

P2)

$$\begin{aligned}\underline{(a)} \quad P(R) &= P(R|S)P(S) + P(R|\neg S)P(\neg S) \\ &= (0.7)(0.25) + (0.3)(0.75) \\ &= \underline{0.4} \text{ Ans}\end{aligned}$$

$$\begin{aligned}\underline{(b)} \quad P(ue) &= P(ue, R, u_0) + P(ue, R, \neg u_0) + P(ue, \neg R, u_0) \\ &\quad + P(ue, \neg R, \neg u_0) \\ &= P(ue|R, u_0)P(u_0|R)P(R) + P(ue|R, \neg u_0) \\ &\quad P(\neg u_0|R)P(R) + P(ue|\neg R, u_0)P(u_0|\neg R)P(\neg R) \\ &\quad + P(ue|\neg R, \neg u_0)P(\neg u_0|\neg R)P(\neg R) \\ &= (0.12)(0.7)(0.4) + (0.25)(0.3)(0.4) + (0.1)(0.2)(0.6) + \\ &\quad (0.08)(0.8)(0.6) \\ &= \underline{0.114} \text{ Ans.}\end{aligned}$$

$$\underline{(c)} \quad P(M|S) = \frac{P(M, S)}{P(S)}$$

$$P(S) = 0.25$$

$$\begin{aligned}P(M, S) &= P(M, S, ue, u_0, R) + P(M, S, ue, u_0, \neg R) + P(M, S, ue, \neg u_0, R) + \\ &\quad P(M, S, ue, \neg u_0, \neg R) + P(M, S, \neg ue, u_0, R) + P(M, S, \neg ue, u_0, \neg R) + \\ &\quad P(M, S, \neg ue, \neg u_0, R) + P(M, S, \neg ue, \neg u_0, \neg R)\end{aligned}$$

$$\begin{aligned}P(M, S, ue, u_0, R) &= P(M|ue)P(ue|u_0, R)P(u_0|R)P(R|S)P(S) \\ &= (0.02)(0.12)(0.7)(0.7)(0.25) \\ &= 0.000294\end{aligned}$$

$$\begin{aligned}P(M, S, ue, u_0, \neg R) &= P(M|ue)P(ue|u_0, \neg R)P(u_0|\neg R)P(\neg R|S)P(S) \\ &= (0.02)(0.1)(0.2)(0.3)(0.25) \\ &= 0.00003\end{aligned}$$

$$\begin{aligned}
 P(M, S, \neg U_e, \neg R) &= P(M | U_e) P(U_e | \neg R) P(\neg R | S) P(S) \\
 &= (0.02)(0.25)(0.3)(0.7)(0.25) \\
 &= 0.0002625
 \end{aligned}$$

$$\begin{aligned}
 P(M, S, \neg U_e, R) &= P(M | U_e) P(U_e | R) P(R | S) P(S) \\
 &= (0.02)(0.08)(0.3)(0.7)(0.25) \\
 &= 0.000096
 \end{aligned}$$

$$\begin{aligned}
 P(M, S, U_e, R) &= P(M | \neg U_e) P(\neg U_e | R) P(R | S) P(S) \\
 &= (0.42)(0.88)(0.7)(0.7)(0.25) \\
 &= 0.049276
 \end{aligned}$$

$$\begin{aligned}
 P(M, S, \neg U_e, \neg R) &= P(M | \neg U_e) P(\neg U_e | \neg R) P(\neg R | S) P(S) \\
 &= (0.42)(0.9)(0.2)(0.3)(0.25) \\
 &= 0.00567
 \end{aligned}$$

$$\begin{aligned}
 P(M, S, U_e, \neg R) &= P(M | U_e) P(U_e | \neg R) P(\neg R | S) P(S) \\
 &= (0.42)(0.75)(0.7)(0.7)(0.25) \\
 &= 0.0165375
 \end{aligned}$$

$$\begin{aligned}
 P(M, S, \neg U_e, R) &= P(M | U_e) P(U_e | R) P(R | S) P(S) \\
 &= (0.42)(0.92)(0.8)(0.3)(0.25) \\
 &= 0.023184
 \end{aligned}$$

$$P(M | S) = \frac{P(M, S)}{P(S)} = \frac{0.09133}{0.25} = 0.3654 = \text{Ans.}$$

$$(d) P(B | S) = \frac{P(B, S)}{P(S)}$$

$$P(B, S) = P(B, S, \neg R) + P(B, S, R) + P(B, S, \neg R) + P(B, S, R)$$

$$\begin{aligned}
 P(B, S, \neg R) &= P(B | \neg R, S) P(S) P(\neg R | S) \\
 &= (0.8)(0.25)(0.7)(0.7) = 0.098 = \text{Ans}
 \end{aligned}$$

$$\begin{aligned}
 P(B, S, \neg R) &= P(B | \neg R, S) P(S) P(\neg R | S) \\
 &= (0.8)(0.25)(0.2)(0.3) \\
 &= 0.012
 \end{aligned}$$

$$\begin{aligned}
 P(B, S, R) &= P(B | R, S) P(S) P(R | S) \\
 &= (0.4)(0.25)(0.3)(0.7) \\
 &= 0.021
 \end{aligned}$$

$$\begin{aligned}
 P(B, S, \neg R) &= P(B | \neg R, S) P(S) P(\neg R | S) \\
 &= (0.4)(0.25)(0.8)(0.3) \\
 &= 0.024
 \end{aligned}$$

$$\therefore P(B|S) = \frac{0.155}{0.25} = 0.62 = \text{Ans}$$

$$\underline{(e).} \quad P(S|B, \neg R) = \frac{P(S, B, \neg R)}{P(B, \neg R)}$$

$$\begin{aligned}
 P(S, B, \neg R) &= P(S, B, \neg R, R) + P(S, B, \neg R, \neg R) \\
 &= 0.021 + 0.024 \\
 &= 0.045
 \end{aligned}$$

$$\begin{aligned}
 P(B, \neg R) &= P(B, \neg R, R, S) + P(B, \neg R, R, \neg S) + P(B, \neg R, \neg R, S) + \\
 &P(B, \neg R, \neg R, \neg S).
 \end{aligned}$$

$$P(B, \neg R, R, S) = P(B, S, \neg R, R) = 0.021$$

$$\begin{aligned}
 P(B, \neg R, R, \neg S) &= P(B | \neg R, \neg S) P(\neg R | R) P(R | \neg S) P(\neg S) \\
 &= (0.4)(0.3)(0.3)(0.75) = 0.027
 \end{aligned}$$

$$P(B, \neg R, \neg R, S) = P(B, S, \neg R, \neg R) = 0.024$$

$$\begin{aligned}
 P(B, \neg R, \neg R, \neg S) &= P(B | \neg R, \neg S) P(\neg R | \neg R) P(\neg R | \neg S) P(\neg S) \\
 &= (0.4)(0.8)(0.7)(0.75) = 0.168
 \end{aligned}$$

$$\therefore P(S|B, \neg R) = \frac{0.045}{0.24} = 0.1875 = \text{Ans}$$