1. ① $f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + O(\Delta x^3)$

$f(x-\Delta x) = f(x) - \frac{\partial f}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + O(\Delta x^3)$

$\Delta x\, u_x + \frac{1}{2}\Delta x^2\, u_{xx} = u_{j+1} - u_j + O(\Delta x^3)$

$-\Delta x\, u_x + \frac{1}{2}\Delta x^2\, u_{xx} = u_{j-1} - u_j + O(\Delta x^3)$

得 $u_x = \dfrac{u_{j+1} - u_{j-1}}{2\Delta x} + O(\Delta x^2)$, $u_{xx} = \dfrac{u_{j+1} + u_{j-1} - 2u_j}{\Delta x^2} + O(\Delta x)$

② $f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \frac{1}{6}\frac{\partial^3 f}{\partial x^3}\Delta x^3 + O(\Delta x^4)$

$f(x+2\Delta x) = f(x) + 2\frac{\partial f}{\partial x}\Delta x + 2\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \frac{4}{3}\frac{\partial^3 f}{\partial x^3}\Delta x^3 + O(\Delta x^4)$

$f(x+3\Delta x) = f(x) + 3\frac{\partial f}{\partial x}\Delta x + \frac{9}{2}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \frac{9}{2}\frac{\partial^3 f}{\partial x^3}\Delta x^3 + O(\Delta x^4)$

$\frac{1}{6}\Delta x^3\, u_{xxx} + \frac{1}{2}\Delta x^2\, u_{xx} + \Delta x\, u_x = u_{j+1} - u_j + O(\Delta x^4)$

$\frac{4}{3}\Delta x^3\, u_{xxx} + 2\Delta x^2\, u_{xx} + 2\Delta x\, u_x = u_{j+2} - u_j + O(\Delta x^4)$

$\frac{9}{2}\Delta x^3\, u_{xxx} + \frac{9}{2}\Delta x^2\, u_{xx} + 3\Delta x\, u_x = u_{j+3} - u_j + O(\Delta x^4)$

得 $u_x = \dfrac{-11u_j + 18u_{j+1} - 9u_{j+2} + 2u_{j+3}}{6\Delta x} + O(\Delta x^3)$

$u_{xx} = \dfrac{2u_j - 5u_{j+1} + 4u_{j+2} - u_{j+3}}{\Delta x^2} + O(\Delta x^2)$