Chhavi 102103438 CO16 Predictive Analysis

Assignment – Parameter Estimation

Q1.)

mean → θ_1

Variance → θ_2

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Take logarithm.

$$\log L(\theta_1, \theta_2) = \frac{-n}{2}\log(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

For θ_1 differentiate $\log(L(\theta_1, \theta_2))$ w.r.t θ_1 and set it equal to zero.

$$\frac{\partial \log(L)}{\partial\theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1) = 0.$$

$$\sum_{\cancel{x}i=1}^{n}(x_i-\theta_1) = 0$$

$$\sum_{i=1}^{n}(x_i - n\theta_1) = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

M.L.E of θ_1 is sample mean.

For θ_2, differentiate $\log(L(\theta_1, \theta_2))$ w.r.t θ_2 and set equal to zero.

$$\frac{\partial \log(L)}{\partial\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

2.) $B(m,\theta)$ Binomial distribution.

$m \to$ number of trials

$\theta \in (0,1)$ prob. of success.

$$L\theta = \prod_{i=1}^{n} f(x_i; m, \theta)$$

PMF

$$f(x; n, \theta) = {}^{n}C_{x} \cdot \theta^{x} \cdot (1-\theta)^{m-x}$$

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \cdot \theta^{x_i} (1-\theta)^{m-x_i}$$

Take logarithm.

$$\log(L(\theta)) = \sum_{i=1}^{n} \log \binom{m}{x_i} + \sum_{i=1}^{n} x_i \log(\theta) + \sum_{i=1}^{n} (m - x_i) \log(1-\theta)$$

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i) = 0.$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (m - x_i)$$

Multiply both sides by θ & $1-\theta$.

$$(1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{n} (m - x_i)$$

$$\sum_{i=1}^{n} x_i - \theta \sum_{i=1}^{n} x_i = \theta m - \theta \sum_{i=1}^{n} x_i$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$

MLE of θ for $B(n,\theta)$ is $\bar{x}$ where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$