Question1

(a) (i) f: R \ {0} → {−1, 1} given by $f(x) = \begin{cases} \lfloor\frac{X}{|X|}\rfloor, & \text{if } x < 0 \\ \lfloor\frac{X}{X}\rfloor, & \text{if } x > 0 \end{cases}$

(ii) g : R → {0, 1} given by $g(x) = \begin{cases} \text{x div x}, & \text{if } x \epsilon \text{ Z} \\ \text{x} * 0, & \text{if } x \notin \text{ Z} \end{cases}$

(iii) h : N → {0, 1} given by $h(x) = \begin{cases} \lfloor\frac{X}{5}\rfloor, & \text{if } 5|x \\ \text{x \% x}, & \text{if } 5 \nmid x \end{cases}$

(iv) min : R × R → R given by $\min(x, y) = \begin{cases} \text{x div x}, & \text{if x} \leq \text{y} \\ \text{x \% x}, & \text{if x} < \text{y} \end{cases}$

(v) q : R → {0, 1} given by $q(x) = \begin{cases} \lfloor\frac{X}{5}\rfloor, & \text{if x } \epsilon \text{ Z and } 5|\text{x} \\ \text{x} * 0, & \text{otherwise} \end{cases}$

(b) (i)- if 17|n, then n = 17k
assume not 17|n', then n = 10*(n' + 5b) + b = 10n' + 51b
n div 17 = 10n'/17 + 51b/17 = 10n'/17 +3b
because b is integer, so 3b is integer.
Because not n'|17, so 10n'/17 is not a integer.
So not 17|n, this is contradict with 17|n.
Therefore, if 17|n, then 17|n'.

- if 17|n', n' = 17k. n = 10*(17k + 5b) + b = 170kb + 51b
n div 17 = 10kb + 3b.
Because k and b are both integer, so 17|n.
Therefore, if 17|n', then 17|n.

In conclusion, 17|n if and only if 17|n'

(ii) the first step need O(1), the second step need O(1), the third step need O(1), and this will take O(logn) time. Therefore, it need O(logn)(O(1) + O(1) + O(1)) ϵ O(logn)

Question2

(a) (i) if A∩X = ∅ and B∩X = ∅, then A∩X = B∩X, but A is not necessary equal to B.
Counterexample:
A = {1}, B = {2}, X = {3}, A∩X = ∅ and B∩X = ∅, and we have A∩X = B∩X,
However, A ≠ B

(ii) we observe that A ⊕ A = ∅, A ⊕ ∅ = A, A ⊕ (B ⊕ C) = (A ⊕ B) ⊕ C),
So we have
$$\begin{aligned} A &= A \oplus \emptyset \\ &= A \oplus (X \oplus X) \\ &= (A \oplus X) \oplus X \end{aligned}$$

$= (B \oplus X) \oplus X$
$= B \oplus (X \oplus X)$
$= B \oplus \emptyset$
$= B$

Therefore, If there is a set X such that $A \oplus X = B \oplus X$ then $A = B$.

(iii) from assignment1 problem4(c), we get that $A \oplus B = (A \cup B) \cap (A^c \cup B^c)$,

So we have $A \oplus B = (A \cup B) \cap (A^c \cup B^c)$
$= (A \cup B) \cap (A \cap B)^c$ de Morgan's law
$= (A \cup B) - (A \cap B)$ Definition

Therefore, if $A \cap X = B \cap X$ and $A \cup X = B \cup X$,
$A \oplus X = (A \cup X) - (A \cap X)$
$= (B \cup X) - (B \cap X)$
$= B \oplus X$

From (ii), we can get that $A = B$

Therefore, If there is a set X such that $A \cap X = B \cap X$ and $A \cup X = B \cup X$ then $A = B$.

(b) (i) let $x = u \in X, v \in Y, w \in Z$

If $XY = YZ$, then $uv = vw$
$X^* = \{\lambda, u, uu, uuu, \ldots\}$, $Z^* = \{\lambda, w, ww, www, \ldots\}$
$X^*Y = \{v, uv, uuv, uuuv, \ldots\}$, $YZ^* = \{v, vw, vww, vwww, \ldots\}$
Because $uv = vw$, so $uuv = u(uv) = uvw = (uv)w = vww$,
Similarly, $uuuv = vwww$, $uuuuv = vwwww$, …
For each element in X^*Y, we can find the same element in YZ^*,
Therefore, $X^*Y \subseteq YZ^*$
Similarly, $YZ^* \subseteq X^*Y$
Therefore, $X^*Y = YZ^*$

(ii) let $x = u \in X, v \in Y, w \in Z$

$X^*Y = \{v, uv, uuv, uuuv, \ldots\}$, $YZ^* = \{v, vw, vww, vwww, \ldots\}$
If $X^*Y = YZ^*$, $|X^*Y| = |YZ^*|$.
So $uv = vw$, $XY = uv = YZ = vw$
Therefore, if $X^*Y = YZ^*$, $XY = YZ$

Question3

(a) ① From the description, $f(a) = \{x: x \preccurlyeq a\}$, $f(b) = \{x: x \preccurlyeq b\}$

If $a \preccurlyeq b$ for all $b \in A$, then a is a lower bound for A, that is $a \leq b$.

Therefore, $Im(f(a)) \subseteq Im(f(b))$, so $f(a) \subseteq f(b)$

② If $f(a) \subseteq f(b)$, then if $x \leqslant a$, then $x \leqslant b$.

Because $\leqslant$ is transitive, so we can get $a \leqslant b$.

From ① and ②, $a \leqslant b$ if and only if $f(a) \subseteq f(b)$.

(b) if $f(a) = f(b)$, then $f(a) \subseteq f(b)$ and $f(b) \subseteq f(a)$.
From (a), if $f(a) \subseteq f(b)$, $a \leqslant b$, if $f(b) \subseteq f(a)$, $b \leqslant a$.
Because $\leqslant$ is antisymmetric, so $a = b$.
Therefore, if $f(a) = f(b)$, $a = b$. That is when $x \neq y$, $f(x) \neq f(y)$.
Therefore, f is injective.

(c) f is defined as $A \to Pow(A)$ with $f(a) = \{x: x \leqslant a\}$.
So, Dom(f) is A, Im(f) is Pow(A).
For $\emptyset$ in Pow(A), there exists no element 'a' in A that satisfy $f(a) = \emptyset$.

Therefore, there is not aways that $\forall y \in Pow(A): \exists x \in A: f(x) = y$.

Therefore, f is not surjective.

(d) if $c = glb(a, b)$, then c is a lower bound, that is $c \leqslant a$ and $c \leqslant b$,
and c is the greatest of all lower bounds, that is if $a \leqslant b$, $a \leqslant c$; if $b \leqslant a$, $b \leqslant c$.
For $c \leqslant a$ and $c \leqslant b$, from (a), $f(c) \subseteq f(a)$ and $f(c) \subseteq f(b)$.
So we have $f(c) \subseteq f(a) \cap f(b)$.
There must exists $a \leqslant b$ or $b \leqslant a$,
Let us assume that $a \leqslant b$, then $f(a) \subseteq f(b)$, $f(a) \cap f(b) = f(a)$.
If $a \leqslant b$, then $a \leqslant c$, so $f(a) \subseteq f(c)$, so $f(a) = f(a) \cap f(b) \subseteq f(c)$
Therefore, $f(c) = f(a) \cap f(b)$.

Question4

(a) Define $\varphi: (A \times B) \times C \to A \times (B \times C)$
Define $\omega: A \times (B \times C) \to (A \times B) \times C$

Let $f \in x \times y$, $g_f \in f \to C$, $h_f \in f \to A$

$g_f(h_f) = g(f(b, c), a) = h_f$, $h_f(g_f) = h_f(f(a, b), c) = g_f$
Then $\varphi \circ \omega = g_f(h_f)$, $\omega \circ \varphi = h_f(g_f)$
Therefore, $\varphi = \omega^{-1}$
Therefore, there is a bijection between $(A \times B) \times C$ and $A \times (B \times C)$.

(b) ① if $f: S \to T$ is injective, then if $f(x) = f(y)$, $x = y$.

$f \circ g = f(g(u_1)) = f(s_1)$, $f \circ h = f(h(u_2)) = f(s_2)$

So if $f \circ g = f \circ h$, g is necessary equal to h.

② if f ∘ g = f ∘ h implies g = h, let Im(g) = x, Um(h) = y, that is if f(x) = f(y), x = y.
Therefore, f is injective.

From ① and ②, f : S → T is injective if, and only if, for all functions g, h : U → S:
f ∘ g = f ∘ h implies g = h

(c) The asymptotic complexity of the six choice is:

(I) $n^3\sqrt{n} \in O(n^3\sqrt{n})$

(II) $n^3\log(n) \in O(n^3\log(n))$

(III) $T(0) \in O(1)$

$T(n) = T(n - 3) + n^3 \in O(n^4)$

(IV) $T(0) \in O(1)$

$T(n) = 6T(n / 2) + n^3 \in O(n^4)$

(V) $(3^{\log n})^2 = O(3^{2\log n})$

(VI) $3^{(\log n)^\wedge 2} = O(3^{(\log n)^\wedge 2})$

$\lim\limits_{n\to\infty} \sqrt{n} > \lim\limits_{n\to\infty} \sqrt{n} > \lim\limits_{n\to\infty} \log n$, so $O(n^4) > O(n^3\sqrt{n}) > O(n^3\log(n))$.

$\lim\limits_{n\to\infty} 2\log n > \lim\limits_{n\to\infty} (\log n)^\wedge 2$, so $O(3^{(\log n)^\wedge 2}) > O(3^{2\log n})$.

$\lim\limits_{n\to\infty} \frac{(n-3)*n^\wedge 3}{6(n/2)*n^\wedge 3} = \frac{1}{3}$, so (IV) > (III).

$\lim\limits_{n\to\infty} 3^\wedge(2\log n g n) = 3^n > \lim\limits_{n\to\infty} n^\wedge 4$, so $O(3^{2\log n}) > O(n^4)$.

Therefore, the increasing order is : (II) < (I) < (III) ⩽ (IV) < (V) < (VI)

Question5

(a) $O(n^2)$

To search v and v' that v' is linked to v, the searching adjacency matrix time is O(n), and we need to search this for n times, so it can be executed in $O(n^2)$.

Remove a vertex and all incident edges need or prepend a vertex need O(1), and this need to do n times, so they can be executed in O(n) + O(n).

Therefore, the running time is $O(n^2) + O(n) + O(n) \in O(n^2)$.

(b) O(n + m)

G has n vertices, the searching adjacency time is O(n),

G has m edge, the time in removing a vertex is O(1), and we need to remove m times, so the remove time is O(m).

Therefore, the total running time is O(n + m)

(c) O(n + m)
To search v and v' that v' is linked to v, the searching adjacency matrix time is O(m), and we need to search this for n times, so it can be executed in O(mn).
Remove a vertex and all incident edges need or prepend a vertex need O(1), and this need to do n times, so they can be executed in O(n) + O(n).

Therefore, the running time is O(mn) + O(n) + O(n) ϵ O(mn).

Question6

(a) We define height recursively on trees as follows:
- height(τ) = -1
- height([T_{left}, T_{right}]) = max(height(T_{left}), height(T_{right})) + 1

(b)
- balanced(τ) = -1 (when T is empty)
- balanced([T_{left}, T_{right}]) =abs(height(T_{left}) - height(T_{right})) < 2 ? 1: 0

(when the heights of its two subtrees differ by at most 1, it is balanced, else, it is not balanced)

(c) we prove P(T) by induction on T
Base case T = τ, height(T) = height(τ) = -1,

$$\text{count}(T) = \text{count}(\tau) = 0 \leqslant 2^{\text{height}(T) + 1} - 1 = 2^0 - 1 = 0, \text{ so } P(\tau) \text{ holds.}$$

Inductive case: T = [T_{left}, T_{right}], assume P(T_{left}) and P(T_{right}) holds and T_{left} and T_{right} are not empty. That is,

$$\text{count}(T_{left}) \leqslant 2^{\text{height}(Tleft) + 1} - 1 \text{ and count}(T_{right}) \leqslant 2^{\text{height}(Tright) + 1} - 1.$$

We will show that P([T_{left}, T_{right}]) holds. We have:

$$\begin{aligned} \text{count}([T_{left}, T_{right}]) &= 1 + \text{count}(T_{left}) + \text{count}(T_{right}) \quad \text{(def. of count)} \\ &\leqslant 1 + 2^{\text{height}(Tleft) + 1} - 1 + 2^{\text{height}(Tright) + 1} - 1 \\ &= 2^{\text{height}(Tleft) + 1} + 2^{\text{height}(Tright) + 1} - 1 \\ &\leqslant 2^* \max(2^{\text{height}(Tleft) + 1}, \quad 2^{\text{height}(Tright) + 1}) - 1 \\ &= 2^* 2^{\max(\text{height}(Tleft), \text{ height}(Tright)) +1} -1 \\ &= 2^* 2^{\text{height}([Tleft, \text{ } Tright])} -1 \\ &= 2^{\text{height}([Tleft, \text{ } Tright]) + 1} -1 \end{aligned}$$

So P([T_{left}, T_{right}]) holds when T_{left} and T_{right} are not both empty.
By the Principle of Structural Induction, we have that P(T) holds for all trees T.

(d) we prove Q(T) by induction on T
Base case T = τ, obviously T is balanced, height(T) = height(τ) = -1,

$$\text{count}(T) = \text{count}(\tau) = 0 \geqslant 2^{\text{height}(T)/2} - 1 = 2^{(-1/2)} - 1 = \frac{1}{\sqrt{2}} -1,$$

so $Q(\tau)$ holds.

Inductive case: T = [T_{left}, T_{right}], assume Q(T_{left}) and Q(T_{right}) holds and T_{left} and T_{right} are not empty, and T_{left} and T_{right} are balanced. That is,

if balanced(T_{left}) and balanced(T_{right}), then

count(T_{left}) $\geqslant 2^{height(Tleft)/2} - 1$ and count(T_{right}) $\geqslant 2^{height(Tright)/2} - 1$.

We will show that Q([T_{left}, T_{right}]) holds. We have:

if balanced([T_{left}, T_{right}]),

count([T_{left}, T_{right}]) = 1 + count(T_{left}) + count(T_{right}) (def. of count)

$\geqslant 1 + 2^{height(Tleft)/2} - 1 + 2^{height(Tright)/2} - 1$

$= 2^{height(Tleft)/2 + height(Tright)/2} - 1$ ①

Because balanced([T_{left}, T_{right}]),

so height(T_{left}) and height(T_{right}) differ by at most 1.

Consider extreme cases, assume height(T_{left}) > height(T_{right}) ,

so that height(T_{left}) = height(T_{right}) + 1

Then height(T_{left}) + height(T_{right}) =2 height(T_{right}) + 1

$\geqslant$ max(height(T_{left}), height(T_{right})) + 1

So $\frac{\text{height}(T_{left})}{2} + \frac{\text{height}(T_{right})}{2} = \frac{\text{height}(T_{left}) + \text{height}(T_{left})}{2}$

$\geqslant \frac{\max(\text{height}(T_{left}), \text{height}(T_{left})) + 1}{2}$

So ① $\geqslant 2^{(\max(height(Tleft),\ height(Tleft)) + 1)/2} - 1$

$= 2^{(height([Tleft,\ Tright]))/2} - 1$

So Q([T_{left}, T_{right}]) holds when T_{left} and T_{right} are not empty and balanced([T_{left}, T_{right}]).

By the Principle of Structural Induction, we have that Q(T) holds for all trees T.

Question7

(a) (i) C_i : the people is a crew.

P_i : the people is a imposter.

i = {Red, Orange, Green, Blue}

$C_i \wedge \neg P_i$: people i cannot be a crewmate or an imposter at the same time.

we use logical language to represent the four people's statements:

Red: $C_{Red} \rightarrow (C_{Red} \wedge C_{Blue}) \vee (P_{Red} \wedge P_{Blue})$

Orange: $C_{Orange} \rightarrow (P_{Red} \vee P_{Green})$

Green: $C_{Green} \rightarrow P_{Red}$

Blue: $C_{Blue} \rightarrow P_{Red}$

Because there are at least one imposter, so $\neg$(Red $\wedge$ Orange $\wedge$ Green $\wedge$ Blue)(①), $\neg(C_{Red} \wedge C_{Orange} \wedge C_{Green} \wedge C_{Blue})$(②)

if C_{Red}, then red and blue must be the same chatacter, so C_{Blue},

but $C_{Blue} \rightarrow P_{Red}$, so $\neg(C_{Red} \wedge C_{Blue})$(③). And $C_{Green} \rightarrow P_{Red}$, so $\neg(C_{Red} \wedge$

C_{Green})(④)

we need to find a possible combination that let *①∧②∧③∧④ = 1*

(ii) we use truth table to solve the problem, because $C_i \wedge \neg P_i$, so we omit the value of P

C_{Red}	***C_{Orange}***	***C_{Green}***	***C_{Blue}***	*①∧②∧③∧④*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

So there is only one possible combinations:

Red	***Orange***	***Green***	***Blue***
Imposter	crewmate	Crewmate	Crewmate

(b) (i) assume that there exists x = x' in Boolean Algebra.

$1 = x \vee x'$	complementation
$= x \vee x$	hypothesis
$= (x \vee x) \wedge 1$	identity
$= (x \vee x) \wedge (x \vee x')$	complementation
$= x \vee (x \wedge x')$	distributive
$= x \vee 0$	complementation
$= x$	identity

So if x = x', then x = 1

$0 = x \wedge x'$	complementation
$= x \wedge x$	hypothesis
$= (x \wedge x) \vee 0$	identity
$= (x \wedge x) \vee (x \wedge x)$	complementation
$= x \wedge (x \vee x')$	distributive

$= x \wedge 1$ complementation

$= x$ identity

So if $x = x'$, $x = 0$, which is contradiction with $x = 1$ above.

Therefore, there is no x such that $x = x'$

(ii) we use truth table to prove it.

x	x’	y	z	$x \wedge y$	$x' \wedge z$	$y \wedge z$	left	right
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	1	1
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	1	0	0	1	1
1	0	1	1	1	0	1	1	1

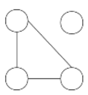
Therefore, $(x \wedge y) \vee (x' \wedge z) \vee (y \wedge z) = (x \wedge y) \vee (x' \wedge z)$

Question8

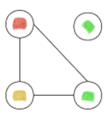
(a) The degree sequence D_i of graph G =<v, e>, where D_i is the number of vertices of degree i. And for G<v, e>,G has an Euler circuit which is a closed Euler path that only contains every edge once. We can get how many times the path contain the edge from the degree sequence. Therefore, we can determine if it has a Euler circuit from the degree sequence information.

(b) disprove

If G is not connected, then we can have a vertex that degree is 0.

Such as , it has even vertices and a Euler circuit, but its edges is 3 is odd.

(c) disprove



We use the graph from (b), we can get that we need at least 3 colours.

Proof:

if G has even number of vertices, so assume that the number of vertices is n(n = 2k and n >= 2).

If G has an Euler circuit, we can get a path that $v_0e_1v_1e_2 \ldots v_me_{m+1}v_0$, so v_0 is attached

to v_1, v_1 is attached to v_2, …. v_m is attached to v_0.
Therefore, $c(v_0) \neq c(v_1)$, $c(v_1) \neq c(v_2)$, … , $c(v_{m+1}) \neq c(v_0)$, m >= 2
So, we need at least 2 colours.
Therefore, it has chromatic number at most 2 is not necessarily true.

(d) To show that a graph G contains a subdivision of H we can do the following.
- Starting with G.
- Repeat any of the following operations as many times as necessary:
  - Delete edges
  - Delete vertices (and all adjacent edges)
  - replace a vertex of degree 2 with an edge connecting its neighbour (delete the vertex and connect its two neighbours with an edge)
- Finish with k_4.

All vertices of k_4 are degree 3, and the vertices of the graph are all degree3, so it contains a subdivision of k_4.

Question9

(a) Because F(n, k) is the number of surjective functions there are from A_n to A_k,
So F(n, n) is the number of surjective functions there are from A_n to $A_{n.}$
We need to select n objects from A_n to construct orders of all objects from A_n and without repetition.
$A_{n.}$ ={1, 2, 3, …, n}, so there are $(n \cdot (n-1) \cdot \cdot \cdot 1)$ surjective functions
Therefore, $F(n, n) = n! = n \cdot (n-1) \cdot \cdot \cdot 1$

(b) F(n, 2) is the number of surjective functions: $A_{n.}$ ={1, 2, 3, …, n} → A_2 {1, 2}
So we need to select 2 objects from A_2 of size of n without repetition.
Therefore, $F(n, 2) = \sqcap(n,2) = n \cdot (n-1) \cdot \cdot \cdot (n-1) = \frac{n!}{(n-2)!}$

(c) E(n) is the number of equivalence relations on A_n, so E(n) can be calculated from the number of partitions.
When k = 1, A= {1}, there is 1 partition. E(n) = 1.
when k = n’, E(k) = E(n’),
if we add an element, there are following situations:
- the added element is alone, n elements(which E(k) = E(n’))
- the added element is a group with any one of the n elements, there are $\sqcap(k,1)$ combinations, and the other (k-1) element has E(k -1) partitions. So there are totally $\sqcap(k,1)E(k-1)$ partitions.
- the added element is a group with any two of the n elements, there are $\sqcap(k, 2)$ combinations, and the other (k - 1) element has E(k - 2) partitions. So there are

totally ⊓(k, 2)E(k - 2) partitions.

……

- the added element is a group with any i of the n elements, there are ⊓(k, i) combinations, and the other (k-1) element has E(k - i) partitions. So there are totally ⊓(k,i)E(k - i) partitions.

Therefore, E(k+1) = E(n') + ⊓(k,1)E(k -1) + ⊓(k, 2)E(k -2) + … + 1

$$= \sum_{i=1}^{k} \sqcap(i, k)E(k)$$

Therefore, E(n) = $\begin{cases} 1, & x = 1 \\ \sum_{i=n'}^{n} \sqcap(i, n)E(n'), & x > 1 \end{cases}$

(d) F(1, 1) = 1

F(n', k') = ⊓(n', k') = n · (n − 1)· · ·(n – k' + 1) = $\frac{n'!}{(n'-k)!}$

F(n, k) = F(n', k')

Each f ϵ A_k^{An} can be viewed as f = g ∪ {(n, y)} where y ϵ A_k and g is function with domain A_{n-1},

so F(n, k) = F(n', k') + ⊓(n – n', k – k').

Therefore, F(n, k) = $\begin{cases} 1, & n = 1, k = 1 \\ F(n', k') + \sqcap(n - n', k - k'), & n > 1, k > 1, \text{and } n > n', k \geq k' \end{cases}$

(e) from(b), F(n, k) = ⊓(n,k),

E(n, k) = ⊓(F(n, k), k).

Question10

(a) P(A\B) = P(A∩B^C)

If A∩B = ∅, P(A∩B^C) = P(A) ⩾ P(A) – P(B)

If A∩B ≠ ∅, P(A∩B^C) = P(A)P(B^c) =P(A)(1 – P(B)) = P(A) – P(A)P(B)

Because P(A) ⩽ 1, so P(A)P(B) ⩽ P(B), so P(A) – P(A)P(B) ⩾ P(A) – P(B).

Therefore, P(A\B) ⩾ P(A) – P(B)

(b) P(A|B) = $\frac{P(A\cap B)}{P(B)}$, P(B|A) = $\frac{P(B\cap A)}{P(A)}$ = $\frac{P(A\cap B)}{P(B)}$

If 0 < P(A) ⩽ P(B), then $\frac{P(AB)}{P(B)}$ ⩽ $\frac{P(AB)}{P(A)}$, so P(A|B) ⩽ P(B|A)

Therefore, if 0 < P(A) ≤ P(B) then P(A|B) ≤ P(B|A)

(c) E(X) = X.P(X)

$X > E(X) \Rightarrow X > X.P(X) \Rightarrow 1 > P(X)$

And $P(X) < 1$ constant establishment.

So $P(X > E(X)) = P(1 > P(X)) = 1 \geqslant \frac{1}{2}$

(d) use contradiction

Assume that if $E(X) > E(Y)$, $P(X>Y) = 0$

Now, we have $P(X>Y) = 0$

$\Rightarrow P((X-Y) > 0) = 0$

$\Rightarrow P(Z > 0) = 0$, let $Z = X - Y$

If $P(Z>0) = 0$, then $Z \leqslant 0$

$\Rightarrow E(Z) \leqslant 0$

$\Rightarrow E(X - Y) \leqslant 0$

$\Rightarrow E(X) \leqslant E(Y)$, which is contradict with the $E(X) > E(Y)$

Therefore, if $E(X) > E(Y)$ then $P(X > Y) > 0$.