DDA 4230 Tutorial 4

Section 0: Outline

1. Implement the multi-arm bandits (MAB) environment
2. Implement & test the ϵ-greedy algorithm
3. Implement & test the Explore-then-commit (ETC) algorithm
4. Implement & test Upper Confidence Bound (UCB) algorithm
5. Discussion

```
# Import libraries
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
```

Section 1: MAB Environment

Input

1. m_arm: Number of arms
2. Mean_reward: Expectation of reward for each arm
3. Std: Standard Deviation of reward for each amr

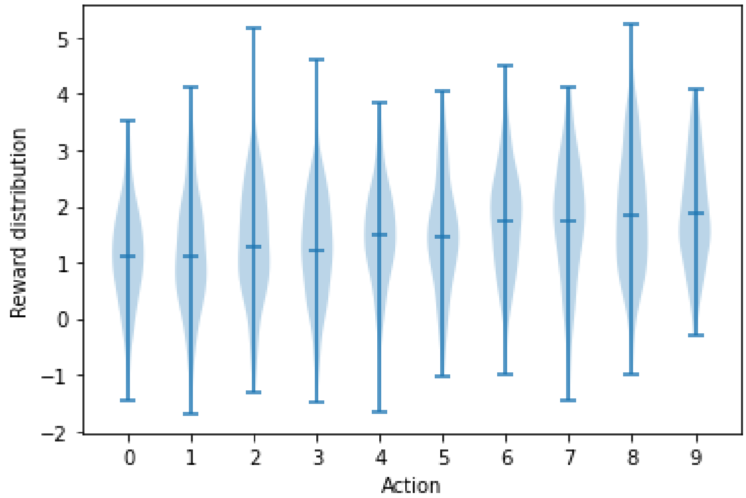
Methods

1. act(action): Select arm 'action' and return the observed reward

```
class Bandit:
    def __init__(self, m_arm=10, mean_reward=[1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6,
1.7, 1.8, 1.9],
                 std=np.ones(10)*1):
        self.true_mean = mean_reward
        self.m = m_arm
        self.std = std
        assert self.m == len(self.true_mean) and self.m == len(
            self.std), 'Number of arms must equal to numer of mean rewards/stds'

    def act(self, action):
        return np.random.randn()*self.std[action]+self.true_mean[action]
```

$std * N(0, 1) + mean$

```python
np.random.seed(1)
m=10
bandit_10=Bandit(m_arm=m)
data_set=np.zeros([200,m])
for i in range(m):
    for j in range(200):
        data_set[j,i]=bandit_10.act(i)
plt.violinplot(data_set,showmeans=True)
plt.xlabel("Action")
plt.ylabel("Reward distribution")
plt.xticks(list(range(1,11)), list(range(10)))
plt.show()
```



Section 2: ϵ-greedy Algorithm

Algorithm 1: The ε-greedy algorithm

Input: $\varepsilon_t, t \in \{0, 1, \ldots, T\}$ the exploration parameters

Output: $\pi(t), t \in \{0, 1, \ldots, T\}$

while $0 \leq t \leq m-1$ **do**

$$\pi(t) = t + 1$$

while $m \leq t \leq T$ **do**

$$\pi(t) \sim \begin{cases} \arg\max_{i \in [m]} \left\{ \frac{1}{N_{t-1,i}} \sum_{t'=0}^{t-1} r_{t'} \mathbb{1}\{a_{t'} = i\} \right\} & \text{with probability } 1 - \varepsilon_t \\ i \quad \text{with probability } \varepsilon_t/m, \text{ for each } i \in [m] \end{cases}$$

Input

1. bandits: MAB environment
2. eps: value of ϵ
3. T: Horizon

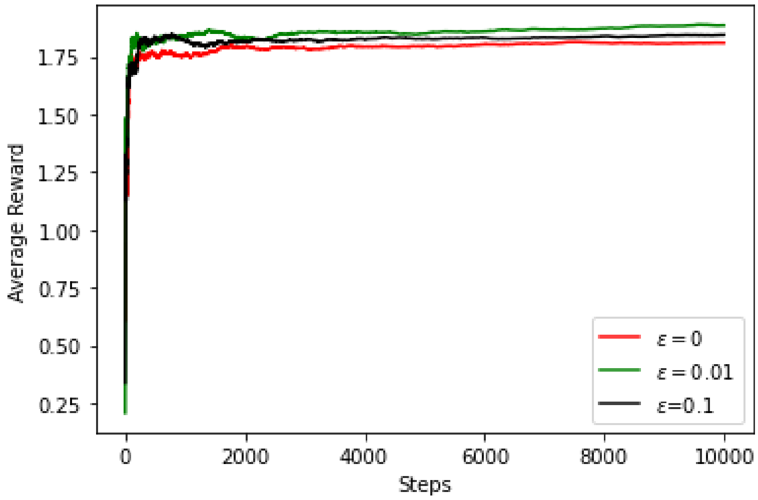
Return

1. q_hat: Reward estimation for each arm
2. avg_return_list: average return over time

A Trick: Incremental Implementation

Let $Q_n = \frac{R_1+R_2+\ldots+R_{n-1}}{n-1}$, then,
$Q_{n+1} = \frac{1}{n}\sum_{i=1}^{n} R_i = \frac{1}{n}(R_n + \sum_{i=1}^{n-1} R_i) = \frac{1}{n}(R_n + (n-1)Q_n) = \frac{1}{n}(R_n - Q_n + nQ_n) = Q_n + \frac{1}{n}[R_n - Q_n]$

```
# Eps-greedy
def eps_greedy(bandits, eps, T=1000):
    m = bandits.m
    N = np.zeros(m)
    q_hat = np.zeros(m)
    avg_return = 0
    avg_return_list = np.zeros(T)
    for t in range(m):
        action = t
        reward = bandits.act(action)
        q_hat[action] = reward
        N[action] += 1
        avg_return += (reward-avg_return)/(t+2)
        avg_return_list[t] = avg_return
    for t in range(m, T):
        if np.random.rand() < eps:
            action = np.random.choice(m)
        else:
            action = np.argmax(q_hat)
        reward = bandits.act(action)
        q_hat[action] = (q_hat[action]*N[action]+reward)/(N[action]+1)
        N[action] += 1
        avg_return += (reward-avg_return)/(t+2)
        avg_return_list[t] = avg_return
    return q_hat, avg_return_list


T = 10000
q_hat, avg_return_list = eps_greedy(bandit_10, eps=0, T=T)
plt.plot(np.arange(T), avg_return_list, c='red')

q_hat, avg_return_list = eps_greedy(bandit_10, eps=0.01, T=T)
plt.plot(np.arange(T), avg_return_list, c='green')

q_hat, avg_return_list = eps_greedy(bandit_10, eps=0.1, T=T)
plt.plot(np.arange(T), avg_return_list, c='black')
plt.legend(["$\epsilon=0$", "$\epsilon=0.01$", "$\epsilon$=0.1"])

plt.xlabel("Steps")
plt.ylabel("Average Reward")
plt.show()
```



Section 3: ETC Algorithm

Algorithm 2: The explore-then-commit algorithm

Input: k: number of exploration on each arm

Output: $\pi(t), t \in \{0, 1, \ldots, T\}$

while $0 \leq t \leq km - 1$ **do**

$$a_t = (t \bmod m) + 1$$

while $km \leq t \leq T - 1$ **do**

$$a_t = \arg\max_{i \in [m]} \frac{1}{k} \sum_{t'=0}^{mk} r_{t'} \mathbb{1}\{a_{t'} = i\}$$

Input

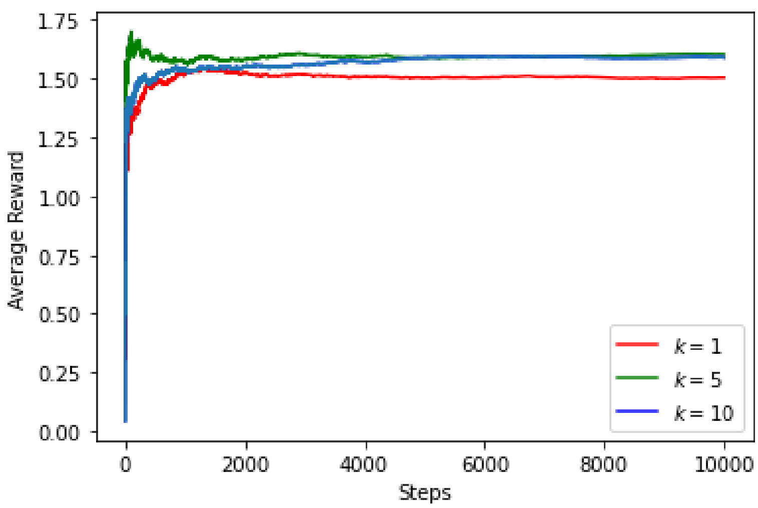
1. bandits: MAB environment
2. k: number of exploration on each arm
3. T: Horizon

Return

1. q_hat: Reward estimation for each arm
2. avg_return_list: average return over time

```
# ETC

def ETC(bandits, k=10, T=1000):
    m = bandits.m
    N = np.zeros(m)
    q_hat = np.zeros(m)
    avg_return = 0
    avg_return_list = np.zeros(T)
    for t in range(k*m):
        action = t % m
        reward = bandits.act(action)
        q_hat[action] = reward
        N[action] += 1
        avg_return += (reward-avg_return)/(t+2)
```

```
            avg_return_list[t] = avg_return
        old_q_hat=q_hat.copy()
        for t in range(k*m, T):
            action = np.argmax(old_q_hat)
            reward = bandits.act(action)
            q_hat[action] = (q_hat[action]*N[action]+reward)/(N[action]+1)
            N[action] += 1
            avg_return += (reward-avg_return)/(t+2)
            avg_return_list[t] = avg_return
        return q_hat, avg_return_list


T = 10000
q_hat, avg_return_list = ETC(bandit_10, k=1, T=T)
plt.plot(np.arange(T), avg_return_list, c='red')

q_hat, avg_return_list = ETC(bandit_10, k=5, T=T)
plt.plot(np.arange(T), avg_return_list, c='green')

q_hat, avg_return_list = ETC(bandit_10, k=10, T=T)
plt.plot(np.arange(T), avg_return_list, c='blue')

plt.legend(["$k=1$", "$k=5$", "$k=10$"])

plt.plot(np.arange(T), avg_return_list)
plt.xlabel("Steps")
plt.ylabel("Average Reward")
plt.show()
```



Section 4: UCB Algorithm

Algorithm 3: The UCB algorithm.

Input: δ: confidence level

Output: $a_t, t \in \{0, 1, \dots, T\}$

while $t \le T-1$ **do**

$$a_t = \arg\max_{i \in [m]} \mathrm{UCB}_i(t-1, \delta),$$

where ties break arbitrarily and for $i \in [m]$,

$$\mathrm{UCB}_i(t-1,\delta) = \begin{cases} \infty\,, & N_{i,t-1} = 0\,, \\ \frac{1}{N_{i,t-1}} \sum_{t' \le t-1} r_{t'} \mathbb{1}\{a_{t'} = i\} + \sqrt{\frac{2\log(1/\delta)}{N_{i,t-1}}}\,, & N_{i,t-1} > 0\,; \end{cases}$$

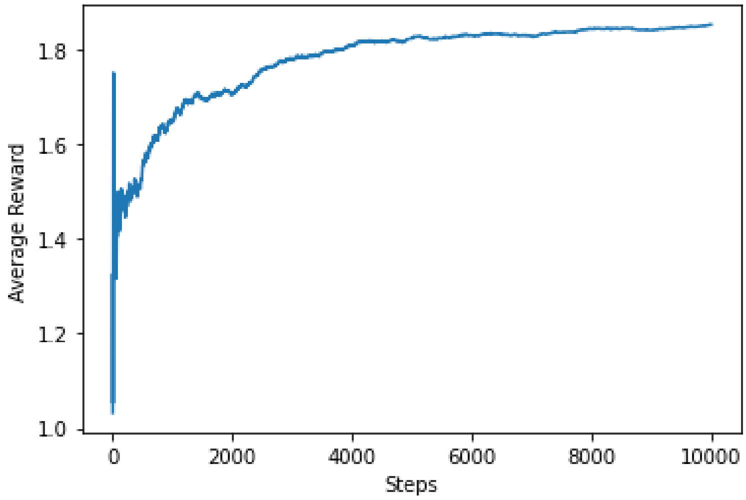
Input

1. bandits: MAB environment
2. T: Horizon

Return

1. q_hat: Reward estimation for each arm
2. avg_return_list: average return over time

```python
# UCB
def UCB(bandits, sigma, T=1000):
    m = bandits.m
    #sigma = bandits.std
    sigma=np.ones(m)*sigma
    N = np.zeros(m)
    q_hat = np.zeros(m)
    UCB = np.zeros(m)+np.inf
    avg_return = 0
    avg_return_list = np.zeros(T)

    for t in range(0, T):
        action = np.argmax(UCB)
        reward = bandits.act(action)
        q_hat[action] = (q_hat[action]*N[action]+reward)/(N[action]+1)
        UCB[action] = q_hat[action] + \
            np.sqrt(2*np.log(1/sigma[action])/(N[action]+1))
        N[action] += 1
        avg_return += (reward-avg_return)/(t+2)
        avg_return_list[t] = avg_return
    return q_hat, avg_return_list

T=10000
q_hat, avg_return_list = UCB(bandit_10,sigma=1/T**2, T=T)
plt.plot(np.arange(T), avg_return_list)
plt.xlabel("Steps")
plt.ylabel("Average Reward")
plt.show()
```



Section 5: Discussion

```
T=10000
q_hat_eps, avg_return_list_eps = eps_greedy(bandit_10, eps=0.1, T=T)
q_hat_etc, avg_return_list_etc = ETC(bandit_10, k=5, T=T)
q_hat_ucb, avg_return_list_ucb = UCB(bandit_10,sigma=1/T**2, T=T)
plt.plot(np.arange(T), avg_return_list_eps, c='red')
plt.plot(np.arange(T), avg_return_list_etc, c='green')
plt.plot(np.arange(T), avg_return_list_ucb, c='black')

plt.legend(["Eps-greedy", "ETC", "UCB"])

plt.xlabel("Steps")
plt.ylabel("Average Reward")
plt.show()
```