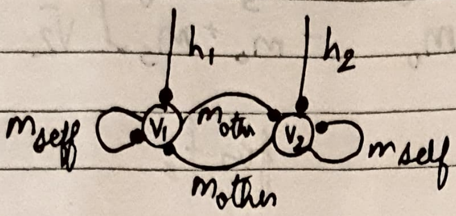
MIT OCW

9.40 PS 7 RECURRENT NETWORKS



eig vectors $\Phi = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ $\quad M = \begin{bmatrix} m_{self} & m_{other} \\ m_{other} & m_{self} \end{bmatrix}$

Assuming linear $F(x) = x$

Q1

$w = m$

$$\tau_n \frac{dv_1}{dt} = -v_1 + h_1 + w_{self} V_1 + w_{other} V_2$$

$$\tau_n \frac{dv_2}{dt} = -V_2 + h_2 + w_{self} V_2 + w_{other} V_1$$

$$\therefore \tau_n \frac{d\bar{v}}{dt} = -\bar{v} + \bar{h} + M\bar{v} \qquad \bar{v} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \bar{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$\therefore \tau_n \frac{d\bar{v}}{dt} = -(I - M)\bar{v} + \bar{h}$$

Q2: $\because M\hat{f}_i = \lambda_i \hat{f}_i$ $\qquad \underbrace{(M - \lambda_i I)}_{0}\underbrace{\hat{f}_i}_{\neq 0} = 0$

Eigen val: $\det(M - \lambda I) = 0$

$$\det\begin{bmatrix} m_{self} - \lambda & m_{other} \\ m_o & m_{self} - \lambda \end{bmatrix} = 0 \qquad m_o = m_{other},\ m_s = m_{self}$$

$$(m_s - \lambda)^2 - m_o^2 = 0$$

$$m_s^2 + \lambda^2 - 2m_s\lambda = m_o^2$$

$$\therefore \lambda = \frac{+2m_s \pm \sqrt{4m_s^2 - 4(m_s^2 - m_o^2)}}{2} \qquad \because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{+2m_s \pm \sqrt{4m_o^2}}{2} = \boxed{+m_s \pm m_o}$$

$$= m_s + m_o,\ m_s - m_o$$

Proof: $\begin{bmatrix} m_s & m_o \\ m_o & m_s \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} m_s + m_o & -m_s + m_o \\ m_s + m_o & -m_o + m_s \end{bmatrix} \frac{1}{\sqrt{2}}$

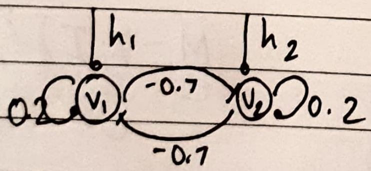
for $\hat{f}_1$ for $\hat{f}_2$

$M \cdot \hat{f}_1 = \underbrace{(m_s + m_o)}_{\lambda_1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$ $M \cdot \hat{f}_2 = \underbrace{(m_s - m_o)}_{\lambda_2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$

Q3: $\bar{h} = \left(\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \cdot \hat{f}_1 \right) \hat{f}_1 + \left(\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \cdot \hat{f}_2 \right) \hat{f}_2$

$= \underbrace{\left(\frac{h_1 + h_2}{\sqrt{2}} \right)}_{\substack{b_1 \\ = h_{common}}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \underbrace{\left(\frac{-h_1 + h_2}{\sqrt{2}} \right)}_{\substack{b_2 \\ = h_{diff}}} \hat{f}_2$

Part 2: $m_s = 0.2$ $m_o = -0.7$

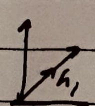


$\bar{h}(t \geq 0) = \begin{pmatrix} 117 \\ 123 \end{pmatrix} Hz$

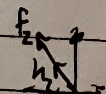
1) Eigen val: $\lambda_1 = -0.5$ $(m_s + m_o)$

$\lambda_2 = 0.9$ $(m_s - m_o)$

2) attenuated : $\lambda_1 = -0.5$, $\hat{f}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$



3) amplified : $\lambda_2 = 0.9$ $f_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}}$



4) $g_{u_1} = \frac{1}{1-(-0.5)} = \frac{1}{1.5} = \frac{2}{3}$ $g_{u_2} = \frac{1}{1-0.9} = \frac{1}{0.1} = 10$

attenuated   amplified

5) $\tau_{eff} = \dfrac{\tau}{1-\lambda}$ $\quad \therefore \tau_{eff_1} = \dfrac{\tau}{1+0.5} = \dfrac{18}{1.5} = 12\,ms$ more rapid

$$\boxed{\tau_{eff_2} = \frac{18}{0.1} = 180ms \text{ less rapidly}}$$

6) $\tau_n \dfrac{dc_1}{dt} = -c_1 + (0.5)c_1 + \cancel{\begin{pmatrix}117\\123\end{pmatrix}} \underbrace{\dfrac{290}{\sqrt{2}}}_{\Phi^T \bar{h}}$

$\tau_n = 18\,ms$

$$c_1 = \frac{1}{\sqrt{2}}(1 \;\; 1)\begin{pmatrix}v_1\\v_2\end{pmatrix} = \frac{v_1+v_2}{\sqrt{2}}$$

$$\tau_n \frac{dc_2}{dt} = -c_2 + 0.9c_2 + \cancel{\begin{pmatrix}117\\123\end{pmatrix}} \underbrace{\frac{6}{\sqrt{2}}}_{\Phi^T \bar{h}}$$

$$\therefore \cancel{c_\infty}\; c_2 = \frac{1}{\sqrt{2}}(-1 \;\; 1)\begin{pmatrix}v_1\\v_2\end{pmatrix} = \frac{-v_1+v_2}{\sqrt{2}}$$

7) $\bar{h} = \underbrace{\dfrac{290}{\sqrt{2}}}_{b_1}\hat{f}_1 + \underbrace{\dfrac{6}{\sqrt{2}}}_{b_2}\hat{f}_2$

$$\tau_n \frac{d\bar{c}}{dt} = -\bar{c} + \begin{pmatrix}-0.5 & \\ & 0.9\end{pmatrix}\bar{c} + \begin{pmatrix}290/\sqrt{2}\\6/\sqrt{2}\end{pmatrix}$$

$$\Downarrow$$

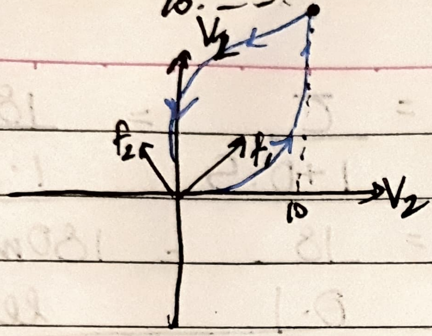
$$0 \quad \bar{c}_\infty = \begin{pmatrix}\frac{1}{1.5} & \\ & \frac{1}{0.1}\end{pmatrix}\begin{pmatrix}290/\sqrt{2}\\6/\sqrt{2}\end{pmatrix}$$

$$\bar{v}_\infty = \Phi \bar{c}_\infty = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}2/3 & \\ & 10\end{pmatrix}\begin{pmatrix}290/\sqrt{2}\\6/\sqrt{2}\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} & \\ & \end{pmatrix} \quad \cancel{160} \quad \begin{matrix}160\\60\end{matrix}$$

$$\bar{v}_\infty = \begin{pmatrix}\cancel{110}\\\cancel{20}\\50\end{pmatrix} \quad \begin{pmatrix}50\\110\end{pmatrix}$$

(8)



This is inaccurate

Nullclines look like