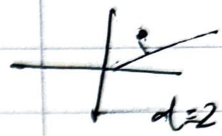
PS4

Q2 if $V = \{\alpha v : \alpha \in \mathbb{R}\}$ — v: unit length

then $f_v(x) = \underset{v \in V}{\operatorname{argmin}} \|x - v\|^2 = \underbrace{\underset{\alpha \in \mathbb{R}}{\operatorname{argmin}} \|x - \alpha v\|_2^2}_{\alpha} \cdot v$ — $f_v(x)$: proj x along v

Show $\underset{u : u^T u = 1}{\operatorname{argmin}} \sum_{i=1}^{n} \|x^{(i)} - f_v(x^{(i)})\|_2^2$ is 1st PC



$d=2$

$$(x - \alpha v)^T (x - \alpha v) = x^T x - x^T \alpha v - (\alpha v)^T \cdot x + (\alpha v)^T (\alpha v)$$

$$\frac{\partial}{\partial \alpha} \quad = x^T x - 2x \cdot \alpha v + (\alpha v)^T (\alpha v)$$

$$= 0 - 2x \cdot v + 2\alpha v^T v$$

$$\therefore \alpha = \underbrace{\frac{x^T v}{v^T v}}_{\in \mathbb{R}^1} = x^T v \quad \because v^T v = \text{unit}$$

$$f_v(x) = (x^T v) v$$

$$\underset{v : v^T v = 1}{\operatorname{argmin}} \sum_{i=1}^{n} \|x^{(i)} - (x^{(i)T} v) v\|_2^2$$

$$(x^{(i)} - (c) v)^T (x^{(i)} - cv)$$

$$\sum_{i=1}^{n} x^{(i)T} x^{(i)} - 2c\, x^{(i)T} v + c^2 \underbrace{v^T v}_{1}$$

$$- 2c\, x^{(i)}$$

$$= \underset{v : v^T v = 1}{\operatorname{argmin}} \sum_{i=1}^{n} \underbrace{x^{(i)T} x^{(i)}}_{\text{irrelevant}} - 2(v^T x^{(i)})^2 + (v^T x^{(i)})^2 \underbrace{v^T v}_{1}$$

$$= \underset{v : v^T v = 1}{\operatorname{argmin}} \sum_{i=1}^{n} -(v^T x^{(i)})^2 \quad = (v^T x)^T (v^T x) = (v^T x)(v^T x) \quad v^T x\, x^T v$$

$$= \underset{v : v^T v = 1}{\operatorname{argmax}}\ v^T \left(\sum_{i=1}^{n} x^{(i)} x^{(i)T} \right) v$$

Q ~~4~~3. $s \in \mathbb{R}^d$ source data $x \in \mathbb{R}^d$ observed data

$x = As$ $\quad s = Wx \quad W = \begin{bmatrix} - w_j - \end{bmatrix}$ $j = 1, \dots, d$

$\underset{\text{mix matrix}}{w}$ $\quad \overset{=}{A^{-1}}$ $\quad w_j \in \mathbb{R}^d$

$\mathbb{R}^{d \times d}$

$\therefore s_j = w_j^T x$

$X \in \mathbb{R}^{n \times d}$ $\begin{bmatrix} - x_i^{(i)} - \\ \\ \end{bmatrix}$

(a) $\ell(W) = \sum_{i=1}^{n} \left(\log|W| + \sum_{j=1}^{d} \log g'(w_j^T x^{(i)}) \right)$

g: cdf

$g' = c\exp\left(-\frac{1}{2}(w_j^T x_{(i)})^2\right)$ $\quad n\log|W| + \sum_{i=1}^{n} \sum_{j=1}^{d} \log g'(w_j^T x^{(i)})$ $\quad g'$: pdf of source

$\nabla_W \ell(W) = nW^{-T} + \nabla_W C - \nabla_W \frac{1}{2} \sum_i \sum_j (w_j^T (x^{(i)}))^2$

$= nW^{-T} + 0 - \sum_i \sum_j w_j^T (x^{(i)} x^{(i)T}) = 0$

$nW^{-T} = W \sum_i x^{(i)} x^{(i)T}$

$= W X^T X$

$W^{-1} W^{-T} = \frac{X^T X}{n}$

$W^T W = \left(\frac{1}{n} X^T X\right)^{-1}$

~~..~~ True also for all $W = QW$ where Q is orthogonal matrix. As $W^T W = W^T Q^T Q W = W^T W$

$\therefore$ W has inherent ambiguity as susceptible to rotation so don't know which source where

(b) $s \sim \mathcal{L}(0,1)$ $\quad f_{\mathcal{L}}(s) = \frac{1}{2}\exp(-|s|)$

$$\ell(W) = \sum \log|W| + \sum_i \sum_j \log \frac{1}{2} e^{-|w_j^T x^{(i)}|}$$

$\nabla_W \ell(W) = \cancel{\Sigma}.$

$$\underbrace{\phantom{\sum_i \sum_j \log \frac{1}{2}}}_{-\frac{1}{2}\sum_i \sum_j |w_j^T x^{(i)}|}$$

$$\nabla_W \ell(W) = \sum_i \left((W^{-1})^T - \sum_j \operatorname{sign}[w_j^T x^{(i)}]\, x^{(i)T} \right)$$

$$W_{t+1}^{(i)} \leftarrow W_t^{(i)} + \alpha \left[W^{-T} - \sum_{j=1}^{d} \operatorname{sign}(w_j^T x^{(i)})\, x^{(i)T} \right)$$

2018 2: $\pi_i(s,a) = p(a \mid s, \pi_i)$ $\quad \pi_0(s,a) = p(a|s)$

π_1 target to evaluate $\quad p(s)$: state distribution

$$\mathbb{E}_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a) = \sum_{s,a} R(s,a)\,p(s,a) \qquad \pi_0(s,a)>0$$

$$R(s,a)\,p(s)\,\underset{\pi_1(s,a)}{p(a|s)}$$

Regression:

(a) $\mathbb{E}_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} \frac{\pi_1(s,a)}{\hat{\pi}_0(s,a)} R(s,a)$ $\quad$ show if $\pi_0 = \hat{\pi}$, imp sc $= \mathbb{E}_{s,a} R(s,a)$

$$= \mathbb{E}_{\substack{s\sim p(s)\\ a\sim\pi_0(s,a)}} \frac{\pi_1(s,a)}{\pi_0(s,a)} R(s,a)$$

$$= \sum_{s,a} \frac{\pi_1(s,a) R(s,a)}{\pi_0(s,a)}\, p(s)\, \pi_0(s,a)$$

$$\sum_{(s,a)} p(s)\,\pi_1(s,a)\, R(s,a) = \mathbb{E}_{\substack{s\sim p(s)\\ a\sim\pi_1(s,a)}} R(s,a)$$

(b) $\frac{\mathbb{E}_{\sim\pi_0} \frac{\pi_1}{\hat{\pi}_0} R}{\mathbb{E}_{\sim\pi_0} \frac{\pi_1}{\hat{\pi}_0}}$ $\quad$ if $\hat{\pi}_0 = \pi_0$ $\quad \mathbb{E}_{\sim\pi_1} R(s,a)$

$$\frac{\sum \frac{\pi_1}{\pi_0} R\, p(s)\, \pi_0}{\sum \frac{\pi_1}{\pi_0} p(s)\, \pi_0} = \frac{\mathbb{E}_{\sim\pi_1} R(s,a)}{\sum_{\substack{s\sim p(s)\\ a\sim\pi_0}} p(s,a)} = \frac{\mathbb{E}_{\sim\pi_1} R(s,a)}{1}$$

(c) $\Rightarrow \frac{\sum_{(s,a)} p(s,a) \frac{\pi_1}{\hat{\pi}_0} R}{\sum_{s,a} p(s,a) \frac{\pi_1}{\hat{\pi}_0}} = \frac{p(s,a) \frac{\pi_1}{\pi_0} R}{p(s,a) \frac{\pi_1}{\pi_0}} = R$ $\quad$ if $\hat{\pi}_0 \neq \pi$, can't correct $\mathbb{E}_{\sim\pi_0} R \neq \mathbb{E}_{\sim\pi_1} R$

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d (i) $\cancel{\mathbb{E}_{a\sim\pi_1} \hat{R} = \sum_{(S,A)}\left(\sum_a \hat{R}\,\hat{p}^{\pi_1}(a|s)\right) p(s)\,\pi_0(a|s)}$

$$\mathbb{E}_{\substack{s\sim p\\ a\sim\pi_0}}\left(\mathbb{E}_{a\sim\pi_1} \hat{R}(s,a)\right) = \mathbb{E}_{\substack{s\sim p\\ a\sim\pi_1}} \hat{R}$$

if $\hat{\pi}_0 = \pi_0$, $\quad \mathbb{E}_{\substack{s\sim p\\ a\sim\pi_0}} \frac{\pi_1}{\pi_0}\left(R(s,a) - \hat{R}(s,a)\right)$

$$\therefore \mathbb{E}_{\substack{s\sim p\\ a\sim\pi_1}} R(s,a) - \hat{R}(s,a) \qquad + \hat{R}$$

$$= \mathbb{E}_{\substack{s\sim p\\ a\sim\pi_1}} R(s,a)$$

(ii) $\mathbb{E}_{\substack{s\sim p\\ a\sim\pi_0}} \mathbb{E}_{a\sim\pi_1} R = \mathbb{E}_{\substack{s\sim p\\ a\sim\pi_1}} R$ $\qquad R(s,a) - \hat{R}(s,a)$

$R(s,a) - R(s,a) = 0$

(e) (i) Importance, Estimate $\hat{\pi}_0$ easier

(ii) Regression, Estimating $\hat{R}(s,a)$ easier

$$\left|\sum_{s'\in S} P_{sa}(s')(V_1(s') - V_2(s'))\right| \leq \sum_{s'\in S} P_{sa}(s')\left|V_1(s') - V_2(s')\right| \leq \|V_1 - V_2\|_\infty$$

$$V'(s) = R(s) + \gamma \max_{a\in A} \sum_{s'\in S} P_{sa}(s') V(s') \qquad \underbrace{\sum_{s'\in S} P_{sa}(s')}_{=1}$$

$V' = B(V)$

(a) $\|B(V_1) - B(V_2)\|_\infty \leq \gamma \|V_1 - V_2\|_\infty$

where $\|V\| = \max_{s\in S} |V(s)|$

$$\|B(V_1) - B(V_2)\|_\infty = \gamma \left\| \max_{a\in A} \sum_{s'\in S} P_{sa}(s')(V_1(s') - V_2(s'))\right\|_\infty$$

$$= \gamma \max_{s'\in S} \left[\max_{a\in A} \sum_{s'\in S} P_{sa}(s')(V_1(s') - V_2(s'))\right]_\infty$$

$$\leq \gamma \|V_1 - V_2\|_\infty$$

: if $\alpha, x \in \mathbb{R}^n$ $\sum_i \alpha_i = 1$ then $\sum_i \alpha_i x_i \leq \max_i x_i$ $\alpha_i \geq 0$

(b) $B(V) = V$ fixed point

$$B(V) = V = V(s) = R(s) + \gamma \max_{a\in A} \sum_{s\in S} P_{sa}(s') V(s')$$

Let V_1 & V_2 be fixed pts

$$\|V_1 - V_2\|_\infty = \|B(V_1) - B(V_2)\|_\infty \leq \gamma \|V_1 - V_2\|_\infty$$

Contradic: if $\|V_1 - V_2\|_\infty > 0$, divide bs to get $\gamma \geq 1$ but false.

$\therefore \|V_1 - V_2\|_\infty = 0$

$\therefore V_1 = V_2$