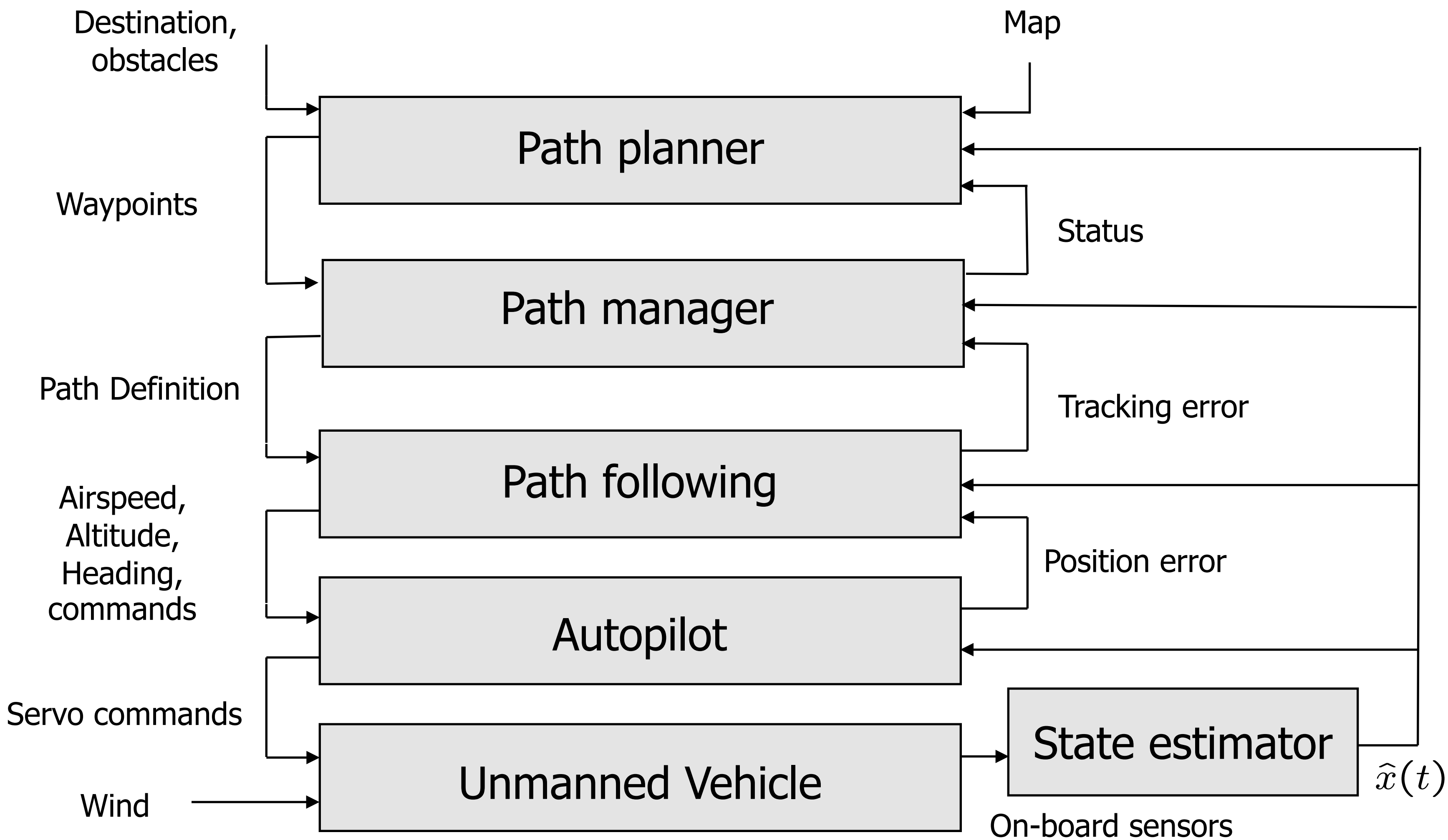
Chapter 7

Sensors

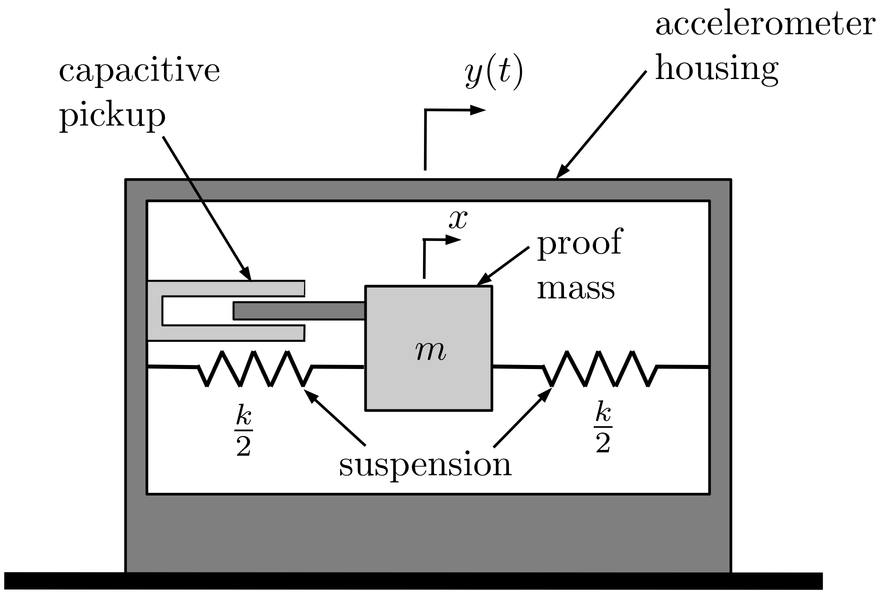
Architecture



Sensors for MAVs

- The following types of sensors are commonly used for guidance and control of MAVs
  - accelerometers
  - rate gyros
  - pressure sensors
  - magnetometers (digital compasses)
  - GPS

MEMS Accelerometer



$$m\ddot{x} + kx = ky(t)$$

$$\delta = y(t) - x$$

Acceleration of proof mass proportional to deflection of suspension

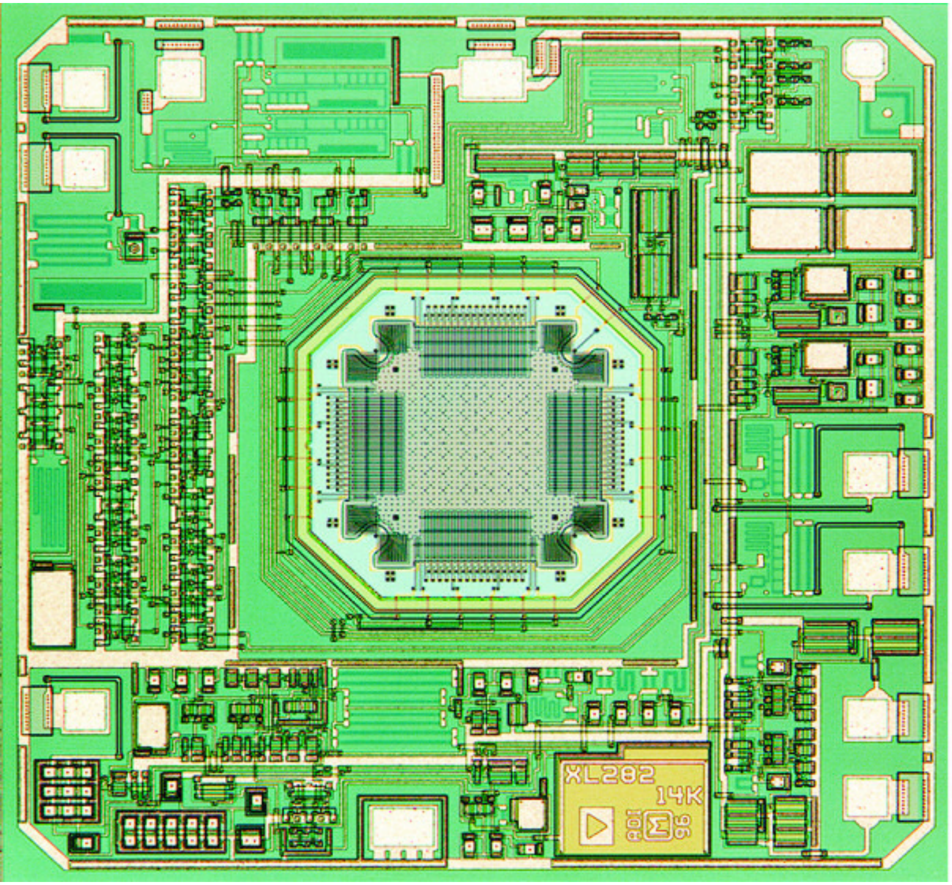
$$\ddot{x} = \frac{k}{m}\delta$$

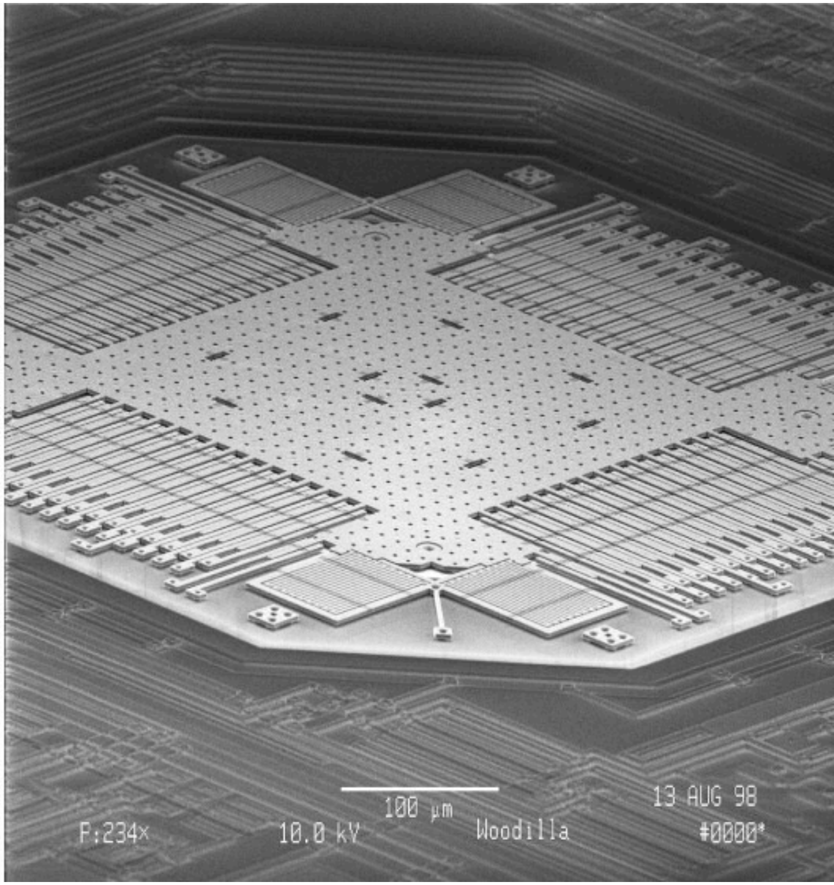
$$\frac{X(s)}{Y(s)} = \frac{1}{\frac{m}{k}s^2 + 1}$$

$$\frac{A_X(s)}{A_Y(s)} = \frac{1}{\frac{m}{k}s^2 + 1}$$

Model $\qquad \Upsilon_{\text{accel}} = k_{\text{accel}}A + \beta_{\text{accel}} + \eta'_{\text{accel}}$

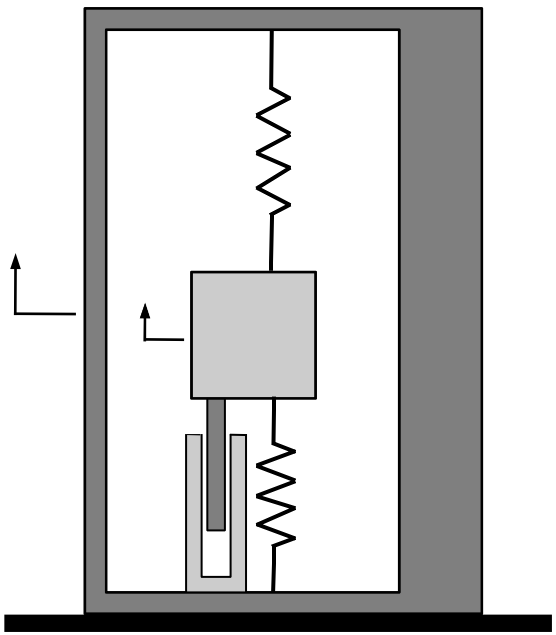
MEMS Accelerometer





Acceleration Measurement

Tricky concept: Measured acceleration is the total acceleration of the accelerometer casing minus the acceleration of gravity



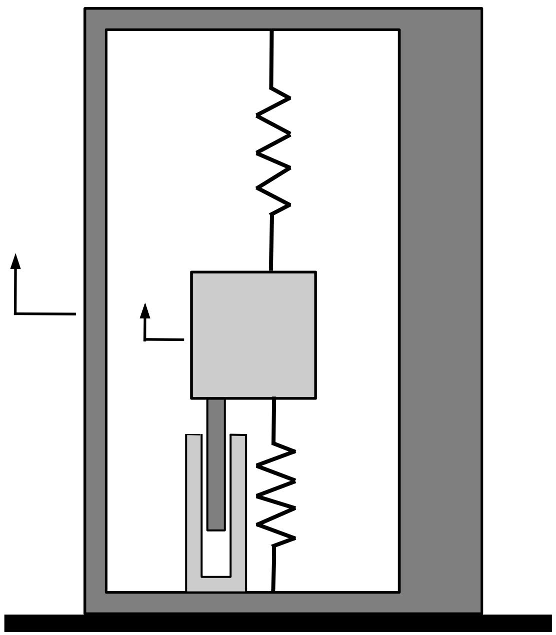
$$\mathbf{a} = \frac{1}{\mathrm{m}} \left(\mathbf{F}_{\mathrm{total}} - \mathbf{F}_{\mathrm{gravity}} \right)$$

Example: Set the accelerometer on a table top. What does it measure?

Accels measure components of linear, coriolis, and externally applied acceleration. They do not measure gravity, since both the proof mass and the casing are acted on by gravity in exactly the same way

Acceleration Measurement

Said another way, accelerometers measure *specific force*, which is defined as the sum of the non-gravitational forces divided by the mass



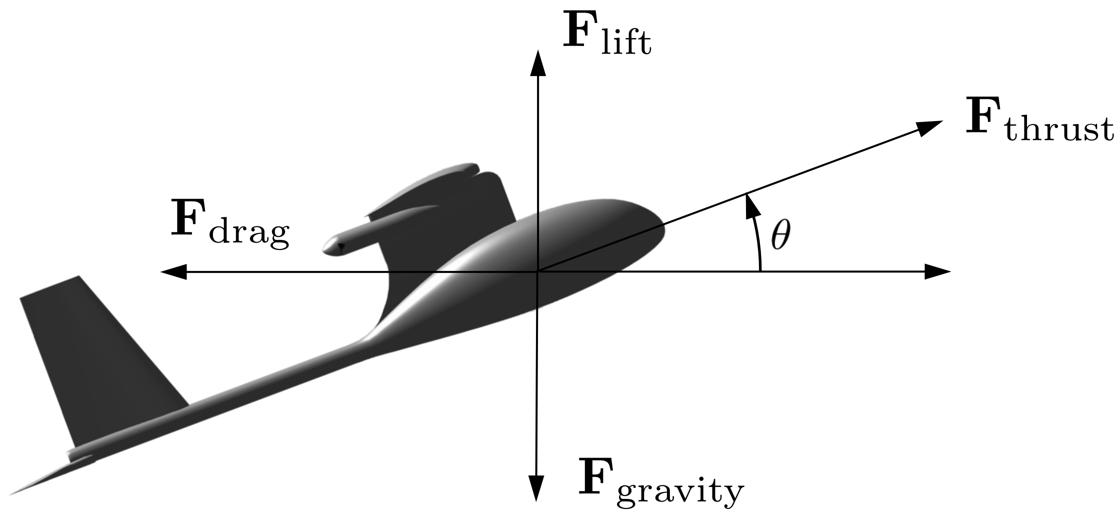
$$\mathbf{a}_{\text{measured}} = \frac{1}{\mathrm{m}}\left(\sum \mathbf{F}_{\text{non-gravitational}}\right)$$
$$= \frac{1}{\mathrm{m}}\left(\sum \mathbf{F} - \mathbf{F}_{\text{gravitational}}\right)$$

Example: Set the accelerometer on a table top.
What does it measure?

Hint: (Draw FBD of accel housing)

What about an accelerometer in free fall?

Acceleration on Fixed-Wing Aircraft



$$
\begin{aligned}
\mathbf{a}_{\text{measured}} &= \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{total}} - \mathbf{F}_{\text{gravity}}\right) \\
&= \frac{1}{\mathsf{m}}\left(\left(\mathbf{F}_{\text{lift}} + \mathbf{F}_{\text{drag}} + \mathbf{F}_{\text{thrust}} + \mathbf{F}_{\text{gravity}}\right) - \mathbf{F}_{\text{gravity}}\right) \\
&= \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{lift}} + \mathbf{F}_{\text{drag}} + \mathbf{F}_{\text{thrust}}\right)
\end{aligned}
$$

Acceleration on Fixed-Wing Aircraft

Recall from Chapter 3, that

$$\mathsf{m}\left(\frac{d\mathbf{v}}{dt_b} + \boldsymbol{\omega}_{b/i} \times \mathbf{v}\right) = \mathbf{F}_{\text{total}}.$$

Using the expression

$$\mathbf{a}_{\text{measured}} = \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{total}} - \mathbf{F}_{\text{gravity}}\right),$$

the output of the accelerometer can be expressed as

$$\mathbf{a}_{\text{measured}} = \frac{d\mathbf{v}}{dt_b} + \boldsymbol{\omega}_{b/i} \times \mathbf{v} - \frac{1}{\mathsf{m}}\mathbf{F}_{\text{gravity}}.$$

Expressing this relationship in the body frame gives

$$\begin{aligned}
a_x &= \dot{u} + qw - rv + g\sin\theta \\
a_y &= \dot{v} + ru - pw - g\cos\theta\sin\phi \\
a_z &= \dot{w} + pv - qu - g\cos\theta\cos\phi
\end{aligned}$$

Accelerometer Models

$$\boxed{\begin{aligned} y_{\text{accel},x} &= \dot{u} + qw - rv + g\sin\theta + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \dot{v} + ru - pw - g\cos\theta\sin\phi + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \dot{w} + pv - qu - g\cos\theta\cos\phi + \eta_{\text{accel},z} \end{aligned}}$$

or

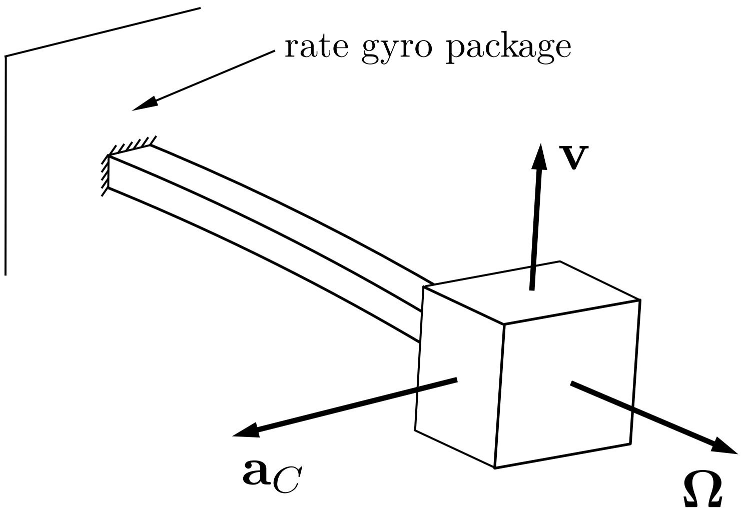
$$\boxed{\begin{aligned} y_{\text{accel},x} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_X(\alpha) + C_{X_q}(\alpha)\frac{\bar{c}q}{2V_a} + C_{X_{\delta_e}}(\alpha)\delta_e\right] \\ &\quad + \frac{\rho S_{\text{prop}} C_{\text{prop}}}{2\text{m}}\left[(k_{\text{motor}}\delta_t)^2 - V_a^2\right] + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{bp}{2V_a} + C_{Y_r}\frac{br}{2V_a}\right. \\ &\quad \left. + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right] + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_Z(\alpha) + C_{Z_q}(\alpha)\frac{\bar{c}q}{2V_a} + C_{Z_{\delta_e}}(\alpha)\delta_e\right] + \eta_{\text{accel},z} \end{aligned}}$$

Accelerometer Models

or

$$\boxed{\begin{aligned} y_{\text{accel},x} &= \frac{f_x}{\mathsf{m}} + g\sin\theta + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \frac{f_y}{\mathsf{m}} - g\cos\theta\sin\phi + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \frac{f_z}{\mathsf{m}} - g\cos\theta\cos\phi + \eta_{\text{accel},z} \end{aligned}}$$

MEMS Rate Gyro



Point translating on a rotating rigid body experiences a coriolis acceleration:

$$\mathbf{a}_C = 2\boldsymbol{\Omega} \times \mathbf{v}$$

MEMS rate gyro – resonating proof mass:

$$\mathbf{v} = A\omega_n \sin(\omega_n t)$$

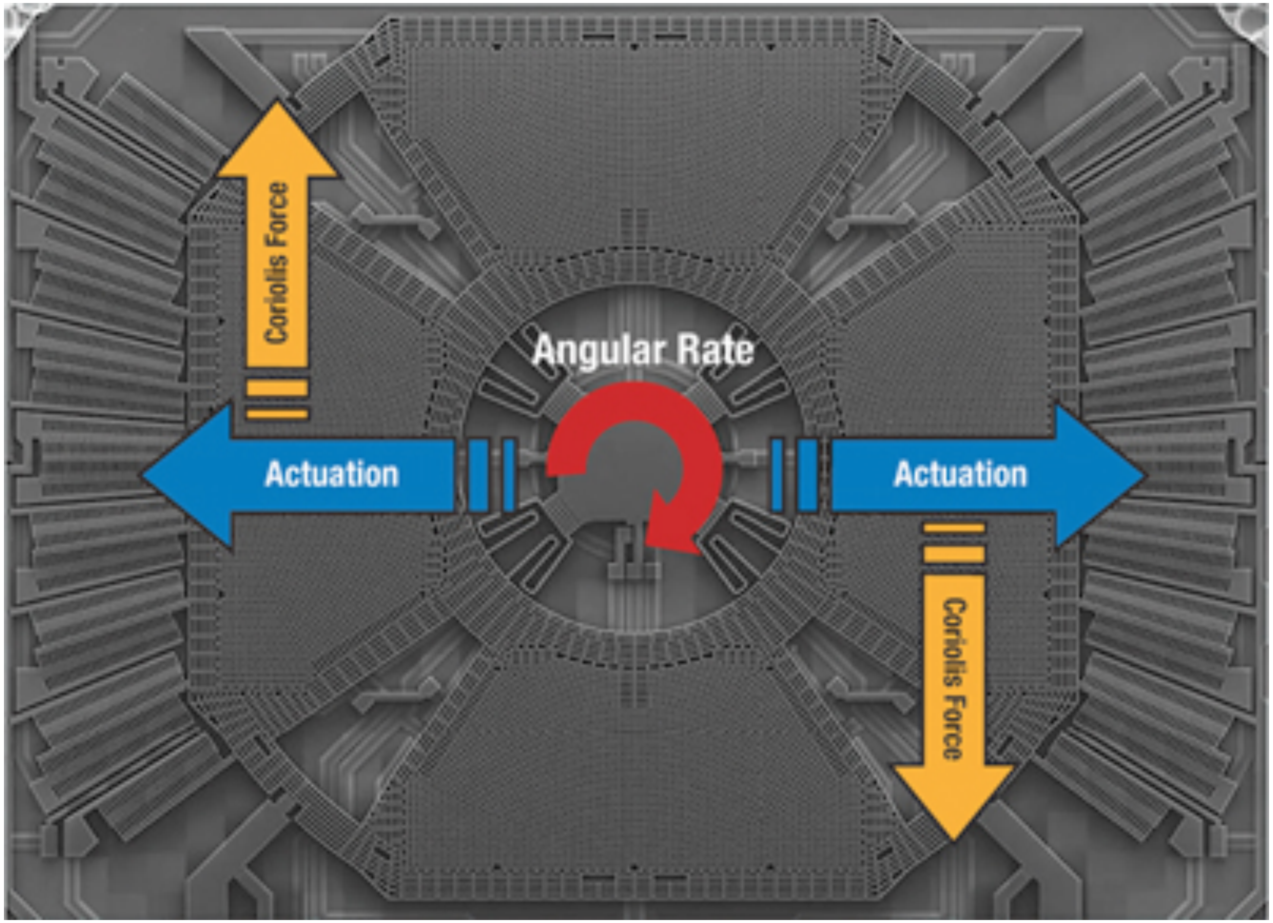
Sensor measures deflection of proof mass due to coriolis acceleration

$$\begin{aligned} V_{\text{gyro}} &= k_C |\mathbf{a}_C| \\ &= 2k_C |\boldsymbol{\Omega} \times \mathbf{v}| \end{aligned}$$

$$|\boldsymbol{\Omega} \times \mathbf{v}| = \Omega |\mathbf{v}|$$

$$\begin{aligned} V_{\text{gyro}} &= 2k_C \Omega |A\omega_n \sin(\omega_n t)| \\ &= 2k_C A\omega_n \Omega \\ &= K_C \Omega \end{aligned}$$

MEMS Rate Gyro



Rate Gyro Model

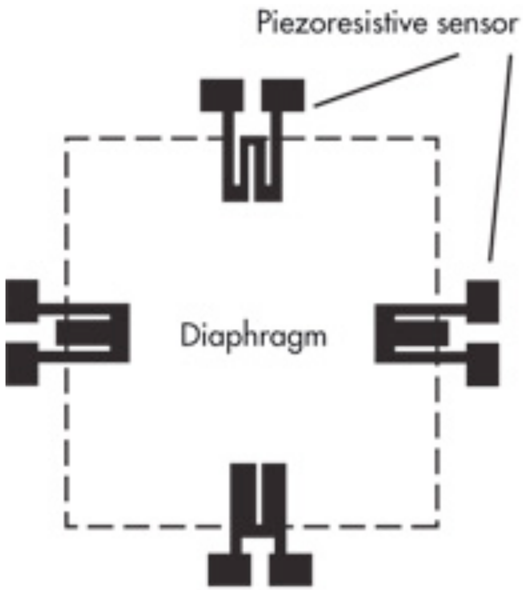
$$\Upsilon_{\rm gyro} = k_{\rm gyro}\Omega + \beta_{\rm gyro} + \eta'_{\rm gyro}$$

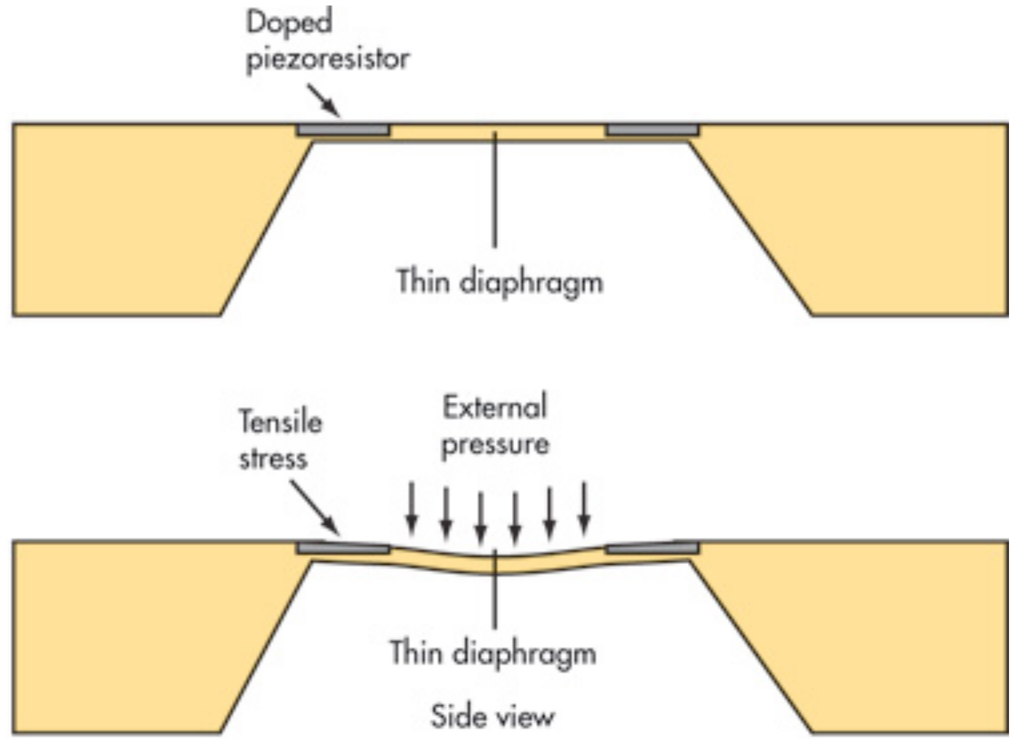
voltage output proportional to angular rate

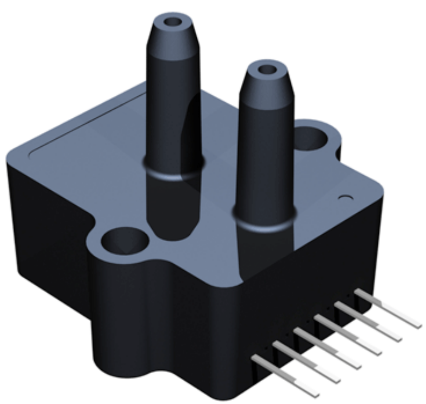
$$\begin{aligned} y_{{\rm gyro},x} &= p + \eta_{{\rm gyro},x} \\ y_{{\rm gyro},y} &= q + \eta_{{\rm gyro},y} \\ y_{{\rm gyro},z} &= r + \eta_{{\rm gyro},z} \end{aligned}$$

rate gyro measurements expressed in units of rad/s

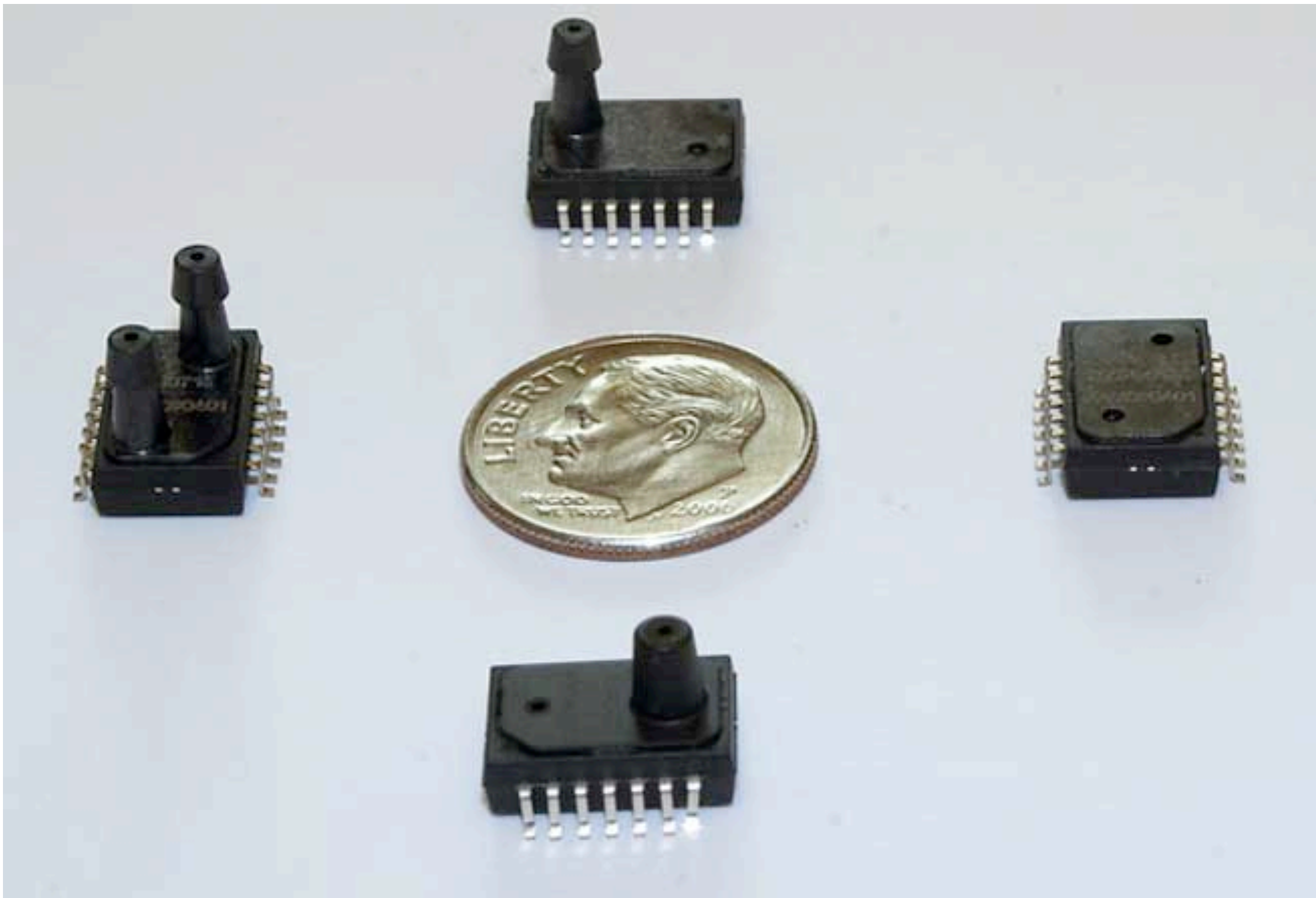
Pressure Measurement







Pressure Measurement

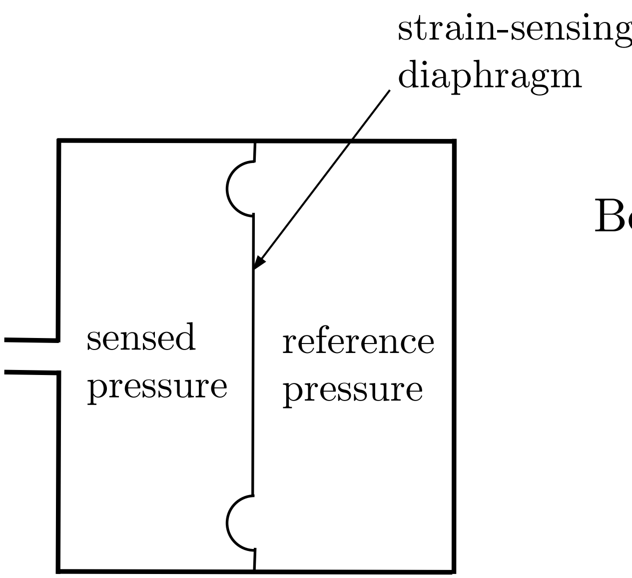


Altitude Measurement

Basic equation of hydrostatics: $P_2 - P_1 = \rho g (z_2 - z_1)$

$$P - P_{\text{ground}} = -\rho g (h - h_{\text{ground}})$$
$$= -\rho g h_{\text{AGL}}$$

(assumes constant density)



Below 11,000 m, can use barometric formula:

$$P = P_0 \left[\frac{T_0}{T_0 + L_0 h_{\text{ASL}}} \right]^{\frac{gM}{RL_0}}$$

P_0: standard pressure at sea level
T_0: standard temperature at sea level
L_0: rate of temperature decrease
g: gravitational constant
R: universal gas constant for air
M: standard molar mass of atmospheric air

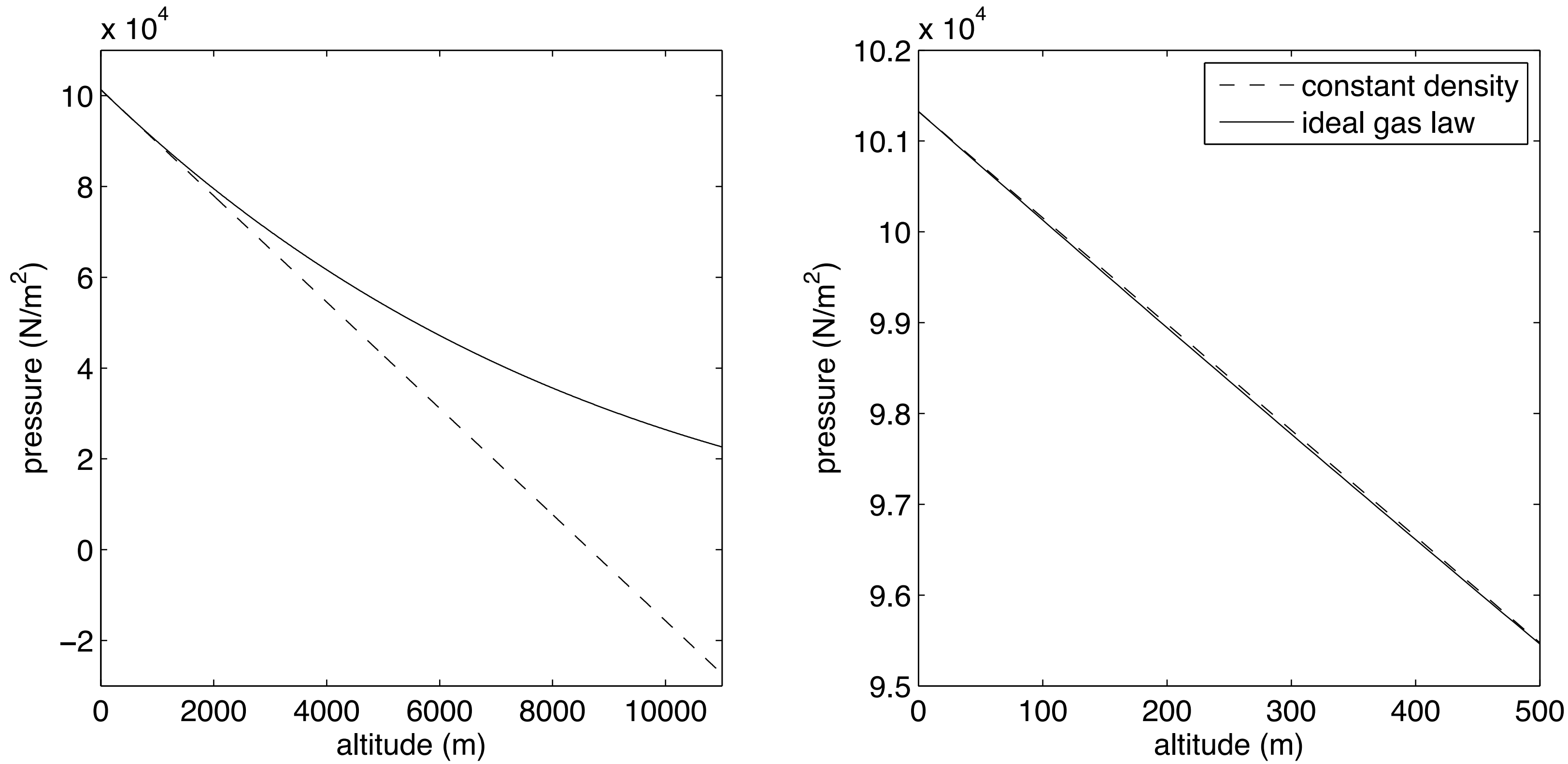
Takes into account change in density with altitude and temperature

Altitude Measurement

We usually assume density is constant:

$$\begin{aligned} y_{\text{abs pres}} &= (P_{\text{ground}} - P) + \beta_{\text{abs pres}} + \eta_{\text{abs pres}} \\ &= \rho g h_{\text{AGL}} + \beta_{\text{abs pres}} + \eta_{\text{abs pres}} \end{aligned}$$

Is this valid?



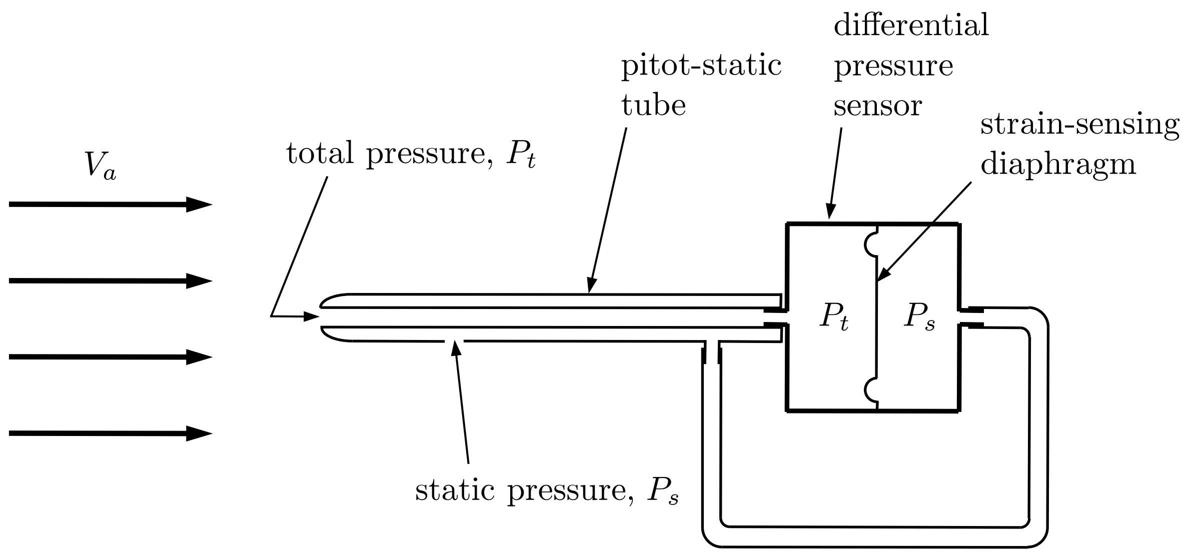
Airspeed Measurement

From Bernoulli's equation:

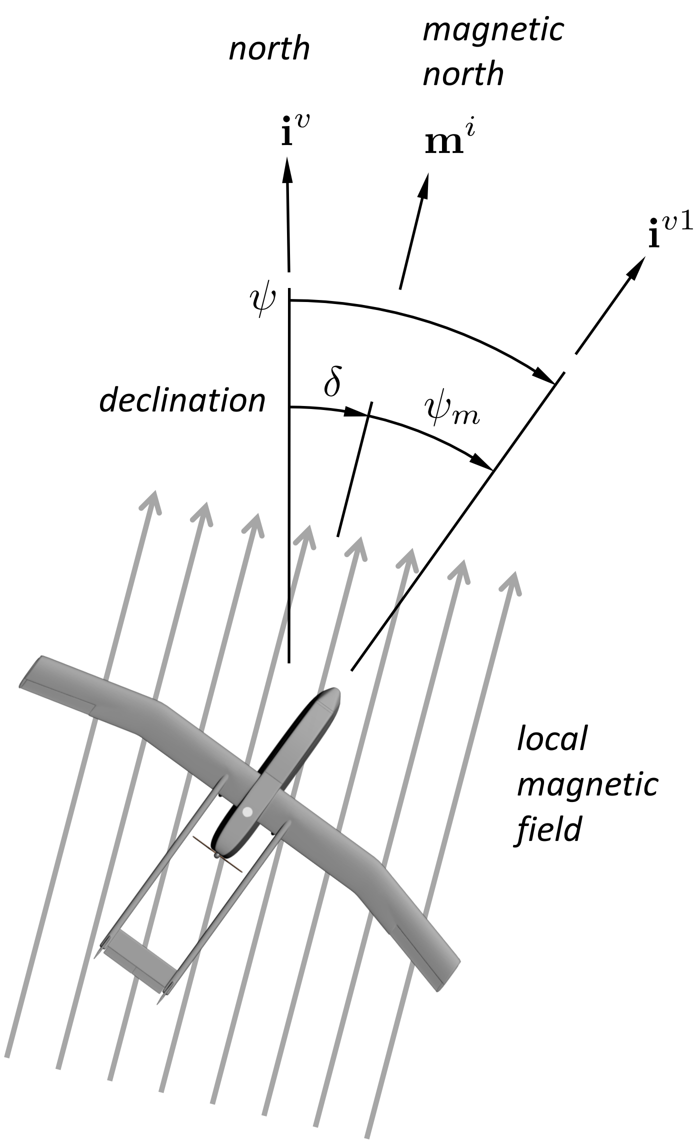
$$P_t = P_s + \frac{\rho V_a^2}{2} \quad \text{or} \quad \frac{\rho V_a^2}{2} = P_t - P_s$$

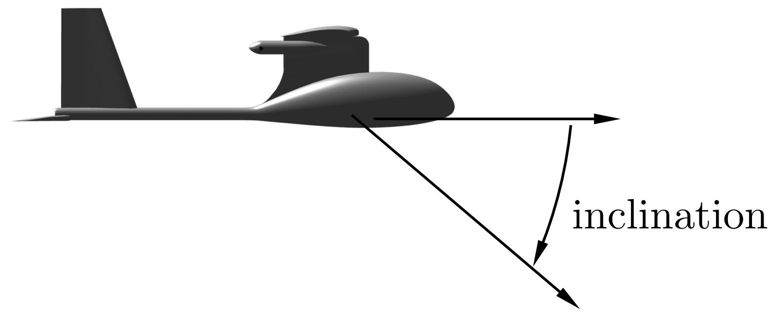
Pitot-static pressure sensor measures dynamic pressure:

$$y_{\text{diff pres}} = \frac{\rho V_a^2}{2} + \beta_{\text{diff pres}} + \eta_{\text{diff pres}}$$



Magnetometer





Let $\mathbf{e}_1 = (1, 0, 0)^\top$ be a unit vector pointing north.

Let $R(0, -\iota, \delta)$ be the rotation matrix from the magnetic frame to the inertial frame. Then

$$\mathbf{m}^i = R^\top(0, -\iota, \delta)\mathbf{e}_1$$

is the unit vector that points in the direction of the magetic field, resolved in the inertial frame.

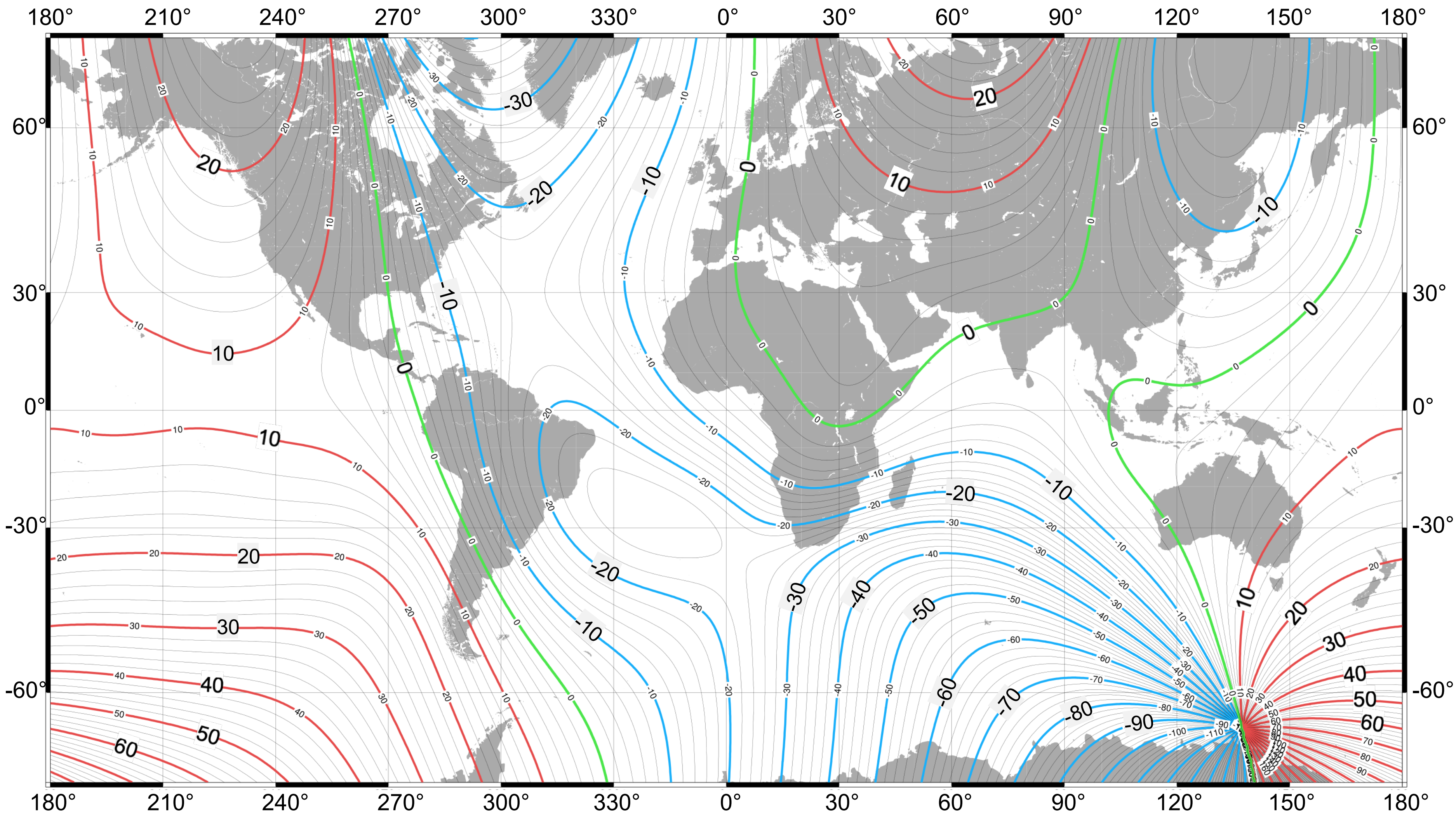
The normalized magnetic field measured in the body frame is

$$\mathbf{m}^b = R_b^{i\top}\mathbf{m}^i.$$

The measurement is given by

$$y_{mag} = \mathbf{m}^b + \boldsymbol{\eta}.$$

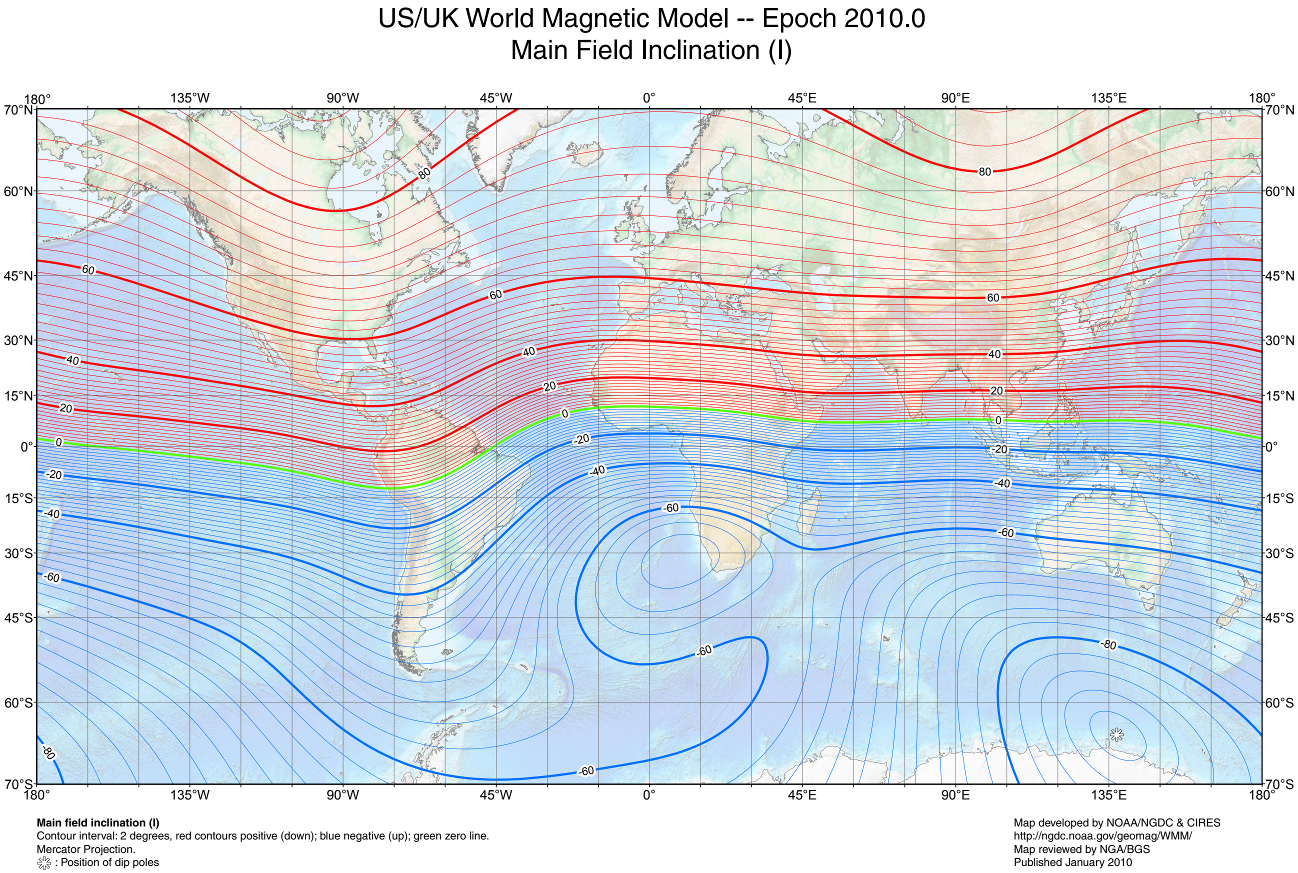
Magnetic Declination Variation



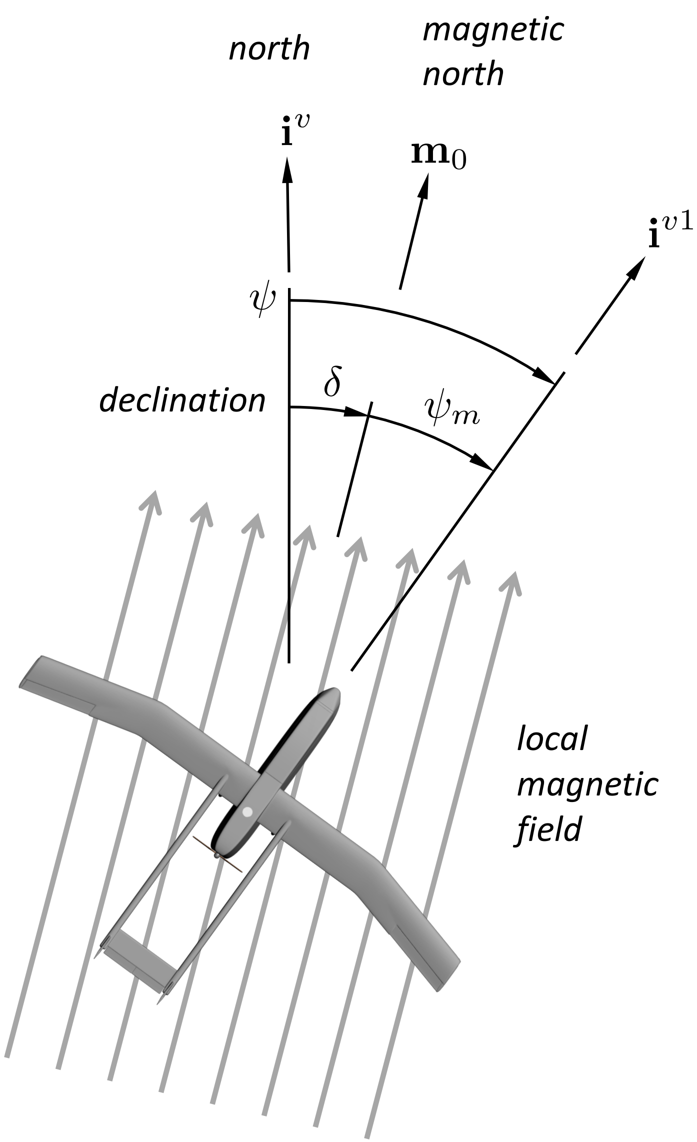
World Magnetic Model, National Geophysical Data Center

Magnetic Inclination

From Wikipedia: "Magnetic dip or magnetic inclination is the angle made by a compass needle with the horizontal at any point on the Earth's surface. Positive values of inclination indicate that the field is pointing downward, into the Earth, at the point of measurement.



Magnetometers & Digital Compasses



Heading is sum of magnetic declination angle and magnetic heading

$$\psi = \delta + \psi_m$$

Magnetic heading determined from measurements of body-frame components of magnetic field projected onto horizontal plane

$$\mathbf{m}_0^{v1} = \begin{pmatrix} m_{0x}^{v1} \\ m_{0y}^{v1} \\ m_{0z}^{v1} \end{pmatrix} = \mathcal{R}_b^{v1}(\phi, \theta)\mathbf{m}_0^b$$

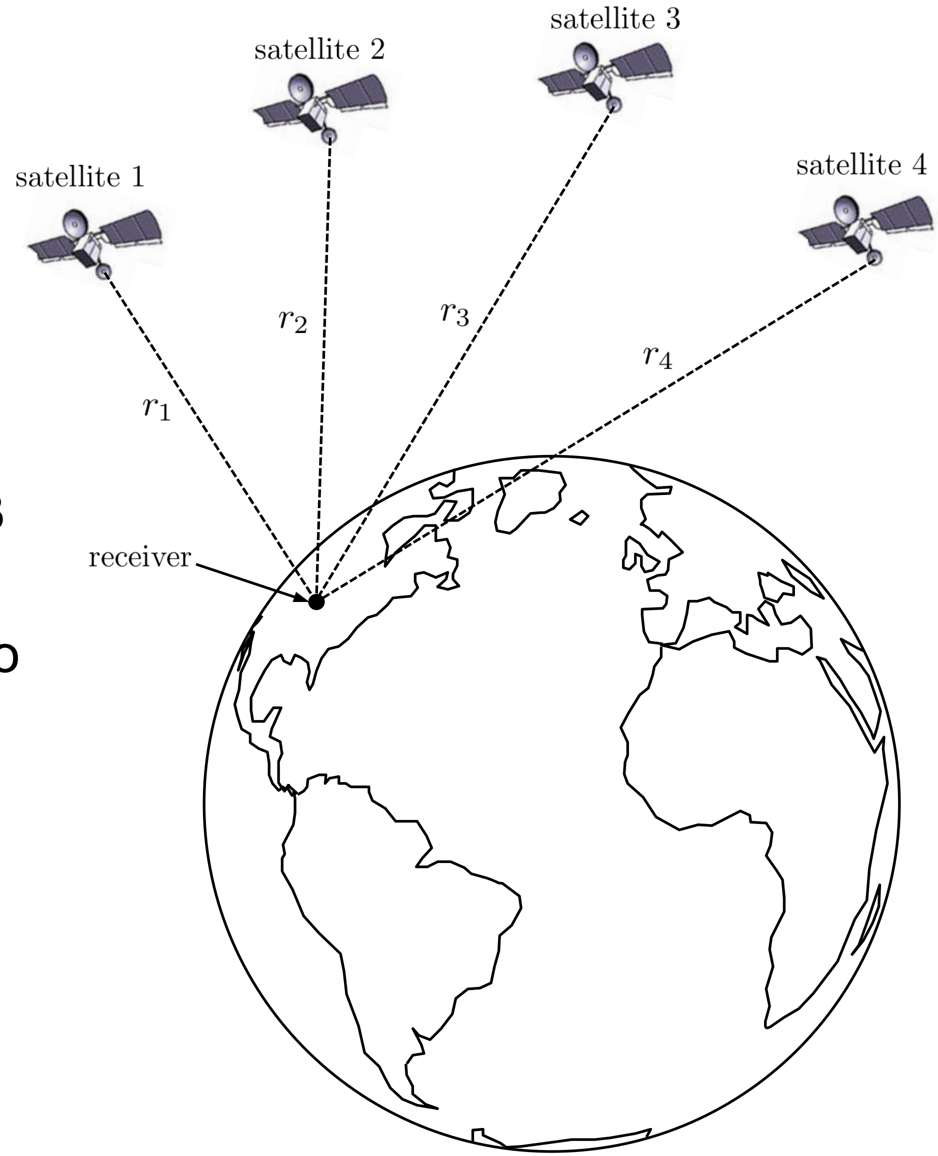
$$= \mathcal{R}_{v2}^{v1}(\theta)\mathcal{R}_b^{v2}(\phi)\mathbf{m}_0^b$$

$$\begin{pmatrix} m_{0x}^{v1} \\ m_{0y}^{v1} \\ m_{0z}^{v1} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta s_\phi & s_\theta c_\phi \\ 0 & c_\phi & -s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{pmatrix} \mathbf{m}_0^b$$

$$\psi_m = -\text{atan2}(m_{0y}^{v1}, m_{0x}^{v1})$$

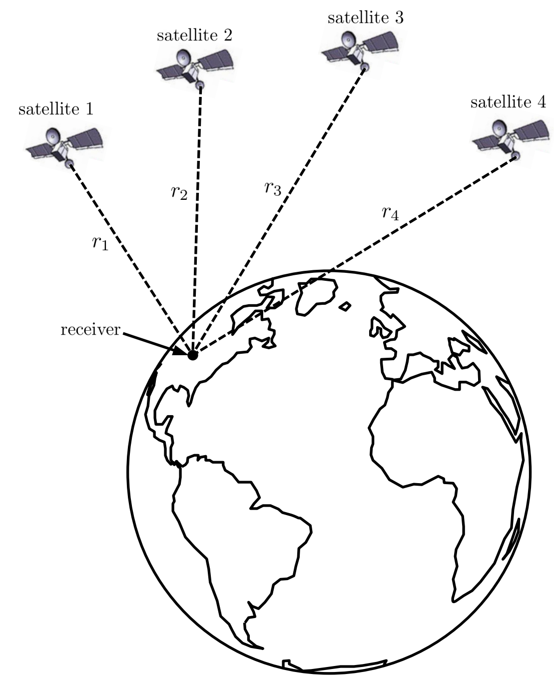
Global Positioning System

- 24 satellites orbiting the earth
- Altitude 20,180 km
- Any point on Earth's surface can be seen by at least 4 satellites at all times
- Time of flight of radio signal from 4 satellites to receiver used to trilaterate location of receiver in 3 dimensions
- 4 range measurements needed to account for clock offset error
- 4 nonlinear equations in 4 unknowns results:
  - latitude
  - longitude
  - altitude
  - receiver clock time offset



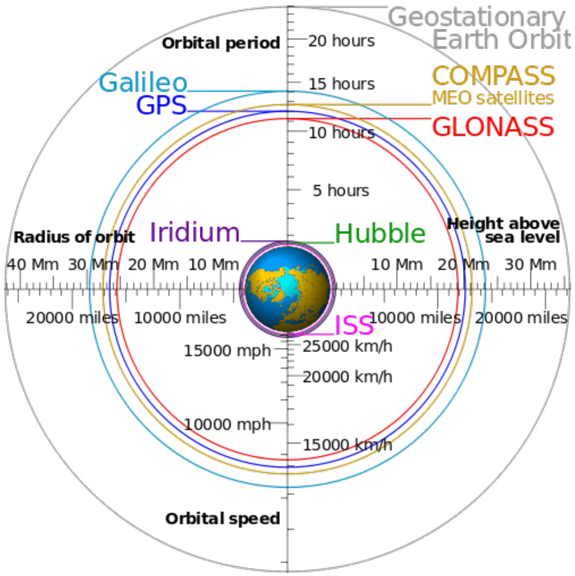
Homage to GPS

- Perhaps greatest enabling technology in emergence of UAS as practically useful technology
- Truly exquisite engineering and science achievement
- Operating principle: Range measurements based on time of flight
  - Trilateration requiring astounding precision timing
  - 10 ns timing error → 3 m pseudorange error
- 30 satellites at 20,200 km altitude, six orbital planes, transmitted signal power same as common light bulb
- Estimated four billion receivers in use – ubiquitous
- Amazing breadth of applications
- Creators of US GPS system recognized with UK's 2019 Queen Elizabeth Prize for Engineering



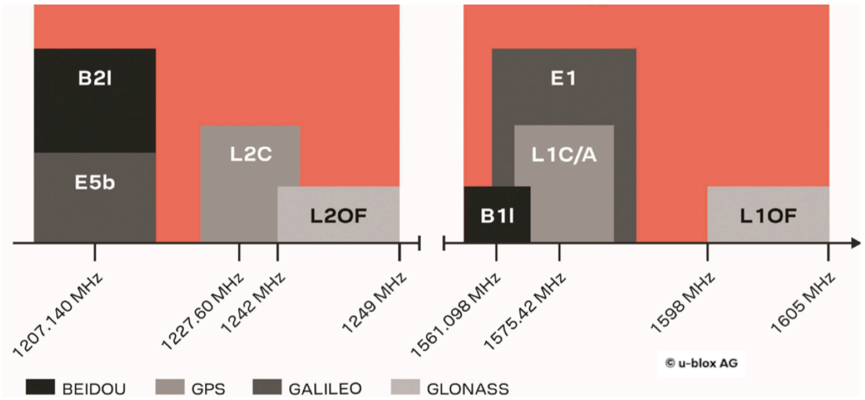
GPS to GNSS

- There are four major GNSS constellations
  - GPS - USA - 30 satellites
  - GLONASS - Russia - 24 satellites
  - Beidou - China - 35 satellites
  - Galileo - Europe - 22 satellites
- All produce signals in L1 and L2 frequency bands



L2 1.2276 GHz

L1 1.56542 GHz



Free information from the sky!

GPS Error Sources

- Time of flight of radio signal from satellite to receiver used to calculate pseudorange
  - Called pseudorange to distinguish it from true range
- Numerous sources of error in time-of-flight measurement:
  - Ephemeris Data – errors in satellite location
  - Satellite Clock – due to clock drift
  - Ionosphere – upper atmosphere, free electrons slow transmission of GPS signal
  - Troposphere – lower atmosphere, weather (temperature and density) affect speed of light, GPS signal transmission
  - Multipath Reception – signals not following direct path
  - Receiver Measurement – limitations in accuracy of receiver timing
- Small timing errors can result in large position errors
  - 10 ns timing error → 3 m pseudorange error

GPS Error Characterization

- Cumulative effect of GPS pseudorange errors is described by user equivalent range error (UERE)
- UERE has two components
  - Bias
  - Random

1-σ, in meters

Error source	Bias	Random	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	2.0	0.7	2.1
Ionosphere	4.0	0.5	4.0
Troposphere monitoring	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver measurement	0.5	0.2	0.5
UERE, rms	5.1	1.4	5.3
Filtered UERE, rms	5.1	0.4	5.1

GPS Error Characterization

- Effect of satellite geometry on position calculation is expressed by dilution of precision (DOP)
- Satellites close together → high DOP
- Satellites far apart → low DOP
- DOP varies with time
- Horizontal DOP is smaller than vertical DOP
- Nominal HDOP = 1.3
- Nominal VDOP = 1.8

Total GPS Error (RMS)

Standard deviation of RMS error in the north-east plane:

$$
\begin{aligned}
E_{\text{n-e,rms}} &= \text{HDOP} \times \text{UERE}_{\text{rms}} \\
&= (1.3)(5.1 \text{ m}) \\
&= 6.6 \text{ m}
\end{aligned}
$$

Standard deviation of RMS altitude error:

$$
\begin{aligned}
E_{\text{h,rms}} &= \text{VDOP} \times \text{UERE}_{\text{rms}} \\
&= (1.8)(5.1 \text{ m}) \\
&= 9.2 \text{ m}
\end{aligned}
$$

GPS Error Model

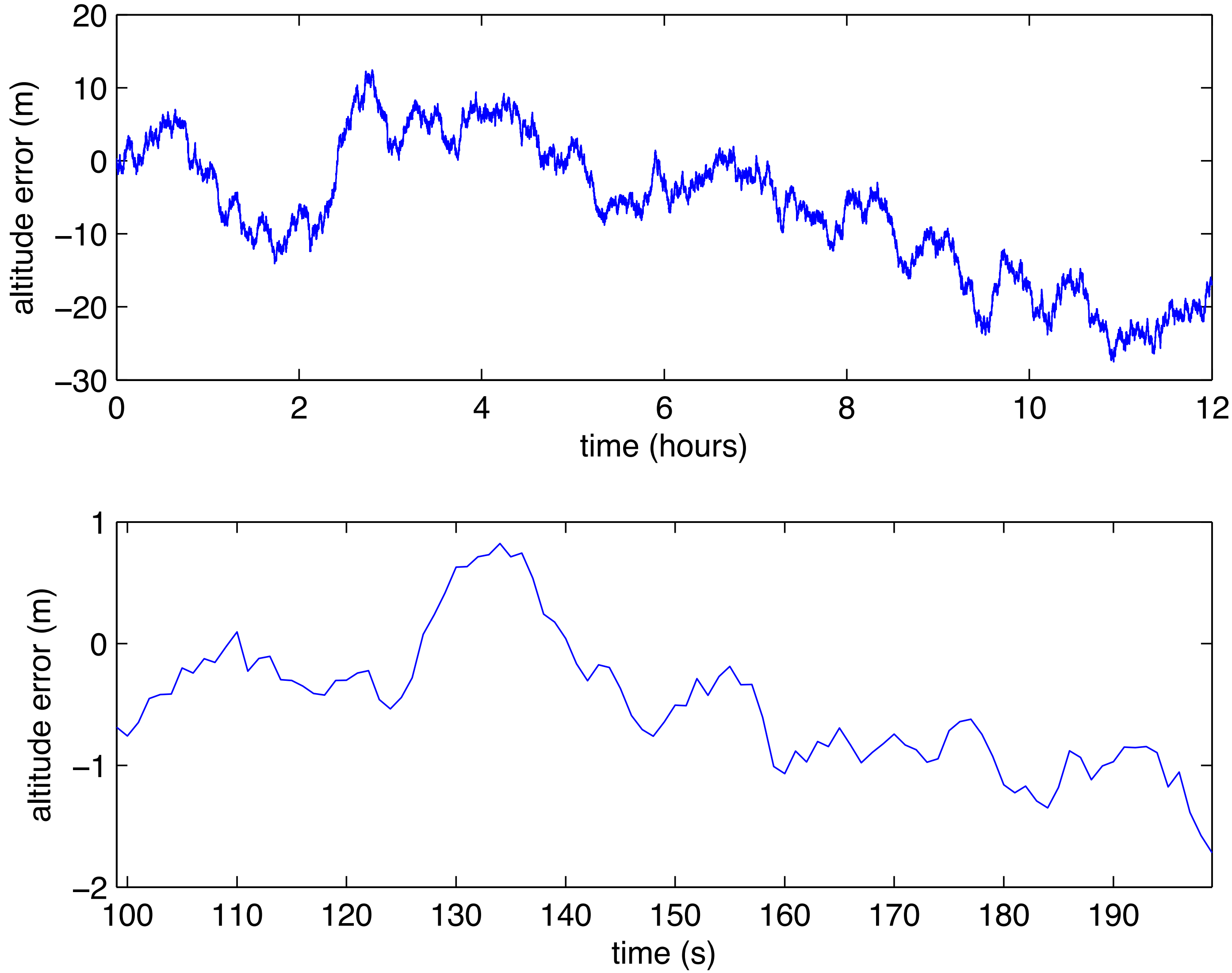
- Interested in transient behavior of errors – how does GPS error change with time
- We use Gauss-Markov error model proposed by Rankin

$$\nu[n+1] = e^{-k_{\mathrm{GPS}} T_s} \nu[n] + \eta_{\mathrm{GPS}}[n]$$

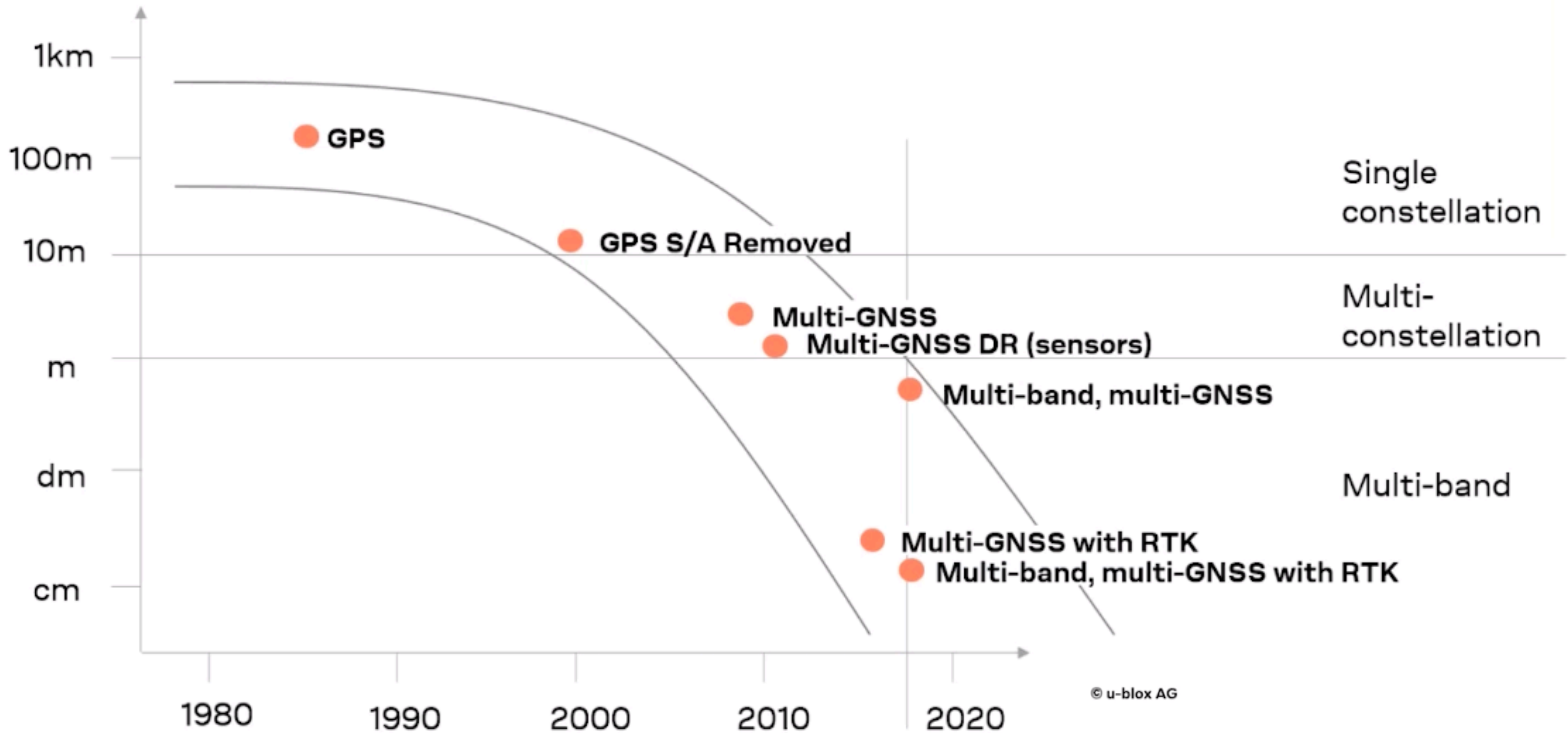
	Nominal 1-σ error (m)		Model Parameters		
Direction	Bias	Random	Std. Dev. η_{GPS} (m)	$1/k_{\mathrm{GPS}}$ (s)	T_s (s)
North	4.7	0.4	0.21	1100	1.0
East	4.7	0.4	0.21	1100	1.0
Altitude	9.2	0.7	0.40	1100	1.0

$$y_{\mathrm{GPS},n}[n] = p_n[n] + \nu_n[n]$$
$$y_{\mathrm{GPS},e}[n] = p_e[n] + \nu_e[n]$$
$$y_{\mathrm{GPS},h}[n] = -p_d[n] + \nu_h[n]$$

GPS Gauss Markov Process Error Model



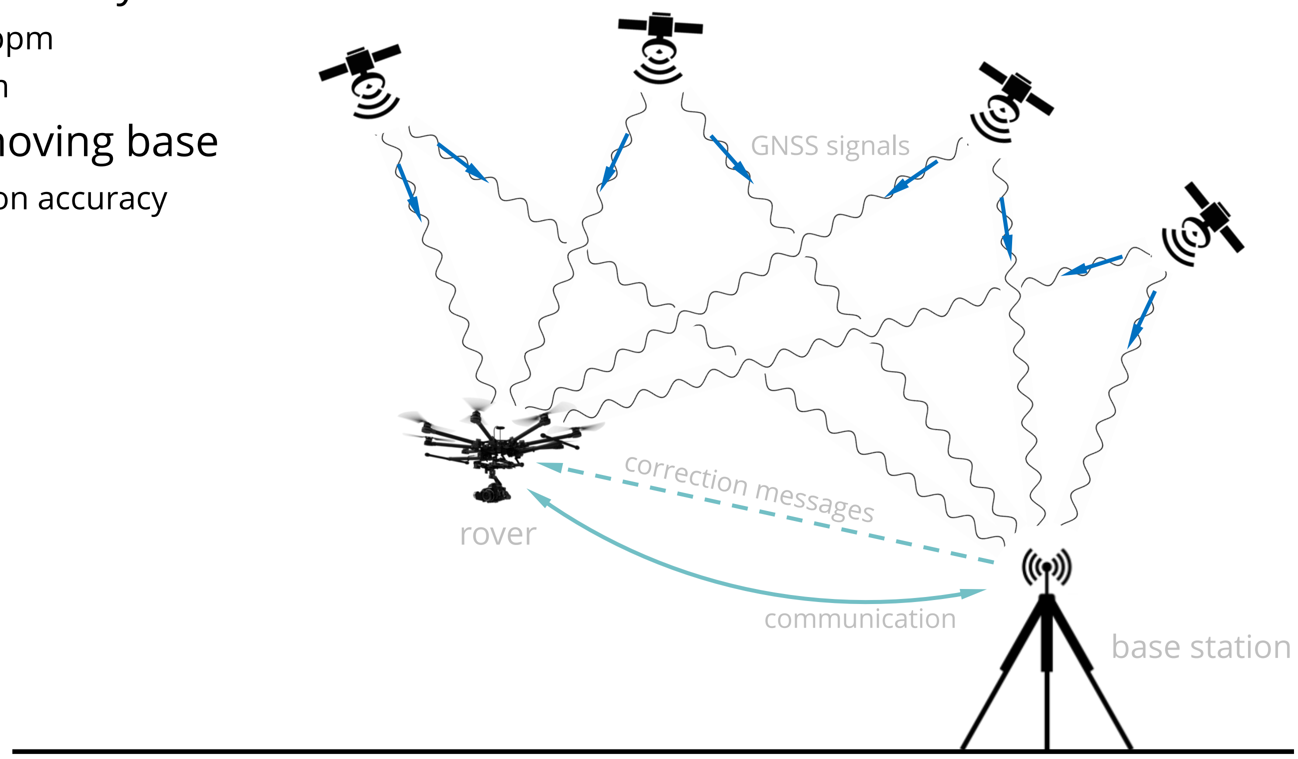
Evolution of GNSS position accuracy



Multi-band, multi-GNSS with RTK offers greater accuracy *and robustness*

Real-time Kinematic GNSS

- Exploits multi-band, multi-GNSS, and carrier phase measurements
    - More signals gives better accuracy, shorter convergence times, higher RTK availability
- Absolute position accuracy
    - Horizontal: 1 cm ± 2 ppm
    - Vertical: 2 cm ± 2 ppm
- Can function with moving base
    - Similar relative position accuracy



Assignment

- Model the noise characteristics and dynamic behavior of sensors
  - Rate gyros
  - Accelerometers
  - Absolute pressure sensor
  - Differential pressure sensor
  - GPS
- Modify mav_dynamics.py