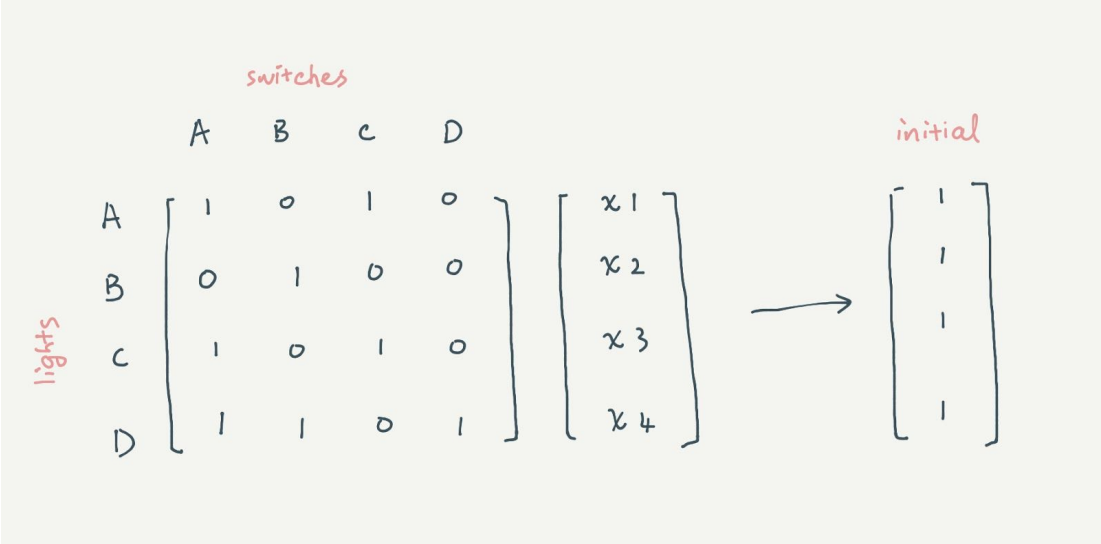
Etude 9 Report

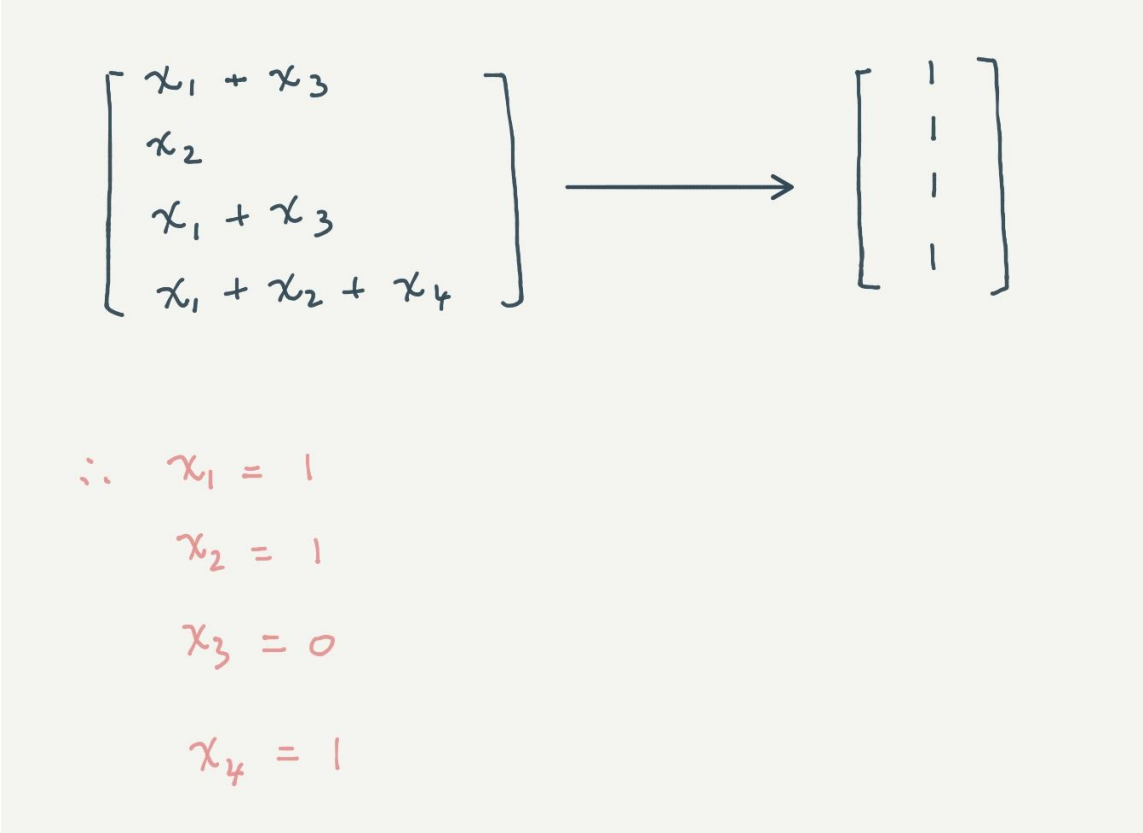
Orlando Yuan Usman Shah

There are two Boolean states of the lights, either ON or OFF. Assume that all lights are ON at the start, and we know the set of edges, eg. AB BC CD DA which means that pressing A will turn both C and D to OFF, pressing B will turn only D to OFF, and pressing C will turn only A to OFF. The method we implemented is using recursion to perform backtrace, {[A, 1], [B, 1], [C, 1], [D, 1]} represents all lights are ON at the beginning. The process is that traversing the way similar to the Probabilistic Group Theory from the last switch to the first switch to try every possible combinations until it reaches {[A, 0], [B, 0], [C, 0], [D, 0]} so that the result would be BD but not AC, both two are the optimal solutions.

However, there is a enhancing way to solve this problem. We can create a ***n * n*** matrix for this problem using only 0 and 1. 0 means the light is OFF, and 1 means the light is ON. In an another case A B C D with the set of edges AC AD BD CA, the matrix at its initial state would be a size of ***4 * 4***. By backtracking from the state where we start at, the equation is $\boldsymbol{AX = B}$, where the solution of X would be the solution of this problem.



As the above figure shows, the x-axis of the matrix represents switch and the y-axis of the matrix represents lights. [A, A] = 1 means light A is controlled by the switch of itself, [A, C] = 1 means light C is also controlled by switch A, and so on. 0 means there is no relation between the certain light and switch. The vector on the right side is the initial state for assuming all lights are on at the beginning.



Through the calculation, $x2 = 1$, $x1$ and $x4$ are both equal to 1 since $x1 + x2 + x4 = 1$, $x3 = 0$ since $x1 + x3 = 1$ and $x1$ is equal to 1. The result of X then for this case is [1, 1, 0, 1] which means we have to press A, B and D to turn all lights off. B and C would be the output it produced if we are using recursion.