \quad 6$

mod 6 = 0 $\quad$ mod 6 = 0 $\quad$ mod 6 = 0

0.3.19 eg $A_n = \{n\} \quad n \in \mathbb{N}$

$A_1 = \{1\} \quad A_2 = \{2\} \dots$

countably infinite

can put in sequence.

6) $|\{q \in \mathbb{Q} : q > 0\}| = |\mathbb{N}|$

rational st. > 0

1) if $\frac{4}{15} = \frac{2^2}{3^1 \cdot 5^1} = 2^{2\cdot 2} \cdot 3 \cdot 5$

$16 \quad \cancel{20} \quad 15$

$= 60 \times 4 = 240$

$2\,|\,108 \quad 2\,|\,54 \quad 3\,|\,27 \quad 3\,|\,9 \quad 3$

$3\,|\,108 \quad 3\,|\,36 \quad 2\,|\,12 \quad 2\,|\,6 \quad 3$

2e $\quad 108 = 2^2\, 3^3$

$2^{2\cdot(1)}\, 3^{2\cdot(3/2)}$ or $2^{2(1)} \cdot 3^{2(2)-1}$

$\therefore \frac{2}{3^2} = \frac{2}{9} \qquad f(2/9) = 108$

2) injectivity (one to one) Then for a q, unique primes

if $f(q_1) = f(q_2)$ then $q_1 = q_2$

onto $\because$ every $n \in \mathbb{N}$ is a q

$\therefore$ has ~~a~~ unique $f(q)$

$\therefore$ bijective