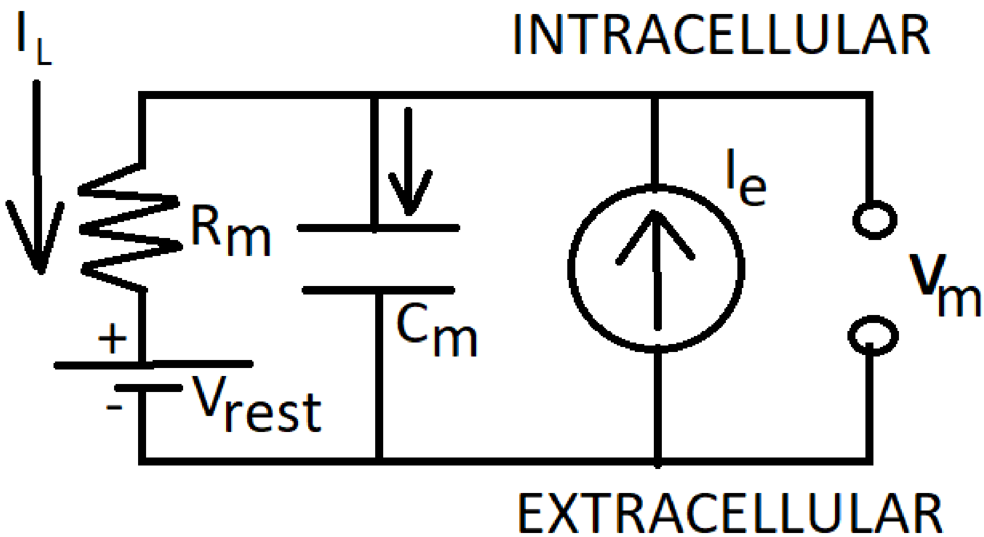
Problem Set 1

Question 2

Shaurya Goyal

January 9, 2022

1



2

Specific membrane conductance $g_m = 1\ \mu S/mm^2$

Radius of spherical neuron $r = 0.06\ mm$

Surface area of a sphere $A = 4\pi r^2 = 144\pi \times 10^{-4}\ mm^2$

Total membrane conductance $\boxed{G_m = g_m \times A = 144\pi \times 10^{-10}\ S}$

Total membrane resistance $\boxed{R_m = \frac{1}{G_m} = \frac{10^{10}}{144\pi}\ \Omega}$

3

Specific membrance capacitance $c_m = 10\ nF/mm^2$

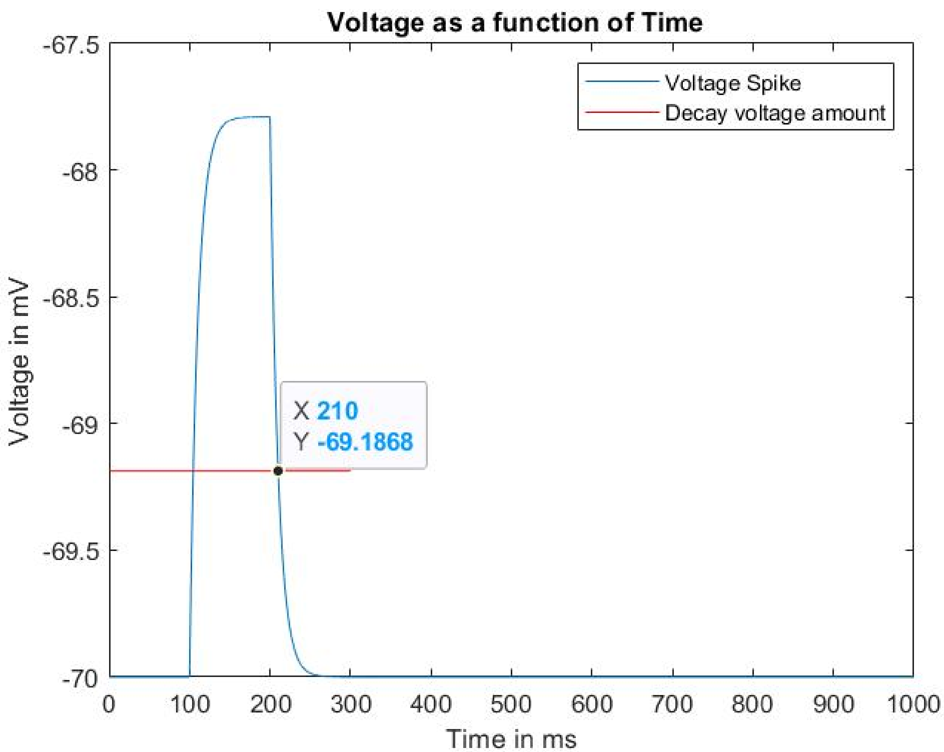
Total membrane capacitance $\boxed{C_m = c_m \times A = 144\pi \times 10^{-12}\ F}$

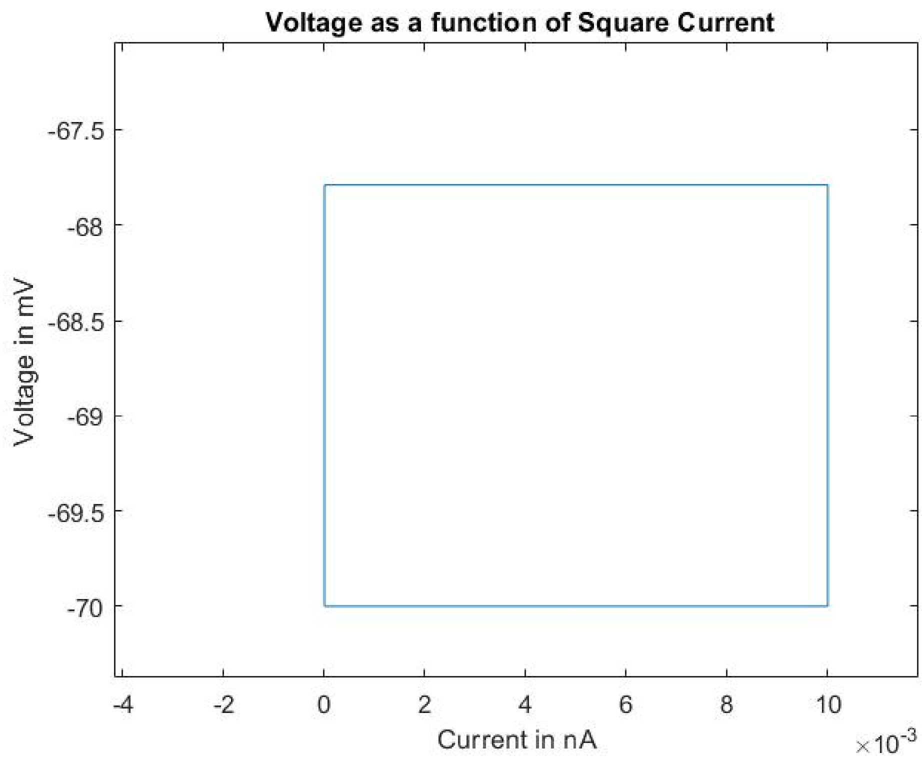
4

Time constant $\boxed{\tau_m = R_m C_m = 10^{-2}\ seconds}$

5 MATLAB code in RCpassive.m

6





7

From plotting the decayed voltage amount (i.e $\frac{1}{e}$ times the total voltage change) in the "Voltage as a function of Time" graph in part 6, it is observed that the decayed voltage amount line intersects the the voltage graph at time = 210 ms

The neuron spikes at time = 200 ms

Therefore we can calculate the time constant from the graph as

$$\tau = \text{Time of intersection} - \text{Time of Spiking}$$
$$\boxed{\tau = 10\ ms}$$

8

This answer in part 7 obtained from simulation matches the time constant that was calculated analytically in part 4

9

The differential equation of the neuron is

$$\boxed{V(t) + \tau_m \frac{dV(t)}{dt} = V_{rest} + R_m T_e}$$

The equation can be rearranged and rewritten as

$$\boxed{\frac{dV(t)}{dt} = \frac{V_{rest} + R_m T_e - V(t)}{\tau_m}}$$

where, t is integrated from t_0 to t
V is integrated from V_0 to $V(t)$

On separating the variables and integrating within the limits, we get

$$\boxed{\log \frac{V_{rest} + R_m T_e - V(t)}{V_{rest} + R_m T_e - V_0} = -\left(\frac{t - t_0}{\tau_m}\right)}$$

This can be rearranged to get the equation in the question

$$\boxed{V(t) = V_{rest} + R_m I_e + (V_0 - V_{rest} - R_m I_e)\ exp\left(\frac{t - t_0}{-\tau_m}\right)}$$

□

10

At $t = t_0$, $V(t) = V_0$

On substituting these values of t and $V(t)$ in the final equation from part 9, we get,

$$\boxed{V_0 = V_{rest} + R_m I_e + (V_0 - V_{rest} - R_m I_e)\ \exp\left(\frac{t_0 - t_0}{-\tau_m}\right)}$$

It is observed that both the sides of the above equation cancel out.

Hence $V(t_0) = V_0$ satisfies the equation. $\square$

11

As $t \to \infty$, $\quad \exp\left(\frac{t_0 - t_0}{-\tau_m}\right) \to 0.$

Therefore

$$\boxed{V(t) \to V_{rest} + R_m I_e = V_\infty}$$

$\square$

12

$I_e = 0.5\ nA$

$V(t) = -60\ mV$

The equation is rewritten in terms of time t as

$$\boxed{t = \tau_m \ln\left(\frac{R_m I_e}{R_m I_e - V(t) + V_{rest}}\right)}$$

On substituting the values, we get

$$\boxed{t = 23.5\ ms}$$

The steady state voltage formula is

$$\boxed{V_\infty = V_{rest} + R_m I_e}$$

On substituting the values, we get

$$\boxed{V_\infty = -58.9476\ mV}$$

13

The resistance in parallel (R_p) is added as follows to the original resistance R

$$\text{Combined membrane resistance } \boxed{R_m = \frac{R \times R_p}{R + R_p}}$$

Any addition of resistance in parallel causes the total resistance of the membrane to drop.

Since the time constant $\boxed{\tau_m \propto R_m}$

A decrease in membrane resistance causes the time constant to decrease i.e the time constant becomes faster.