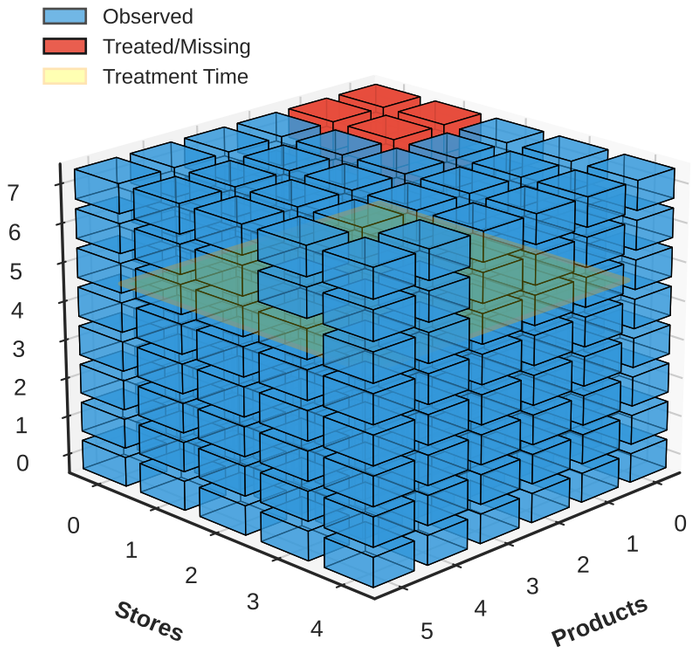
Tensor Completion for Three-Way Marketing Panels

Panel A: Three-Way Tensor (Products × Stores × Time)



Panel B: CP Decomposition (Rank-R Factorization)

$\mathcal{Y}$ *(3-way tensor)* $\approx \sum_{r=1}^{R}$ $\mathbf{a}_r$ *Products* $\otimes$ $\mathbf{b}_r$ *Stores* $\otimes$ $\mathbf{c}_r$ *Time*

CP Decomposition: $Y_{ijk} \approx \sum_{r=1}^{R} a_{ir} \cdot b_{jr} \cdot c_{kr}$

Product loadings capture product-level heterogeneity
Store loadings capture store-level heterogeneity
Time factors capture common temporal patterns

Panel C: Counterfactual Imputation (Treatment Effect Estimation)

Observed Cells (Controls + Pre-Treatment)

Estimate factors: $\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}$

Low-Rank Structure (R factors estimated)

Impute Missing Cells
$\hat{Y}_{ijk}(0) = \sum_r \hat{a}_{ir}\hat{b}_{jr}\hat{c}_{kr}$

Key: Tensor structure enables imputation across all three modes (not just time as in 2D)

Treatment Effect: $\hat{\tau}_{ijk} = Y_{ijk} - \hat{Y}_{ijk}(0)$

Panel A: Observed (blue) and treated/missing (red) cells in a products × stores × time tensor.
Panel B: CP decomposition factorizes the tensor into product loadings, store loadings, and time factors.
Panel C: Impute counterfactuals for treated cells using the estimated low-rank structure.