MDM1 REP0: Part A

Shavarsh Melikyan

October 12, 2023

1 The logistic model

The exponential growth, discussed in a previous part of an assignment, is considered valid if and only if early stages of an epidemic are examined exclusively. For a bit more realistic results we take a look at another model called a logistic model with an additional parameter k, which shows population level at a point the disease is saturated.

Let us define $\lambda = \alpha/h$, the discrete time process is equivalent in the limit of small h to the continuous time equation

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \lambda\,(n - \kappa n^2), \qquad n(0) = 10, \qquad \kappa = 0.001 \tag{1}$$

A plot of the solution to the ODE (1) is shown in Fig. 1 below.

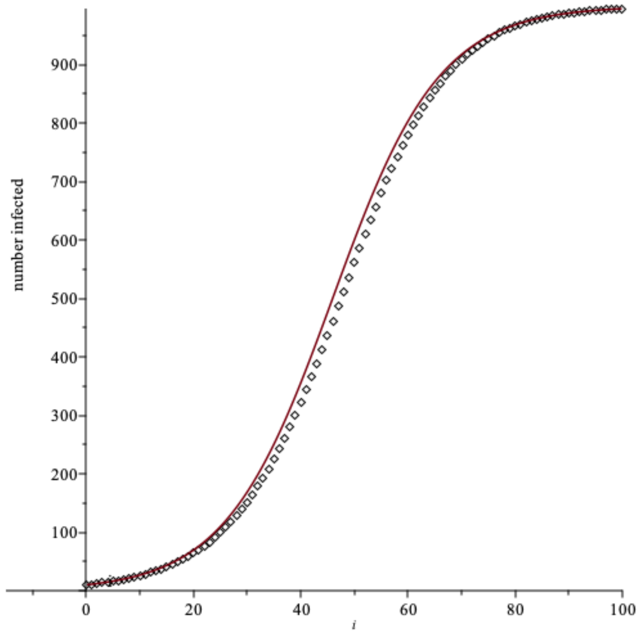

Figure 1: Comparison between discrete- and continuous-time logistic growth of infection. The continuous curve is the solution $n(t)$ to (1) plot against $i = t/h$. Points represent the solution $Inf_{i+1} = Inf_i + \alpha(Inf_i - \kappa Inf_i^2)$ to the discrete model. Parameter values used are $\alpha = 0.1$, $\lambda = \alpha/h$, $\kappa = 0.001$ and $h = 0.1$. The initial condition for both models: $\mathrm{Inf}_0 = n(0) = 10$.

Notice that there is good agreement between the discrete and continuous models.

The limit of the number of Infected people as $t \to \infty$ is around 1000 under given parameters. If to play with κ parameter, we will see that generically the above mentioned limit will stay around constant.