DDA 4230 Tutorial 8

Section 0: Outline

1. Linear Value-function Approximation (Linear VFA)
2. Brief Introduction to Neural Network

```python
import numpy as np
import matplotlib.pyplot as plt
from gridWorldGame import standard_grid, negative_grid,print_values,
print_policy

SMALL_ENOUGH = 1e-3
GAMMA = 0.9
ALL_POSSIBLE_ACTIONS = ('U', 'D', 'L', 'R')
ALPHA = 0.1
SA2IDX = {}
IDX = 0
```

Section 1: Linear Value Function Approximation

We have represented value function by a lookup table where each state has a corresponding entry $V(s)$ or each state-action pair has an entry $Q(s, a)$. However, the above approach might not generalize well to problems with very large state and action spaces. To solve this problem, a large number of approximation methods are proposed.

The benefits of value-function approximation are obvious.

- Smaller storage requirements since only the parameters w need to be stored along with a compact description of the functional form of the architecture. In general, for most approximation architectures, the storage needs are independent of the size of the state space and/or the size of the action space.
- No restriction on the state space to be a finite set for most approximation architectures;
- Good generalization abilities. Function approximation has larger flexibility and may speed up learning in finite problems, due to the fact that the algorithm can generalize earlier experiences to unseen states.

Consider a Linear function approximation. Each state/ state-action pair is represented/encoded to a feature vector $x(s) = (x_1(s), x_2(s), \ldots, x_d(s))^T$ where d is the dimensionality of the feature space. We then approximate our value functions using a linear combination of features as $x(s)w = \sum_{j=1}^{d} x_j(s)w_j$

Since the choice of feature vectors is beyond of our course, the feature vectors are given in this tutorial. If you are interested in such topic, please refer to the area of feature embedding.

1.1 Model Construction

Now, we turn to Q-learning with Linear function approximation which has update rule
$w_{t+1}(s_t, a_t) = w_t(s_t, a_t) + \alpha_t(r_t + \gamma \max_{s,a} Q(s,a) - Q(s_t, a_t))\nabla Q_t(s_t, a_t)$

```
class LinearVFA:
    def __init__(self):
        self.theta = np.random.randn(25) / np.sqrt(25)

    def sa2x(self, s, a):
        return np.array([
        s[0] - 1              if a == 'U' else 0,
        s[1] - 1.5            if a == 'U' else 0,
        (s[0]*s[1] - 3)/3     if a == 'U' else 0,
        (s[0]*s[0] - 2)/2     if a == 'U' else 0,
        (s[1]*s[1] - 4.5)/4.5 if a == 'U' else 0,
        1                     if a == 'U' else 0,
        s[0] - 1              if a == 'D' else 0,
        s[1] - 1.5            if a == 'D' else 0,
        (s[0]*s[1] - 3)/3     if a == 'D' else 0,
        (s[0]*s[0] - 2)/2     if a == 'D' else 0,
        (s[1]*s[1] - 4.5)/4.5 if a == 'D' else 0,
        1                     if a == 'D' else 0,
        s[0] - 1              if a == 'L' else 0,
        s[1] - 1.5            if a == 'L' else 0,
        (s[0]*s[1] - 3)/3     if a == 'L' else 0,
        (s[0]*s[0] - 2)/2     if a == 'L' else 0,
        (s[1]*s[1] - 4.5)/4.5 if a == 'L' else 0,
        1                     if a == 'L' else 0,
        s[0] - 1              if a == 'R' else 0,
        s[1] - 1.5            if a == 'R' else 0,
        (s[0]*s[1] - 3)/3     if a == 'R' else 0,
        (s[0]*s[0] - 2)/2     if a == 'R' else 0,
        (s[1]*s[1] - 4.5)/4.5 if a == 'R' else 0,
        1                     if a == 'R' else 0,
        1
      ])

    def predict(self, s, a):
        x = self.sa2x(s, a)
        return self.theta.dot(x)


    def grad(self, s, a):
        return self.sa2x(s, a)

def getQs(model, s):
    '''
    Return all Q value w.r.t given state s
    used for choosing action and finding policy
    '''
    Qs = {}
    for a in ALL_POSSIBLE_ACTIONS:
        q_sa = model.predict(s, a)
        Qs[a] = q_sa
    return Qs
```

```
def max_dict(d):
    # Find the largest Q value and the corresponding action
    max_key = None
    max_val = float('-inf')
    for k, v in d.items():
        if v > max_val:
            max_val = v
            max_key = k
    return max_key, max_val

def random_action(a, eps=0.1):
  # Epsilon-soft to ensure all states are visited
    p = np.random.random()
    if p < (1 - eps):
        return a
    else:
        return np.random.choice(ALL_POSSIBLE_ACTIONS)
```

1.2 Evaluation

```
grid = negative_grid(step_cost=-0.1)
print("Rewards Map:")
print_values(grid.rewards, grid)
```

```
Rewards Map:
---------------------------
-0.10|-0.10|-0.10| 1.00|
---------------------------
-0.10| 0.00|-0.10|-1.00|
---------------------------
-0.10|-0.10|-0.10|-0.10|
```

Note: In general, before feature encoding and function approximation, we need first convert state/state-values to some numerical values. Then, these numerical values can be used for indexing or feature embedding.

```
# Convert state-action to index
states = grid.all_states()
for s in states:
    SA2IDX[s] = {}
    for a in ALL_POSSIBLE_ACTIONS:
        SA2IDX[s][a] = IDX
        IDX += 1
print(SA2IDX)
```

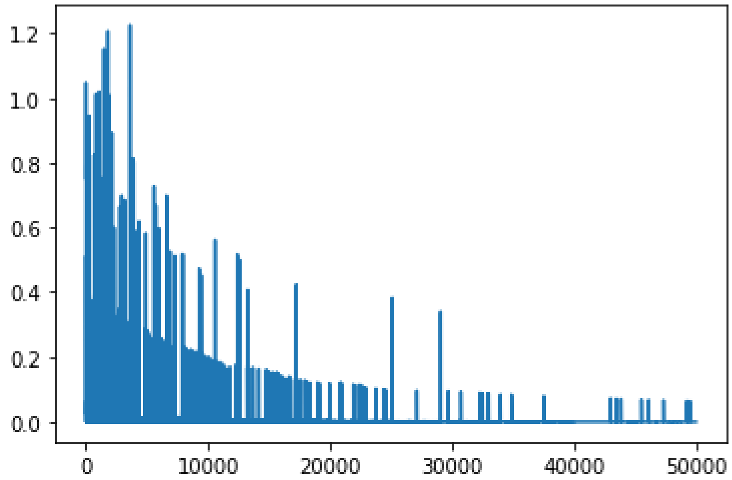
```
{(0, 1): {'U': 0, 'D': 1, 'L': 2, 'R': 3}, (1, 2): {'U': 4, 'D': 5, 'L': 6, 'R':
7}, (0, 0): {'U': 8, 'D': 9, 'L': 10, 'R': 11}, (1, 3): {'U': 12, 'D': 13, 'L':
14, 'R': 15}, (2, 1): {'U': 16, 'D': 17, 'L': 18, 'R': 19}, (2, 0): {'U': 20,
'D': 21, 'L': 22, 'R': 23}, (2, 3): {'U': 24, 'D': 25, 'L': 26, 'R': 27}, (2, 2):
{'U': 28, 'D': 29, 'L': 30, 'R': 31}, (1, 0): {'U': 32, 'D': 33, 'L': 34, 'R':
35}, (0, 2): {'U': 36, 'D': 37, 'L': 38, 'R': 39}, (0, 3): {'U': 40, 'D': 41,
'L': 42, 'R': 43}}
```

```
# Initialize model
model = LinearVFA()
```

```
# repeat until convergence
t = 1.0 # Shrink parameter for eps-greedy
t2 = 1.0 # Shrink parameter for Learning rate
deltas = []
for it in range(50000):
    if it % 100 == 0:
        t += 0.01
        t2 += 0.01
    if it % 1000 == 0:
        print("iteration:", it, end='->')
    alpha = ALPHA / t2 # Learning rate shrink over time
    s = (2, 0) # start state
    grid.set_state(s)

    Qs = getQs(model, s)

    a = max_dict(Qs)[0] # First step
    a = random_action(a, eps=0.5/t) # epsilon-greedy
    biggest_change = 0
    while not grid.game_over():
        r = grid.move(a)
        s2 = grid.current_state()
        old_theta = model.theta.copy()
        # Terminal state has Q value 0
        if grid.is_terminal(s2):
            model.theta += alpha*(r - model.predict(s, a))*model.grad(s, a)
        else:
            # not terminal
            Qs2 = getQs(model, s2)
            a2, maxQs2a2 = max_dict(Qs2)
            a2 = random_action(a2, eps=0.5/t) # epsilon-greedy

            model.theta += alpha*(r + GAMMA*maxQs2a2 - model.predict(s,
a))*model.grad(s, a)
            s = s2
            a = a2

    biggest_change = max(biggest_change, np.abs(model.theta - old_theta).sum())
    deltas.append(biggest_change)
print("Done")
```

```
iteration: 0->iteration: 1000->iteration: 2000->iteration: 3000->iteration: 4000-
>iteration: 5000->iteration: 6000->iteration: 7000->iteration: 8000->iteration:
9000->iteration: 10000->iteration: 11000->iteration: 12000->iteration: 13000-
>iteration: 14000->iteration: 15000->iteration: 16000->iteration: 17000-
>iteration: 18000->iteration: 19000->iteration: 20000->iteration: 21000-
>iteration: 22000->iteration: 23000->iteration: 24000->iteration: 25000-
>iteration: 26000->iteration: 27000->iteration: 28000->iteration: 29000-
>iteration: 30000->iteration: 31000->iteration: 32000->iteration: 33000-
>iteration: 34000->iteration: 35000->iteration: 36000->iteration: 37000-
>iteration: 38000->iteration: 39000->iteration: 40000->iteration: 41000-
>iteration: 42000->iteration: 43000->iteration: 44000->iteration: 45000-
>iteration: 46000->iteration: 47000->iteration: 48000->iteration: 49000->Done
```

```
plt.plot(deltas)
plt.show()
```



Note: As discussed in LN15, the convergence of such a Linear VFA based Q-learning is not guaranteed.

```
# determine the policy from Q*
# find V* from Q*
policy = {}
V = {}
Q = {}
for s in grid.actions.keys():
    Qs = getQs(model, s)
    Q[s] = Qs
    a, max_q = max_dict(Qs)
    policy[s] = a
    V[s] = max_q
```

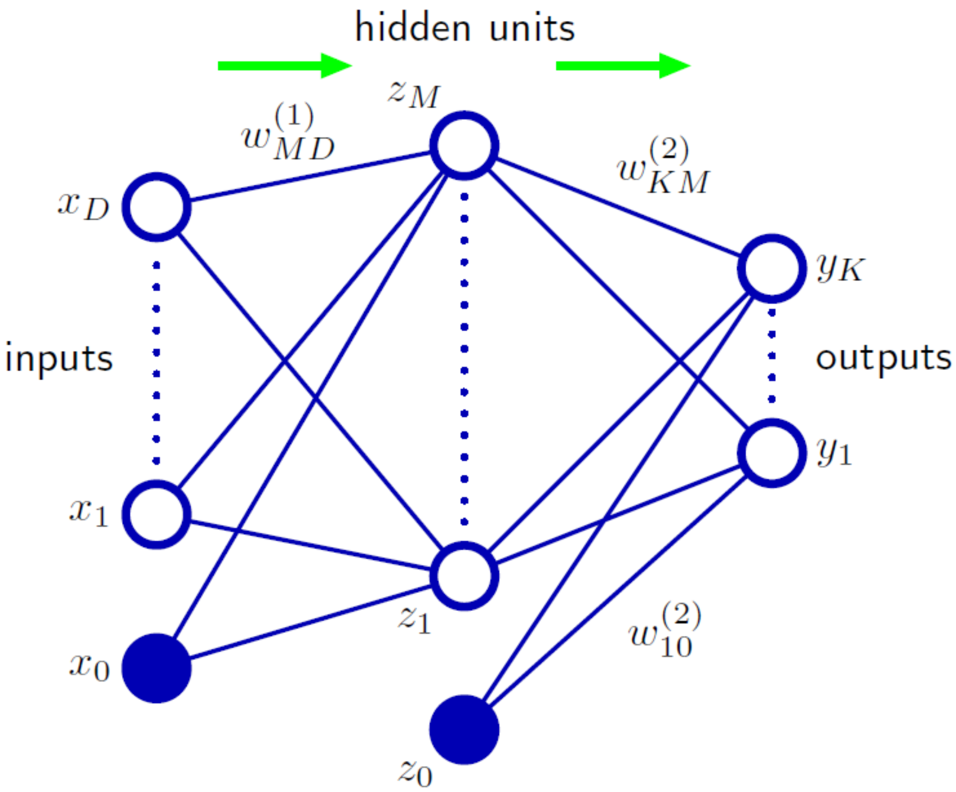
```
print("final values:")
print_values(V, grid)
print("final policy:")
print_policy(policy, grid)
```

```
final values:
---------------------------
```

```
 0.62| 0.80| 1.00| 0.00|
---------------------------
 0.46| 0.00| 0.71| 0.00|
---------------------------
 0.31| 0.17| 0.33| 0.80|
final policy:
---------------------------
  R  |  R  |  R  |     |
---------------------------
  U  |     |  U  |     |
---------------------------
  U  |  U  |  U  |  U  |
```

Section 2: Brief Introduction to Neural Network

- What is (Artificial) Neural Network?
    - In general, Neural Network is a class of non-linear function $f(x)$. Example



We have $z_j = \sigma\left(\sum_{i=0}^{D} x_i w_{i,j}^{(1)}\right), \forall j = 1, 2, \ldots M$ $y_k = \sigma\left(\sum_{j=0}^{M} z_j w_{j,k}^{(2)}\right), \forall k = 1, 2, \ldots M$ So, $f_k(x) = y_k = \sigma\left(z_0 w_{0,k}^{(2)} + \sum_{j=1}^{M} \sigma\left(\sum_{i=0}^{D} x_i w_{i,j}^{(1)}\right) w_{j,k}^{(2)}\right), \forall k = 1, 2, \ldots M$ where σ is a nonlinear function called activation function such as Sigmoid and Relu.

Each node in the graph is called a neuron.

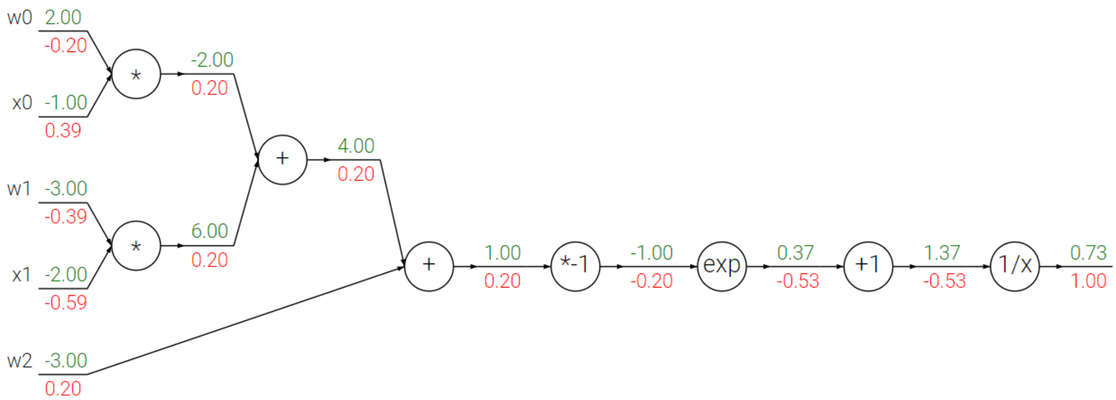
- Why do we need Neural Network?
    - Theoretically, ANN can approximate any function by universal approximation theorems.

2.1 Forward & backward Propogation

- Forward Propogation
    - Given x, return $f(x)$
- Back Propogation:

- Calculate the gradient of each parameter using Chain Rule.

Example: Consider a neural network as below:



Forward Propogation: Green

Backward Propogation: Red

```
# Last Neuron
1*-1/(1.37)**2*np.exp(-1)
```

```
-0.1960037514899261
```

2.2 Regression using Neural Network

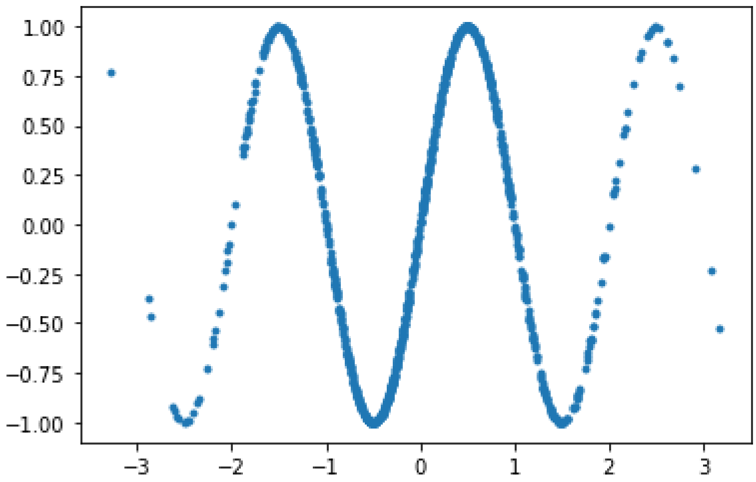
Find f minimize (f(X)-Y)^2

```
def sigmoid(x):
    s=1.0/(1.0+1.0/np.exp(x))
    ds=s*(1-s)
    return s

N, D_in, H, D_out = 1000, 1, 200, 1

x = np.random.randn(N, D_in)
y=np.sin(np.pi*x)
w1 = np.random.randn(D_in, H)
w2 = np.random.randn(H, D_out)

learning_rate = 1e-5
for it in range(20000):
    # Forward pass
    h = x.dot(w1) # N * H
    h_s=sigmoid(h) # Sigmoid
    y_pred = h_s.dot(w2) # N * D_out

    # compute loss
    loss = np.square(y_pred - y).sum()
    print(it, loss)
```

```
    # Backward pass
    # compute the gradient
    grad_y_pred = 2.0 * (y_pred - y)
    grad_w2 = h_s.T.dot(grad_y_pred)
    grad_h_s = grad_y_pred.dot(w2.T)
    grad_s= h_s*(1-h_s)*grad_h_s
    grad_w1 = x.T.dot(grad_s)

    # update weights of w1 and w2
    w1 -= learning_rate * grad_w1
    w2 -= learning_rate * grad_w2
```

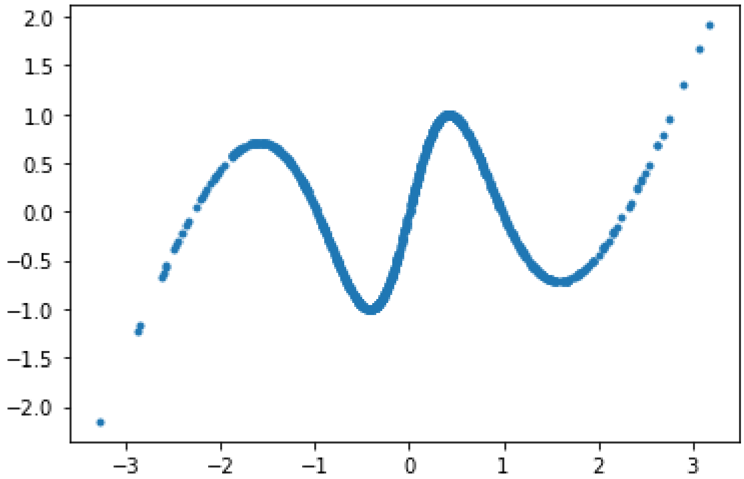
```
plt.plot(x,y,'.')
```

```
[<matplotlib.lines.Line2D at 0x7f9f42aee0f0>]
```




```
plt.plot(x,y_pred,'.')
```

```
[<matplotlib.lines.Line2D at 0x7f9f42add668>]
```



Acknowledge

Part of this tutorial is adapted from https://github.com/lazyprogrammer/machine_learning_examples/tree/master/rl and https://cs231n.github.io/optimization-2/