Parameter Estimation via the Method of Moments

Required definitions and equations:

Estimator	Description	Notes
$m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$	1st sample moment	This is the equation for the sample mean, $\bar{x}$.
$m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2$	2nd sample moment	The second sample moment is <u>sometimes</u> equal to the estimator for the sample variance. This is NOT always the case.
$\mu_1 = E[X]$	1st population moment	This equation is an estimator of the population mean.
$\mu_2 = E[X^2]$	2nd population moment	This equation is <u>sometimes</u> an estimator of the population variance. This is NOT always the case.

Procedure:

1.) Calculate:

- m_1
- m_2
- $E[X]$
- $E[X^2]$

2.) Equate each sample moment with the associated population moment:

(e.g., set the first sample moment equal to the first population moment, the second sample moment to the second population moment, and so on.)

Sample Moments		Population Moments
m_1	=	μ_1
m_2	=	μ_2

$\downarrow$

Sample Moments		Population Moments
$\frac{1}{n}\sum_{i=1}^{n} x_i$	=	$E[X]$
$\frac{1}{n}\sum_{i=1}^{n} x_i^2$	=	$E[X^2]$

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Procedure (continued):

3.) Solve the system of equations for μ_1 and μ_2 in terms of x_i.

4.) When μ and μ_2 are "alone" on one side of their respective equations, top them off with a tilde and conclude that they are method of moments estimators:

- $\widetilde{\mu_1}$ is a method of moments estimator for the first population moment, the population mean.
- $\widetilde{\mu_2}$ is a method of moments estimator for the second population moment.