THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY
Machine Learning
Homework 1

Due Date: See course webpage.

Your answers should be typed, not handwritten. You can submit a Word file or a pdf file. Submissions are to be made via Canvas. Note that penalty applies if your similarity score exceeds 40. To minimize your similarity score, don't copy the questions.

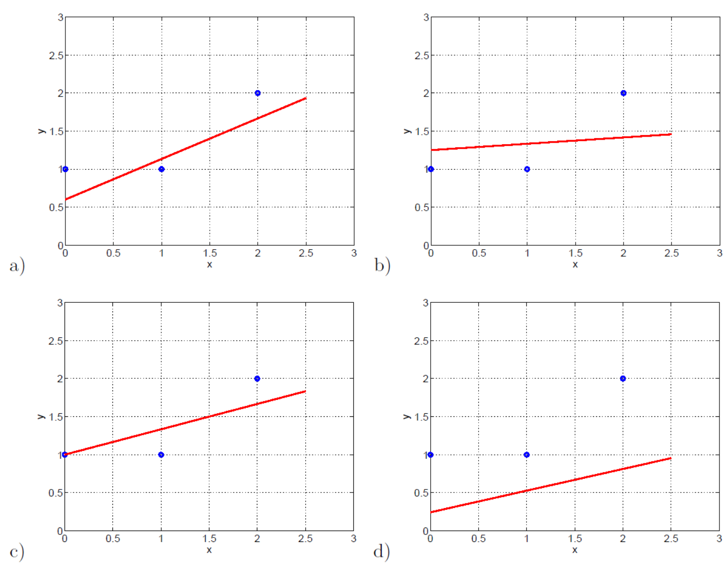
Question 1: Suppose a dataset $\mathcal{D} = \{\mathbf{x}_i\}_{i=1}^N$ is generated from some unknown distribution $p(\mathbf{x})$ and we learn from $\mathcal{D}$ a distribution $q_\theta(\mathbf{x})$ with parameters θ. What is the KL divergence $KL(p||q_\theta)$ of q_θ from p? What is the cross entropy $H(p, q_\theta)$ between p and q_θ? How are they related?

What is the log-likelihood of $l(\theta|\mathcal{D})$? How is maximizing $l(\theta|\mathcal{D})$ related to minimizing the cross entropy and the KL divergence ?

Question 2 Consider carrying out linear regression on the following dataset. Manually compute the ordinary least squares solution.

x_1	0	0	1	1	1
x_2	1	1	1	0	0
y	0	1	2	3	4

Question 3 The following figures show linear regression results on a dataset of only three data points (marked blue).



The results were obtained using following regularization schemes:

1. $\frac{1}{3}\sum_{i=1}^{3}(y_i - w_0 - w_1 x_i)^2 + \lambda w_1^2$ where $\lambda = 1$.
2. $\frac{1}{3}\sum_{i=1}^{3}(y_i - w_0 - w_1 x_i)^2 + \lambda w_1^2$ where $\lambda = 10$.
3. $\frac{1}{3}\sum_{i=1}^{3}(y_i - w_0 - w_1 x_i)^2 + \lambda(w_0^2 + w_1^2)$ where $\lambda = 1$.
4. $\frac{1}{3}\sum_{i=1}^{3}(y_i - w_0 - w_1 x_i)^2 + \lambda(w_0^2 + w_1^2)$ where $\lambda = 10$.

Match the regularization schemes with the regress results. Briefly explain your answers.

Question 4 Consider applying logistic regression to the following dataset:

x_1	0	0	1	1
x_2	0	1	0	1
y	0	0	0	1

The target is to learn a model of the form $p(y = 1|\mathbf{x}, \mathbf{w}) = \sigma(w_0 + w_1x_1 + w_2x_2)$.

Suppose $w_0 = -2$, $w_1 = 1$ and $w_2 = 1$ initially and $\alpha = 0.1$. Manually run the batch gradient descent algorithm for one iteration. Give the weights and training error (i.e., fraction of misclassified examples) after the iteration.

Question 5 Consider applying logistic regression to the following dataset:

x_1	0	0	1	1
x_2	0	1	0	1
y	1	0	0	1

1. If we use raw feature x_1 and x_2, the model is

$$p(y = 1|\mathbf{x}, \mathbf{w}) = \sigma(w_0 + w_1x_1 + w_2x_2).$$

   What is the minimum achievable training error in this case? Give weights that achieve the minimum error.

2. Next consider using an additional feature x_1x_2 in addition to the raw feature x_1 and x_2. The model now is

$$p(y = 1|\mathbf{x}, \mathbf{w}) = \sigma(w_0 + w_1x1 + w_2x_2 + w_3x_1x_2).$$

   What is the minimum achievable training error in this case? Give weights that achieve the minimum error.

Question 6 Consider the gradient vector in logistic regression $\nabla J(\mathbf{w}) = (\frac{\partial J(\mathbf{w})}{\partial w_0}, \frac{\partial J(\mathbf{w})}{\partial w_1}, \ldots, \frac{\partial J(\mathbf{w})}{\partial w_D})$ where

$$\frac{\partial J(\mathbf{w})}{\partial w_j} = -\frac{1}{N}\sum_{i=1}^{N}[y_i - \sigma(z_i)]x_{i,j}.$$

Suppose the feature x_1 is binary and, in the training set, it takes value 1 only in a small number of training examples with class label 1 (i.e., $y = 1$), and it takes value 0 in all training examples with class label 0 (i.e., $y = 0$). What will happen to the weight w_1 if we update it repeatedly using the following rule:

$$w_1 \leftarrow w_1 + \alpha\frac{1}{N}\sum_{i=1}^{N}[y_i - \sigma(\mathbf{w}^\top\mathbf{x}_i)]x_{i,1}$$

What if we use the following update rule instead:

$$w_1 \leftarrow w_1 + \alpha[-\lambda w_1 + \frac{1}{N}\sum_{i=1}^{N}[y_i - \sigma(\mathbf{w}^\top\mathbf{x}_i)]x_{i,1}],$$

where λ is the regularization constant?