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Homework 3

1.
   a. Write in your report the minimum number of 3D points needed to sample in an iteration to compute a putative model.
      The minimum number of points needed to compute a putative model is 3, as the model is in the form of the plane, and with any fewer than 3 points the solution space is infinite. 3 points are required to define a plane..

   b. Determine the probability that the data picked for to fit the putative model in a single iteration fails, assuming that the outlier ratio in the dataset is 0.5 and we are fitting 3D planes.
      Since we are picking 3 points, the probability of all 3 points being inliers is (1-0.5)^3 = 0.125 so the probability of at least 1 being an outlier is 1-0.125=0.875

   c. Determine the minimum number of RANSAC trials needed to have ≥ 98% chance of success, assuming that the outlier ratio in the dataset is 0.5 and we are fitting planes. Hint: You can do this by explicit calculation or by search/trial and error with numpy.

      If 0.875 is the probability of failure for a single trial, we can find N by doing:
      $0.98 \geq 1 - 0.875^N.$ $To\ solve\ for\ N\ the\ equation\ is \lceil log_{0.875}0.02 \rceil = \lceil 29.30 \rceil$
      Therefore we need at least 30 trials.

2.
   a. Suppose we are fitting a linear transformation, which can be parameterized by a matrix M ∈ R^{2x2} (i.e., [x′, y′]T = M[x,y]T). Write in your report: the number of degrees of freedom M has and the minimum number of 2D correspondences that are required to fully constrain or estimate M.
      nk = p, n = 2 eqn/point
      M has:
      - 4 degrees of freedom
      - 2 sets of 2D correspondences needed to fully constrain

   b. Suppose we want to fit [x′ i,y′ i]T = M[xi,yi]T. We would like you formulate the fitting problem in the form of a least-squares problem of the form
      $argmin\,||Am - b||_2^2$
      where m ∈ R^4 contains all the parameters of M, A depends on the points [xi,yi] and b depends on the points [x′ i, y′ i]. Write the form of A, m, and b in your report
      m=[m1,m2,m3,m4],

A = [[x1,y1,0,0],
[0,0,x1,y1],
...
[xk,yk,0,0],
[0,0,xk,yk]]
b = [x’1, y’1, x’2, y’2, …, x’k, y’k]

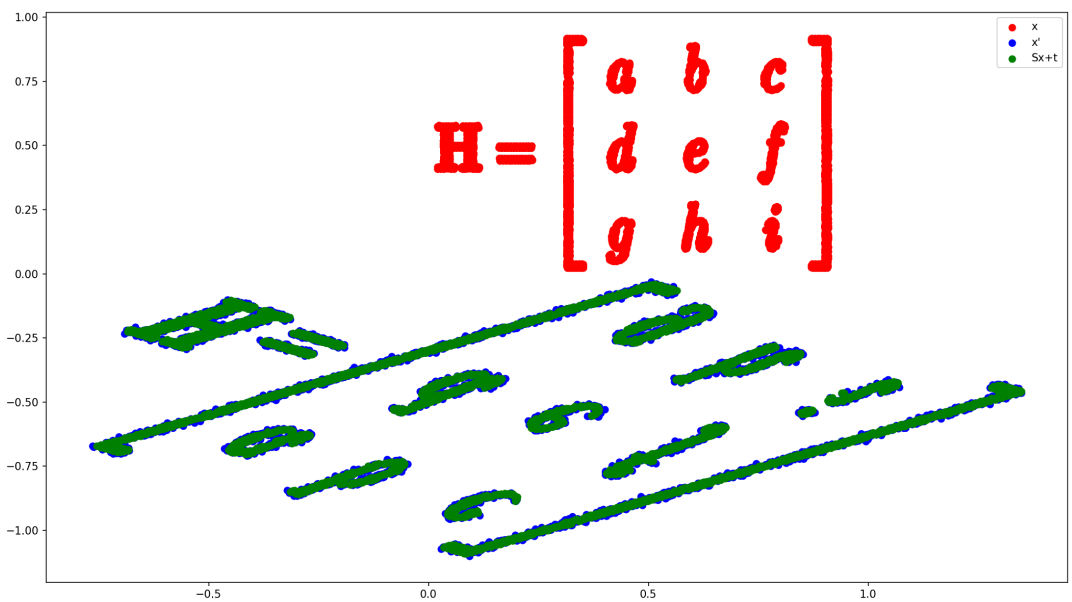
3.

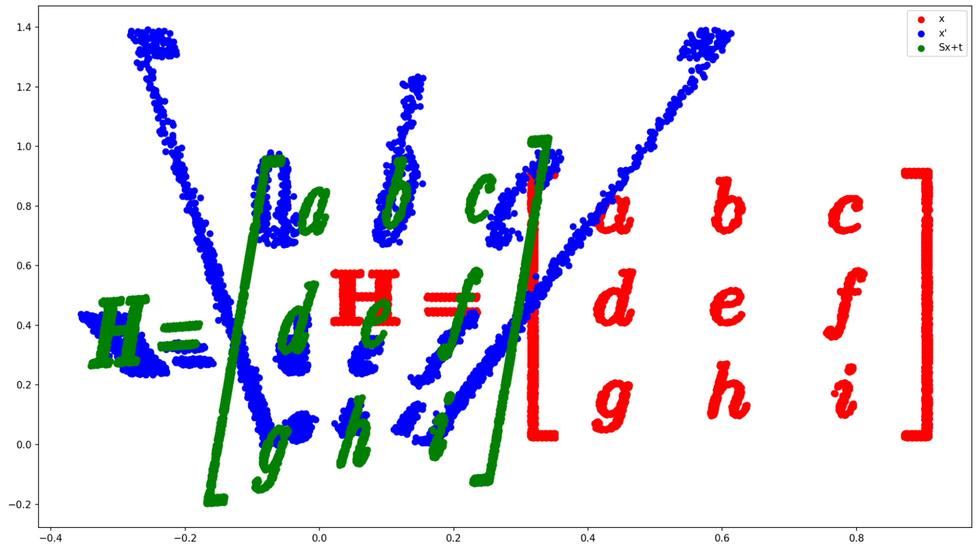
a. Fit a transformation of the form
$[x', y']^T = S[x, y]^T + t, \quad S \in R^{2x2}, t \in R^{2x1}$
by setting up a problem of the form
$argmin\ ||Av - b||_2^2$
$v \in R^6$
and solving it via least-squares
Report (S,t) in your report for points.case_1.npy.

S = [[1.41444296, -1.41424374]
[-0.70762108, -0.70690933]]

t = [0.09998617,
0.20014656]

b. Make as scatterplot of the points [xi,yi], [x′ i,y′ i] and S[xi,yi]T +t in one figure with different colors. Do this for both points_case_1.npy and point_case_2.npy. In other words, there should be two plots, each of which contains three sets of N points. Save the figures and put them in your report 2.npy.





c. Write in the report your answer to: how well does an affine transform describe the relationship between [x,y] ↔ [x′,y′] for points_case_1.npy and points_case_2.npy. You should describe this in two to three sentences. Hint: what properties are preserved by each transformation?
An affine transform can accurately describe the relationship between [x,y] ↔ [x′,y′] as long as the transformation is limited to rotation, translation, scaling, and shearing as is the case in points_case_1.npy. However, if the transformation involves non-linear distortion or changes in topology, an affine transform may not be sufficient to describe the relationship between the two sets of points accurately, which is what we see in points_case_2.npy..

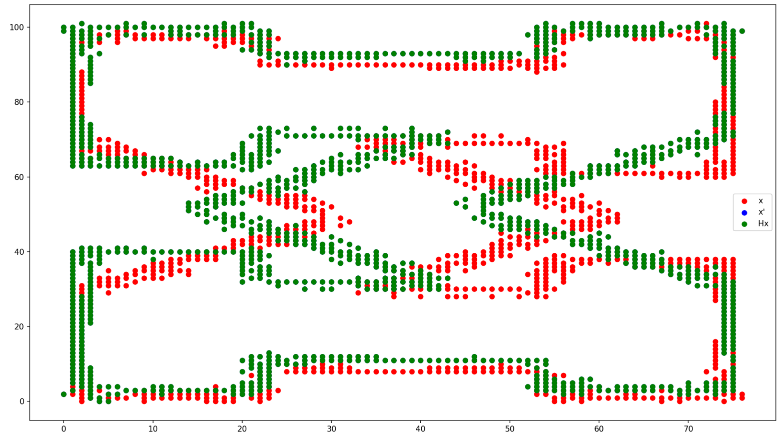
4.

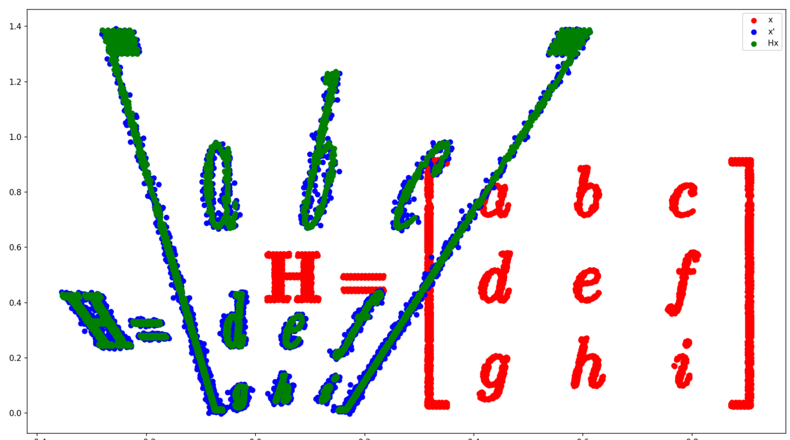
a. *code*
b. Report H for cases points_case_1.npy and points_case_4.npy. You must normalize the last entry to 1.

```
[[ 1.00555949e+00  1.61370672e-03 -1.35143989e-01]
 [ 2.56045861e-03  6.22536404e-01 -7.35872070e-01]
 [ 4.51704286e-05  3.59823762e-05  1.00000000e+00]]
```

```
[[ 3.39641716e-14  1.00000000e+00 -5.65623841e-13]
 [ 1.00000000e+00  5.15606107e-15 -2.29202898e-13]
 [ 9.61481343e-17  9.61481343e-17  1.00000000e+00]]
```

c. Visualize the original points [xi,yi], target points [x′ i,y′ i] and points after applying a homography transform T(H,[xi,yi]) in one figure. Please do this for points case_5.npy and points-case_9.npy. Thus there should be two plots, each of which contains 3 sets of N points. Save the figure and put it in the report.

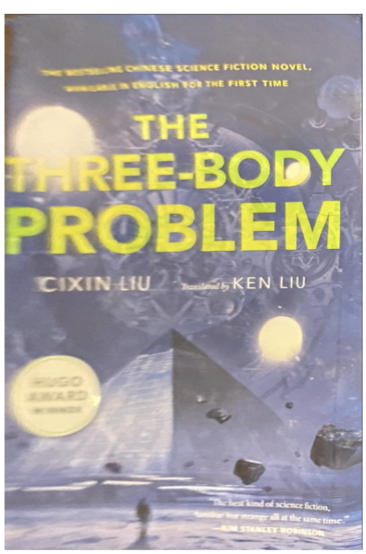


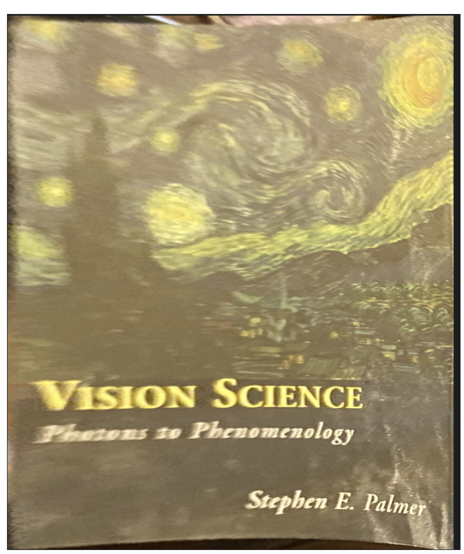


5.

a. *code*

b. Put a copy of both book covers in your report.

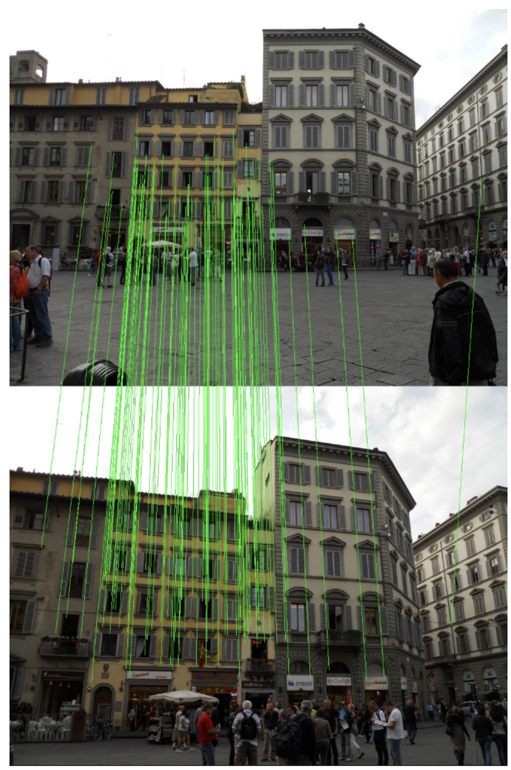




c. One of these images doesn't have perfectly straight lines. Write in your report why you think the lines might be slightly crooked despite the book cover being roughly a plane. You should write about 3 sentences.
In the image we are given for *Vision Science* the top and bottom sides already appear mostly bent. The homography depends upon the location of the corners, so the bending we see in the original image will be preserved. If we could use more points along the sides of the book and fit the homography to the whole perimeter, this may result in straighter lines.

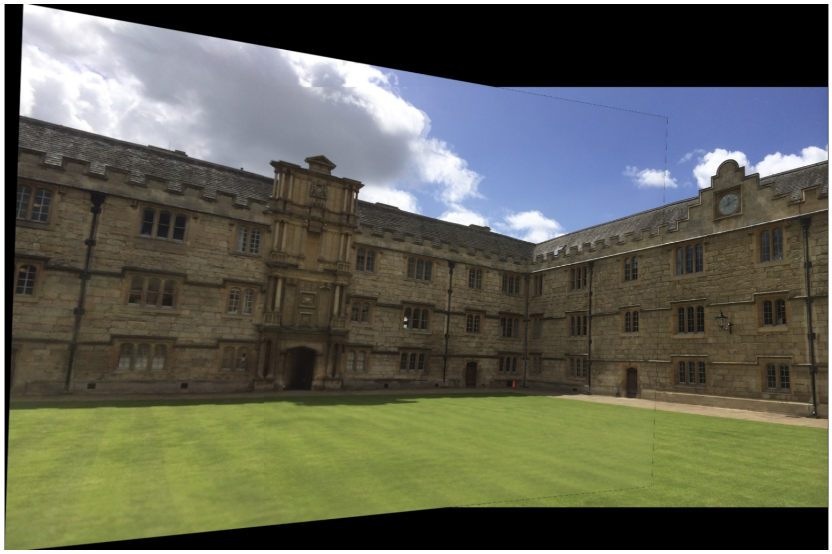
6.

a. *code*
b. *code*
c. *code*
d. Put a picture of the matches between two image pairs of your choice in your report.



e. *code*
f. *code*

g. Put merges from two of your favorite pairs in the report. You can either choose an image we provide you or use a pair of images you take yourself.



h. *code*