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Contents

What are good reduction procedures?

Perspectives from proof-theoretic semantics and type theory

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What are ‘good’ reduction procedures and why is it important to distinguish these from ‘bad’ ones? As (Schroeder-Heister/Tranchini, 2017) argue, from a philosophical point of view, or more specifically a standpoint of proof-theoretic semantics (see Schroeder-Heister (2018)), reduction procedures are closely connected to the question about identity of proofs: If we take proofs to be abstract entities represented by (natural deduction) derivations, then derivations belonging to the same equivalence class induced by the reflexive, symmetric, and transitive closure of reducibility represent the same proof object.[1] As they show, accepting certain reductions, more specifically accepting the so-called *Ekman-reduction* (see below), would lead to a trivialization of identity of proofs in the sense that every derivation of the same conclusion would have to be identified. They suggest such a property of reductions as a criterion to disallow them. I will argue that the question, which reductions we accept in our system, is not only important if we see them as generating a theory of proof identity but is also decisive for the more general question whether a proof has meaningful content. Therefore, we need to be careful: We cannot just accept any reduction procedure, i.e. any procedure eliminating some kind of detour in a derivation.

I think it is advantageous for this question to exploit the Curry-Howard-correspondence (see, e.g., Sørensen/ Urzyczyn (2006)) and look at proof systems annotated with λ-terms (in Curry-style). These make the structure of our derivations explicit and it can be much easier to show what is wrong with potential reductions and why they should not be admitted in our system. The question then shifts to which reduction procedures for *terms* can be allowed. The λ-calculus and some well-known properties thereof can provide us with directions as to what could be (un)desirable features of reductions.

The reductions for our usual connectives, corresponding to β-reductions in λ-calculus, are meant to eliminate unnecessary detours of the following form: There is a formula, called *maximal formula*, which is both the conclusion of an application of an introduction rule of a connective as well as the major premise of an applied elimination rule governing the same connective. It can and has been argued, however, that there are more reductions than the ones that are usually considered (see, e.g., Tennant (1995)). One of those, presented in (Ekman, 1994, 1998)[2], is the following:

$$\dfrac{B \to A \qquad \dfrac{A \to B \qquad \begin{matrix}\mathscr{D}\\ \vdots\\ A\end{matrix}}{B}\;{\scriptstyle \to E}}{A}\;{\scriptstyle \to E} \qquad \rightsquigarrow_{Ekman} \qquad \begin{matrix}\mathscr{D}\\ \vdots\\ A\end{matrix}$$

The trivialization that (Schroeder-Heister/Tranchini, 2017) show is a consequence of failure of the *confluence* property if we accept this reduction next to our usual stock. If confluence fails this means that a term (or resp. derivation) can be reduced to two different terms (resp. derivations) which are *both in normal form*, i.e. not further reducible. I will argue that annotating Ekman-reduction with terms shows something else that is essentially wrong with this reduction and that allowing it would be equal to allowing a reduction for `tonk`, i.e. a reduction for a derivation consisting of a `tonk`-introduction rule followed by its elimination rule: from A

[1] This goes back to (Prawitz, 1971); many others have defended this view since.

[2] The context of this reduction is that with this it can be shown that we can get a non-normalizable derivation of $\perp$, i.e. Tennant’s (1982) proof-theoretic characterization of paradoxes, which is *not paradoxical* in nature, though.

to A tonk B to B (see Prior (1960)). It is generally agreed upon, though, that there *cannot* be a sensible reduction for this connective. I want to show that in both cases allowing these reductions would not only trivialize identity of proofs of the same conclusion but that it would allow to reduce a term of one type to the term of an *arbitrary* other. If we take reductions as inducing an identity relation then that would force us to identify proofs of different *arbitrary* formulas. But even if we reject this assumption (some people do not find this theory of proof identity very compelling), I will argue that allowing such reductions would render derivations in such a system *meaningless*, just like tonk is considered an inherently meaningless connective.

I will briefly sketch out the reasoning. Consider the following annotated rules for tonk and the resulting reduction procedure with a constructor term k and a destructor k':

$$\frac{t : A}{kt : A \;\texttt{tonk}\; B}\,{\scriptstyle tonkI} \qquad \frac{t : A \;\texttt{tonk}\; B}{k't : B}\,{\scriptstyle tonkE} \qquad k'kt \rightsquigarrow_{\texttt{tonk}} t$$

The problem with allowing such a reduction can be shown by type reconstruction of the non-normal term $k'kt$. If we assign $k'kt$ an arbitrary type B, then the only information this gives us for kt is that its type must be of the form "? tonk B" (? meaning that this part is undetermined). Consequently, t can be assigned an arbitrary type then. This means that the types of redex and contractum are completely independent of each other, which is exactly the core of the problem with a reduction for tonk.

An Ekman-derivation and -reduction annotated with terms would be the following:

$$\frac{y : B \to A \qquad \dfrac{x : A \to B \qquad \begin{matrix}\mathscr{D}\\ \vdots \\ t : A\end{matrix}}{App(x,t) : B}\,{\scriptstyle \to E}}{App(y, App(x,t)) : A}\,{\scriptstyle \to E} \qquad App(y, App(x,t)) \rightsquigarrow_{Ekman} t$$

With type reconstruction it becomes evident that the same problem as with tonk prevails here. If we assign $App(y, App(x,t))$ an arbitrary type A, then we can reconstruct bottom-up the following derivation in which a new type variable is used whenever it is independent from the ones already used:

$$\frac{y : B \to A \qquad \dfrac{x : C \to B \qquad \begin{matrix}\mathscr{D}\\ \vdots \\ t : C\end{matrix}}{App(x,t) : B}\,{\scriptstyle \to E}}{App(y, App(x,t)) : A}\,{\scriptstyle \to E}$$

Again, such a reduction allows reducing a term of one type to one of an *arbitrary* other; one that is completely independent in the type reconstruction from the type of the term that is reduced. If we consider reductions to induce identity of proofs, and therefore in our setting, of terms, this is certainly undesirable. What can be clearly observed in both cases is that *subject reduction* (or *type preservation*) and *typechecking* do not hold for these reductions (they do for the β-reductions), i.e. it is not the case that whenever $t : A$ and $t \rightsquigarrow_{Ekman/\texttt{tonk}} t'$, then $t' : A$ and there are cases in which it is *impossible* to assign t' the type assigned to t. Failure of subject reduction and typechecking is, however, only the symptom here, it is not the cause. The cause is, rather, the arbitrariness in the type reconstruction and if we take the terms occurring within a derivation as making explicit how the derivation is built up and what is inferred from what, and therefore, telling us something about the content and meaning of the derivation, then type reconstruction should not yield such arbitrary results.

This arbitrariness cannot arise with the β-reductions and, importantly, there are also other non-standard reductions which are also well-behaved with respect to this feature, i.e. this is not simply to say that β-reductions are the only acceptable reductions. Reductions for paradoxical connectives would be one example or also the one that (Ripley, 2020) introduces in his system of core type theory (for Core Logic). Although the latter would not be suitable if we would like the equivalence relation induced by reductions to give us proof identity - since it trivializes identity of terms and subject reduction also fails in this system -, it is at least partially well-behaved in that the cases in which subject reduction fails are not completely arbitrary concerning the types (a term cannot reduce to a term of an arbitrarily different type).

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▸ KATALIN BIMBÓ, *Computation in logic*.

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Logic's connections to computation go back at least to Aristotle and his system of syllogistic reasoning. Leibniz imagined that a calculus of concepts would be invented at some time in the future; then Hilbert and Frege fulfilled this inkling by supplying calculi to compute theorems in logic, arithmetic and geometry. This thread of achievements establishes a link between computation and steps in a formal proof carried out in a proof system.

Depending on the expressive capacity of a logic it may be possible to use that logic to talk about computer programs or the process of computation. To mention quickly an example or two, the operation of a Turing machine can be described in a predicate language, and it is possible to reason about the execution steps of a high-level programming language in first-order dynamic logic. A more recent illustration is a connection between proofs in linear logic (in a sequent calculus formulation of **LL**) and steps of a certain type of counter machines.

In the main part of this talk, I will quickly overview the central components of the original Curry–Kripke method that Kripke used to establish the decidability of $\mathbf{E}_{\rightarrow}$ and $\mathbf{R}_{\rightarrow}$ in 1959. Curry introduced proof-search trees, which are constructed in a bottom-up fashion. He reformulated the operational rules with built-in mandatory contractions, and proved a lemma that is customarily called Curry's lemma nowadays. To deal with relevance logics, Kripke went a step further and replaced mandatory contractions with optional contractions in operational rules. And then, to prove the finiteness of the proof-search tree, he appealed to a lemma which is now often called Kripke's lemma.

Next, I will look at some applications of the Curry–Kripke method in the direction of lattice-**R** and beyond, namely, applications to **LL**, **MELL** and **NLL**. The case of **LL** (and **NLL**) leads to a quandary of finding at least one mistake in at least one (alleged) proof. In [3], Dunn and I proved several lattice-based modal logics decidable—including **LL**. We also pointed out that one of the problems with *all* the undecidability claims for **LL** is the direction of the computation that is modeled by sequent calculus proofs.

We might think that it does not matter whether we run an abstract machine in the forward direction or backward. Turing machines certainly can move left and right on their tape, and even some versions of finite state automata (FSA's) permit back and forth movements. However, the reversal of the computational steps is different from these examples of pseudo-physical motion. I will briefly look at the simple case of FSA's and finite state transducers; this will facilitate the definition of a suitable (non-deterministic) notion of reverse computation for abacus machines. These latter machines were described by Lambek in 1961. I prove that abaci cannot *reverse compute* primitive recursive functions—using the most permissive yet reasonable notion of reverse computation. This resolves the problem of seemingly contradictory claims about the decidability of **LL**—in favor of [3].

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Naïve Comprehension in HYPE

Maria Beatrice Buonaguidi

The project of a mathematically tenable naïve set theory is inevitably confronted with the difficulty of finding a suitable nonclassical logic. The proof-theoretic weakness of most naïve set theories and the seeming ad-hocness of some of their tenets, which entail commitment to paracompleteness or para-consistency, make a theory based on a combinatorial or iterative notion of set look like the best option. I argue that there are some fundamental advantages in adopting a naïve notion of set, drawing from considerations in category theory and on the possibility, in a naïve framework, of expressing self-referential and universal entities in a natural and simple way. Also, the adoption of a "paradox-proof" naïve set theory would revive the logicist project, and provide a pre-theoretic, logical basis for the foundations of mathematics. I argue that such a naïve set theory must ultimately be based on a framework which allows paradox in a controlled way, retaining classicality in the domain of ordinary mathematics, thus gaining the expressive power given by naïve sets without entirely sacrificing the classical structure and proof-theoretic strength.

A suitable framework is provided by the impossible world approach of the hyperintensional logic HYPE of Leitgeb, which allows paracomplete or paraconsistent submodels while maintaining classicality locally in well-behaved situations. Building on recent developments in theories of truth, which show that the theory of truth **KFL** based on HYPE has the same proof-theoretic strength as its classical counterpart **KF**, I build an arithmetical theory of naïve comprehension, **HYAC**. This theory serves as a proof-theoretic comparison between theories of truth and theories of naïve comprehension as property instantiation, or intensional set formation. I highlight the similarities between a notion of set thus construed and a notion of disquotational truth, and show that the theory **HYAC** based on HYPE has at least the same strength as **KFL**. Moreover, I set to show that **HYAC** can express compositionality for membership, i.e. iterative or mathematical set formation, hinting that a naïve set theory based on HYPE and built from first principles might achieve a strength similar to that of the classical iterative set theories. I demonstrate that this is possible in a restricted form which does not however cripple the strength of the theory. Lastly, I show that, by applying reflection principles to a weakened version of **HYAC**, transfinite induction for the language of membership up to the Feferman-Schütte ordinal Γ_0 can be proved. Although the use of the HYPE framework needs to be better justified because of some modal features of the HYPE conditional, the interesting interplay between nonclassicality and strength is worth considering for future developments.

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The Dialogical Roots of Deduction

Abstract: In this talk, I offer a précis of my recently published book *The Dialogical Roots of Deduction* (CUP, 2020). The book offers an account of the concept and practices of deduction by bringing together perspectives from philosophy, history, psychology and cognitive science, and mathematical practice. I draw on all of these perspectives to argue for an overarching conceptualization of deduction as a dialogical practice: deduction has dialogical roots, and these dialogical roots are still largely present both in theories and in practices of deduction. The account also highlights the deeply human and in fact social nature of deduction, as embedded in actual human practices.

Relevant Predication and Contextual Relevance

Nicholas Ferenz

Abstract

The project of relevant predication, introduced by J. Michael Dunn [2], is one that aims to distinguish *relevant* from *irrelevant* predications. A relevant predication is the attribution of a property to an object such that the object has the property *relevantly* (naturally, essentially, intrinsically, etc). The distinction drawn makes use of relevant logic (particularly **R**), and Dunn identifies some irrelevant properties by the use of implications rejected by relevant logicians. This project has had a closer relationship with more recent developments in identity in relevant logic. (For example, see [3].) Yaroslav Shramko [5] argues that relevance is a three place relation between an object, a property, *and a context*. While the taste of my coffee is relevant in the context where I am sitting at my desk writing, it is not relevant in the context where a small fire has broken out beside my desk. Here I will try to put out the fire with the mug of coffee. The project of relevant predication ought to take into account this contextual nature of relevance.

Shramko develops a formal account of context-sensitive relevance. This account, however, still requires further development. As it stands, Shramko's account appears to be useful for only one-shot uses. That is, in each context one must recreate or re-specify the prerequisites of the formal construction, and then employ it (in much the same way a one-shot belief revision system can be re-employed repeatedly with sufficient preparatory work for each repetition). While this may offer an adequate account in each context, I believe that the relations between contexts (and the relevance of properties in those contexts) can be better captured in a system of relevant predication.

Here I offer the beginnings of an alternative approach to Shamko's which takes into account the contextual nature of the relevance of properties, but which captures the relations between contexts. Some relations between contexts include expansions, contractions, and various ways of combining contexts. I will focus on the particular contexts determined by arguments (premises, conclusions, and the words from which they are made), because (i) I can use the relevant logician's tools for considering premise groupings/combinations, and (ii) I can limit the influence of context on relevance to focus on a simpler case from which to generalize. Moreover, for the sake of simplicity, I will use only a *binary relation of relevance* between properties and contexts, leaving the third place of the relation (i.e., objects) for future generalizations.

The proposed system treats a context as a combination of a *situation* (from situational semantics for relevant logics), a total pre-order of a set of properties, and set of relevant properties (closed under the pre-order). The primary aims of this paper are to (i) give the foundation of this formal account, and (ii) examine potential conditions that combinations/expansions/contractions of contexts should satisfy.

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The Algebra of Logical Atomism

Peter Fritz (based on joint work with Andrew Bacon)

At the core of logical atomism, along the lines of Wittgenstein and Russell, is the following idea about propositions:

(LA) Every proposition is a truth-functional combination of elementary propositions.

In the literature on logical atomism, it is often assumed that propositions form a Boolean algebra under operations like negation, conjunction, and disjunction. Interestingly, different authors make different assumptions about the structure of the algebra of propositions according to logical atomism: some authors assume that it is a free Boolean algebra, while others assume that it is isomorphic to a double powerset algebra.

We propose two natural ways of turning LA into an algebraic condition on Boolean algebras, which we call *independent generation* and *internal freeness*. Both of them are easily shown to be equivalent to the standard condition of being a free Boolean algebra. This result may seem to vindicate the first set of authors mentioned above. However, it essentially depends on the fact that the operations of Boolean algebras are finitary. Another component of logical atomism is the idea that quantified propositions are conjunctions or disjunctions of the relevant instances. Such conjunctions and disjunctions may be infinite. This motivates considering complete Boolean algebras, in which infinite conjunctions and disjunctions can be understood as greatest lower bounds and least upper bounds, and adapting independent generation and internal freeness to the setting of complete Boolean algebras.

We show that in the complete setting, independent generation and internal freeness come apart from the standard notion of freeness: It was shown by Gaifman and Hales that there are no infinite free complete Boolean algebras. In contrast, in the complete setting, independent generation and internal freeness are satisfied by every double powerset algebra. This may seem to vindicate the second set of authors mentioned above. However, we show that independent generation is also satisfied by some algebras which are not isomorphic to a double powerset algebra, and provide a full characterization of such algebras. The question whether there are any further algebras satisfying internal freeness is left as an open problem.

Omitting Types Theorem in hybrid-dynamic first-order logic with rigid symbols

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In this talk, I will present an Omitting Types Theorem (OTT) for an arbitrary fragment of hybrid-dynamic first-order logic with rigid symbols (i.e. symbols with fixed interpretations across worlds) closed under *negation* and *retrieve*. The logical framework can be regarded as a parameter and it is instantiated by some well-known hybrid and/or dynamic logics from the literature. The proof of the OTT relies on a *forcing technique* for which we define and study a *forcing property* based on local satisfiability. For uncountable signatures, the result requires compactness, while for countable signatures, compactness is not necessary. We apply the OTT to obtain upwards and downwards Löwenheim-Skolem theorems for our logic, as well as a completeness theorem for its *constructor-based* variant.

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Lax Logic and its Admissible Rules

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Lax Logic is a fascinating intuitionistic modal logic. It has a non-standard modality that combines some properties of a $\Box$ and some properties of a $\Diamond$. This modality is called the lax modality, denoted by $\bigcirc$, and satisfies the following axioms:

$$
\begin{array}{ll}
(C) & A \to \bigcirc A \\
(M) & \bigcirc \bigcirc A \to \bigcirc A \\
(S) & \bigcirc A \wedge \bigcirc B \to \bigcirc (A \wedge B)
\end{array}
$$

Despite this non-standard modality, it can naturally be described by Kripke semantics and sequent calculi. It also has many interesting applications in algebra, topos theory and hardware verification [1]. In this talk, we will see that propositional Lax Logic (PLL) also behaves very natural in terms of its admissible rules.

The admissible rules of a logic are those rules that can be added without changing the set of theorems of the logic. The study of admissible rules is a nice and difficult area in the field of proof theory. They are interesting to study, because they form an invariant for the logic regardless of the chosen axiomatisation. Thereby they give insight in the structure of all possible inferences in a logic. An example of an admissible rule in intuitionistic propositional logic (IPC) is the disjunction property: from $A \vee B$ we conclude A or B. For structurally complete logics, such as classical propositional logic, the story of the admissible rules is very short: all admissible rules are derivable. However, for IPC, intermediate logics, and modal logics the story becomes much more complex and exciting.

The research of admissible rules was inspired by one of Friedman's problems in [2], asking whether admissibility in IPC is decidable. The question was positively answered by Rybakov, who published a series of papers showing that admissibility in IPC, many intermediate logics, and many modal logics above K4 is decidable, see [7]. Later, full descriptions of the admissible rules are established in terms of a basis for many of these logics. The so-called Visser rules form a basis for the admissible rules for IPC, independently shown by Rozière [6] and Iemhoff [3]. And Jeřábek [5] constructed modal Visser rules forming bases for the admissible rules of classical modal logics.

We combine results and methods for IPC and classical modal logic to obtain a full description of the admissible rules of PLL. We describe the admissible rules in terms of a proof system using the same strategy from Iemhoff and Metcalfe [4]. They provide Gentzen-style proof systems for admissibility for IPC and several modal logics above K4. We combine these ideas in order to define a similar proof system for the admissibility of PLL. In contrast to well-known proof systems for logics that reason about formulas or sequents, these admissibility proof systems reason about rules. In other words, they contain rules about rules.

The constructed admissibility proof system for PLL contain Visser-like rules that are closely related to the Visser rules for IPC and the modal Visser rules. We use semantic reasoning to show our results where the structure of the relational semantics will be of great importance. The method that we use does not only apply to PLL, but we have similar results for five other intuitionistic modal logics containing the completeness axiom, $A \to \Box A$. All admit Visser rules that are a fusion of the Visser rules for IPC and classical modal logic.

A big advantage of the admissibility proof systems is that decidability of admissibility immediately follows from the decidability of the logic. Thereby we positively answer Friedman's question, but now for these intuitionistic modal logics.

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Going Stoic With Questions: Connectives and Questions

Roderic A. Girle

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Abstract:

Aristotle began at the complex end of logic, predicate logic. Aristotle's writings have no clear and explicit recognition of the logic of propositional connectives. Geach claimed that Aristotle's greatest failure in logic was his failure to give the propositional connectives their proper place. That had to wait for Stoic logic, the logic of *whole* propositions and propositional connectives. This paper contains a discussion of the logic of *whole* questions, to *go Stoic* as it were, and to look at connectives and whole questions.

There are two approaches to the logic of questions. There is the reductionist approach and the non-reductionist approach. This paper is non-reductionist. Questions are basic. We assume that questions cannot be accounted for in terms of propositions. Query Logic uses possible worlds to reduce questions to qualified propositions. We assume the contrary, that propositions and questions are distinct but equal. But they interact. The approach here is "inclusive" rather than reductive.

Questions are divided into *open* questions (*what/where/when/who/why/how* questions), and *closed* questions (*true-false* questions). Questions are taken to be logically incomplete. Altough Hiz introduced question-answer *sentences* (Hiz 1978), we take up the anthropological terminology of *Question Answer Pair*, QAP for short (Goody 1978). For example: ⟨*How is it outside*?, *It's cold*.⟩ The logic of QAPs is first order.

Two logical issues arise for connectives. One is the operation of connectives on QAPs (*external*), and the operation of connectives on the elements within the QAP (*internal*). The former is simple, as we shall see. The internal operation of connectives is more complex.

The internal operation of connectives might not be exactly the same as the four propositional connectives for negation, conjunction, disjunction, and implication as first explicitly dealt with by the Stoics. But they seem to occur in much the same way, as in:

Is it not raining?
Do you want wine or orange juice?
Did he pick up the tools and put them away?
If Susan did not knock on the door then who did?

The relationship between a question and its answer is *dynamic* in everyday life. Questions and answers occur in an interactive multi-agent context, the dialogue context. So we address dialogical context and introduces a new category of logical operators called *move operators*.

Fragile Knowledge

Simon Goldstein

Sosa 2009 introduces the phenomenon of `dubious assertion', infelicitous utterances concerning higher order ignorance. In dubious assertions, an agent asserts a claim while raising doubts about her higher order epistemic standing with respect to p.

(1) #p, but I don't know whether I know that p.

This paper explains the infelicity of dubious assertion by defending a new principle about knowledge, Fragility. We say that knowledge is fragile, so that it cannot withstand the knowledge of higher order ignorance:

(2) **Fragility**. If S knows that S doesn't know that S knows p, then S doesn't know p.

Fragility implies that (1) is unknowable, and hence infelicitous given a knowledge norm on assertion.

One key point of the paper is that Fragility is weaker than the KK principle:

(3) **KK**. If S knows that p, then S knows that S knows that p.

Defenders of KK have recently used dubious assertions to motivate the validity of KK. This paper suggests that such an argument is inconclusive. Dubious assertion can be explained without resorting to KK, as long as we accept Fragility.

Even though Fragility is logically weaker than KK, this is only interesting if there are plausible models of knowledge that validate Fragility without validating KK. Building on Williamson 2013, we introduce models of Fragility that invalidate KK.

A Modal Semantics in Arthur N. Prior's 'Symbolism and Analogy'

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Abstract

This paper explores a modal semantics that could be found in Arthur N. Prior's publicly delivered 1957 lecture entitled, 'Symbolism and Analogy'. Prior's semantics employs a translational scheme, where the 'meanings' of certain modal axioms are translated as sentences of an easily understood (meta) language. I show that Prior's 'translational semantics' allows us to distinguish between different modal systems (e.g., D, M, $T4$, $S4$, and $S5$) without the use of (accessibility relations among) possible worlds.

1 Introduction

In this paper, I explore a modal semantics found in Arthur N. Prior's 'Symbolism and Analogy', the second of his three public lectures given for the New Zealand Public Broadcasting in 1957. Prior's semantical approach, I argue, is not the standard possible worlds semantics, but is a translational scheme, where the meanings of the sentences of a target language (viz., the axioms of some modal system) are translated as sentences of an easily understood (meta) language (i.e., a language with well-established (quasi-) logical principles). I show that Prior's 'translational semantics' allows us to distinguish between different modal systems (e.g., D, M, T, $S4$, and $S5$) without the use of (accessibility relations among) possible worlds. It only needs a semantic translation machinery for different modal notions.

2 The Target Language

To demonstrate, let us begin first with the language of our target modal language. Let $p, q, r, ...$ stand for atomic sentences. Let C, N be the primitive logical connectives for conditional and negation, respectively. (The other logical connectives are defined by them.) And let L, M stand for the modal notions of 'necessity' and 'possibility', respectively. Compound sentences are recursively defined following standard Polish notation. Thus, $CpNq$ means 'If p, then not-q'; LMp means 'It is necessarily possible that p'; and, $CpMp$ means 'If p, then it is possible that p'.

3 Semantics for Systems M and T

Let us now demonstrate how Prior's semantics works. Suppose that L stands for 'it is necessarily true that' and M for 'it is possibly true that', then $CpMp$, known as the M-axiom, would express the basic (metaphysical) principle that whatever is so, *can* be so. In contrast, $CMpp$ falsely expresses that whatever can be so, is so.

A corollary to the M-axiom is the so-called T-axiom, $CLpp$. Under the interpretation, this axiom expresses the true principle that whatever is necessarily true, is true. $CpLp$ falsely expresses that whatever is so, is necessarily true.

Given this interpretation, Mp and Lp are also interdefinable via the two sets of conditionals: $CLpNMNp$, 'If it is necessarily true that p, then it's not possibly true that p is not true', and its converse $CNMNpLp$, 'If it's not possibly true that p is not true, then it is necessarily true that p'; and, $CMpNLNp$, 'If it is possibly true that p, then it's not necessarily true that p is not true', and $CNLNpMp$, 'If it's not necessarily true that p is not true, then it is possibly true that p'.

4 Semantics for Systems $S4$ and $S5$

Suppose that Mp instead means 'p either is or will be true' and Lp means 'p is and always will be true'. Not only are $CpMp$ and $CLpp$ still basic truths under this interpretation, but so will be the S4-axiom: $CLpLLp$, 'If p is and always will be true, then it is and always be true that p is and always will be true'. $CLpLLp$, however, will not be a basic truth if L and M simply mean 'it is necessarily true that' and 'it is possibly true that', respectively. For 'water is H2O' might be necessarily true, but it is not necessarily true that water is H2O is necessarily true.

Given the temporal interpretation of L and M, the S5-axiom, $CMpLMp$ is false. That there are flying pigs either is or will be true, but it is surely not the case that it is true and always will be true that there are flying pigs. Perhaps, the S5-axiom is true under the interpretation where L and M quantify over *all* or *some* possible states of affairs.

5 Semantics for System D

Suppose finally that L and M mean 'it is obligatory that' and 'it is permissible that', respectively, then, the D-axiom, $CLpMp$, is validated under this interpretation. For the principle that whatever is obligatory, is permissible is true in such an interpretation. Moreover, the other axioms ($M, T, S4, S5$) are also true under this interpretation.

6 Upshot

In the foregoing discussion, I presented what I think of as Prior's 'translational semantics' for modal notions. Such a semantics accounts for the meanings of the basic axioms of different modal systems. Given additional inferential rules, these different modal translations imply different modal logics, which may include or exclude each other. As could be seen, all the deliverables expected of a modal semantics could be achieved in Prior's semantics with no recourse to possible worlds.

ALGEBRA-VALUED MODELS OF LP-SET THEORY

SANTIAGO JOCKWICH MARTINEZ

Abstract

This talk contributes to the study of models for non-classical set theories. We explore the possibility of constructing algebra-valued models of set theory based on Priest's Logic of Paradox (LP). We first show that we can build "straightforward"(following [Bell, 2005] and [Löwe and Tarafder, 2015]) an algebra-valued model based on LP, though, we obtain a model where Leibniz's law of indiscernibility of identicals fails and we loose several basic set-theoretic properties. On top of this, we observe that we end up with a strange ontology since we have an *excessive* duplication of sets in our model. In particular, most sets collapse to the empty set. Because of these reasons we argue that this approach is unfeasible. Then, secondly, we show that we can overcome this difficulties by modifying the interpretation map for $\in$ and $=$ in our algebra-valued model. Given the modified interpretation map we build a non-classical model of ZF which has as internal logic Priest's Logic of Paradox and validates Leibniz's law of indiscernibility of identicals. Even though it was already shown in [Priest, 2017] that set theories based on LP are compatible with ZFC, the validity of Leibniz's law of indiscernibility of identicals opens up the possibility of constructing equivalence classes and thus producing a quotient model based on LP with a rich ontology preserving the entire set-theoretical universe.

Moreover, we compare the resulting algebra-valued model to other existing models of paraconsistent set theory and point out that our construction seems to have less drawbacks than other set-theoretic models based on LP (as presented in [Priest, 2017]). Finally, we end our talk by investigating the feasibility of adding classes to our algebra-valued model in order to obtain a model of both ZF and naïve set theory.

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Carnap is not a pluralist

Teresa Kouri Kissel

Rudolf Carnap is often thought to be a prototype of a logical pluralist. That is, Carnap is thought to hold that more than one logic is correct. I will show in this paper that he cannot be a logical pluralist. I will also show that he cannot be a logical monist or nihilist.

For our purposes, logical pluralism is the view that more than one logic is correct, logical monism is the view that exactly one logic is correct, and logical nihilism is the view that no logics are correct. Deciding between these three views requires that we answer the question ``how many logics are correct?''. What I will show is that Carnap must hold that either this question is illegitimate, and cannot be answered, or the answer is not ``many'', as the pluralist would expect.

We can effectively ask the question three different ways, and none of these versions of the question can be answered in a way that clearly implies pluralism (or monism or nihilism, for that matter).

If we ask "how many logics are there?" externally, the problem is clear. Carnap, then, must hold that the question is a pseudo-question. Since it is a pseudo-question, it cannot be meaningfully answered, and the whole pluralism/monism/nihilism debate is misguided.

If we ask it internally, then we are effectively asking "how many logics are there in this linguistic framework?". Because of how Carnap establishes his linguistic frameworks, the answer here will always be one. And so, we wind up with a very weird kind of monism – definitely not one which typical logical monists would accept as true.

There is a way around this issue (sort of) for the pluralist. She could adopt meta-frameworks. A meta-framework is one which we can use to compare and contrast two object level frameworks. If we had two frameworks, say one classical and the other intuitionist, then we could embed those in a meta-framework to examine the similarities and differences between the two. If we opted for a meta-framework which distinguished between classical and intuitionistic logic (say, one which identifies logics by their theorems), then this meta-framework would have the additional ability to vindicate a kind of pluralism here. If we ask "how many logics are there is this meta-framework?", the answer would be two. So, asking the question internally can sometime result in a weird monism and sometime in a weird pluralism. But never in logical monism or pluralism full stop.

The pluralists best bet is to ask this question as a pragmatic external question. This means asking something like "which of these meta-frameworks should I use to answer this problem?". If we require a meta-framework which has more than one object-level framework which have distinct logics, then we have pluralism. But, if we require a meta-framework which has object-level frameworks which contain identical logics, we have monism. This makes, again, for a very weird logical monism or pluralism.

Effectively, this means that Carnap must hold that the whole pluralism/monism/nihilism debate is misguided.

Depth Substitutions, Hyperformalism, and Contraction

0: Overview. In his 1984 paper on Depth Relevance, Ross Brady pointed out that in some weak relevant logics, there appear to be different levels of entailment that roughly corresponded to how '$\to$-nested' a formula is. This talk has two goals. First, I aim to get clearer on the exact sense in which logics might exhibit different 'levels of entailment'. Second, I will explore the relationship between various forms of contraction and failures to differentiate levels of entailment.

1: Language and Logics. We work in a standard propositional language $\mathcal{L}$ with the set of atomic formulas $\mathsf{At} = \{p_1, p_2, \dots\}$, and with connectives $\wedge$, $\vee$, $\neg$, and $\to$. We write Form for the class of formulas in this language. We will consider the following axioms and rules:

A1. $A \to A$	A8. $(A \to \neg B) \to (B \to \neg A)$
A2. $(A \wedge B) \to A/B$	R1. $A, A \to B \Rightarrow B$
A3. $A/B \to (A \vee B)$	R2. $A, B \Rightarrow A \wedge B$
A4. $A \wedge (B \vee C) \to ((A \wedge B) \vee (A \vee C))$	R3. $A \to B, C \to D \Rightarrow (B \to C) \to (A \to D)$
A5. $\neg\neg A \to A$	R4. $A \to \neg B \Rightarrow B \to \neg A$
A6. $((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$	R5. $A \to B, A \to C \Rightarrow A \to (B \wedge C)$
A7. $((A \to C) \wedge (B \to C)) \to ((A \vee B) \to C)$	R6. $A \to C, B \to C \Rightarrow (A \vee B) \to C$

BB is the logic generated by A1-A5 and R1-R6. B extends BB by replacing R5 and R6 with A6 and A7. DW extends B by replacing R4 with A8.

Finally, since they will play a role below, we single out the strong contraction axiom W: $(A \to (A \to B)) \to (A \to B)$ and the weak contraction axiom WI: $(A \wedge (A \to B)) \to B$

2: Definitions and Initial Results. We define the *depth* of an occurrence of a subformula as follows:

- C occurs at depth 0 in its unique occurrence in the formula C.
- If C occurs at depth n in A, then the corresponding occurrence of C in $\neg A$, in $A \wedge B$, in $B \wedge A$, in $A \vee B$, and in $B \vee A$ is a depth n occurrence of C as well.
- If C occurs at depth n in A, then the corresponding occurrence of C in $A \to B$ and in $B \to A$ is a depth $n+1$ occurrence of C.

A *depth substitution* is a function $f : \mathbb{N} \times \mathsf{At} \to \mathsf{At}$. A *substitution* is a function $f : \mathsf{At} \to \mathsf{Form}$. Given a depth substitution f and a formula ϕ, define $\phi[f]$ to be the formula that replaces each depth n occurrence of p in ϕ with an occurrence of $f(n, p)$. Similarly, given a substitution s, define $\phi[s]$ to be the formula that replaces each occurrence of p in ϕ with $s(p)$. We call $\phi[s]$ a substitution instance of ϕ.

A set of formulas S is said to be closed under (depth) substitutions when it satisfies the following: if $\phi \in S$ and f is a (depth) substitution, then $\phi[f] \in S$ as well. Since we typically require that *formal* logics be closed under *substitutions*, we will call logics that are closed under both depth substitutions and regular substitutions *hyperformal* logics. I think it's clear that hyperformal logics respect levels of entailment in the strongest possible sense—indeed, in a hyperformal logic, no atom at any level is treated like it is 'the same as' any atom at any other level.

Theorem 1. *If L is generated from some subset of A1-A8 and R1-R6 and f is a depth substitution, then if $\vdash_L \phi$ and f is a depth substitution then $\vdash_L \phi[f]$.*

Proof. By induction on the length of the proof witnessing $\vdash_L \phi$. □

Corollary 1. *BB, B, and DW are hypformal.*

3: Non-hyperformal Logics and Contraction. Non-hyperformalism is connected to failures of contraction in striking ways. In fact, as the following results show, contraction in the presence of essentially any amount of logic immediately rules out hyperformalism:

Theorem 2. *Let L be a nontrivial hyperformal logic that contains every instance of A1 and is closed R1. Then L does not contain every instance of W.*

Proof. Let L be a hyperformal logic that contains every instance of A1 and every instance of W and suppose L is closed under R1. Let $\phi = (p_1 \to (p_1 \to p_2)) \to (p_1 \to p_2)$. By our assumptions, $\vdash_L \phi$. Consider the following depth substitution, f:

$$\begin{aligned} \langle p_1, 3\rangle &\mapsto p_2 \\ \langle p_2, 2\rangle &\mapsto p_3 \\ \langle p_2, 3\rangle &\mapsto p_4 \\ \langle q, n\rangle &\mapsto q \text{ for all other } q \text{ and } n \end{aligned}$$

Note that since L is closed under depth substitutions, $\vdash_L \phi[f] = (p_1 \to (p_2 \to p_4)) \to (p_1 \to p_3)$. Now let s be the following substitution:

$$\begin{aligned} p_1 &\mapsto p_4 \to p_4 \\ p_2 &\mapsto p_4 \\ p_n &\mapsto p_n \text{ for all other } n \end{aligned}$$

As before, since L is closed under substitutions it follows that $\vdash_L (\phi[f])[s] = ((p_4 \to p_4) \to (p_4 \to p_4)) \to ((p_4 \to p_4) \to p_3)$. But of course $(p_4 \to p_4) \to (p_4 \to p_4)$ and $p_4 \to p_4$ are both instances of A1 and thus theorems of L. So by two applications of R1, we see that $\vdash_L p_3$.

Now letting ψ be any formula at all and t be any substitution that maps p_3 to ψ. Since L is closed under substitutions, $\vdash_L p_3[t] = \psi$. So L is trivial. Thus, if L satisfies the hypotheses of the theorem, it must not contain every instance of W. □

Theorem 3. *Let L be a nontrivial hyperformal logic that contains every instance of A1 and is closed R1, and R2. Then L does not contain every instance of WI.*

Proof. Similar to the previous proof. □

We can summarize these results succinctly as follows: no nontrivial hyperformal logic can simultaneously (a) contain enough machinery for $\to$ and $\wedge$ to play their usual roles while (b) accepting any version of contraction stated using these.

4: Conclusion. The philosophical upshot of all this is as follows. Work on weak relevance logics has led us to expect there to be a viable notion of 'levels of entailment' and, further, that there might be logics that respect the difference between these levels. Respecting this difference seems to require that the logics in question be closed under depth substitutions. Of course, closure under *ordinary* substitutions has long been thought to be a requirement of formality itself, and thus a necessary condition for counting as a logic at all. Taking these on board, Theorems 2 and 3 above make clear that no remotely plausible logic can (a) be formal, (b) respect levels of entailment, and (c) admit contraction.

Why metainferences matter

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In this talk, I will present new arguments that shed light on the importance of (i) metainferences of every level, and (ii) metainferential standards of every level, when (semantically) characterizing a logic. This might probably imply that a logic cannot be defined just by a satisfaction standard for inferences. Nevertheless, I my focus will be to prove that a logician cannot be agnostic about metainfereces, metametainferences, etc. The arguments I will introduce show why a thesis that Dave Ripley defends in [1] and [2] is false. This is how he presents it.

> Note that a meta^{0}counterexample relation X [i.e., a counterexample relation for inferences, which is (in most contexts) equivalent to a satisfaction relation for inferences], on its own, says nothing at all about validity of meta^{n}inferences for $0 < n$. Despite this, there is a tendency to move quickly from X to [X] [i.e., a full counterexample relation for every metainferential level], at least for some purposes... For example, [3] (p. 360, notation changed) says "[A]bsent any other reasons for suspicion one should probably take [X] to be what someone has in mind if they only specify X." I don't think this tendency is warranted. Most of the time, when someone has specified a meta^{0}counterexample relation (which is to say an ordinary counterexample relation), they do not have the world of all higher minferences [i.e., metainferences of any level], full counterexample relations, etc, in mind at all. They are often focused on validity for meta^{0}inferences (which is to say inferences). ([1], page 12.)

Though I do think that, in a sense, people do have in mind [X] when they say X, I will not argue for that. I just want to defend that they should have something like that in mind. Specifically, I will try to show why the following position should be revised:

> As I've pointed out, an advocate of **ST** as a useful meta^{0}counterexample relation has thereby taken on no commitments at all regarding meta^{n}counterexample relations for $1 \leq n$. ([1], page 16).)

Or, as Ripley puts in somewhere else:

> ... if someone specifies just a meta^{n}consequence relation, they have not thereby settled on any particular meta^{n+1} consequence relation. ([2]).)

If Ripley's statements are true, then two different logicians may count as advocates of the same inferential logic (or any metainferential logic of level n), despite adopting quite different criteria regarding what counts as a valid metainference (or a valid metainference of level $n+1$). If Ripley is right, then not only can a supporter of a logic like **ST** accept or reject the metainference corresponding to (some version of) the Cut rule, but also she can admit a metainferential counterexample relation that correspond to a trivial or an empty metainferential consequence relation. And, moreover, this might have repercussions on the inferential level, as an empty metainferential logic invalidates any metainference with an empty set of premises and a valid **ST**-inference as a conclusion. Thus, the only available option is to admit that inferences and metainference with an empty set of premises and that inference as its only conclusion, are not only different, but also non-equivalent things. Moreover, something similar happens if we chose a trivial metainferential counterexample relation while adopting **ST** at the inferential level. There will be invalid **ST**-inferences that turns out valid in its metainferential form, forcing this logician to chose between one of the options that we have specified before.

This is a particular strong result, and we will show that it is even stronger than what might initially seem, in two senses: (1) it does not depend on the notion of metainferential validity being favoured—e.g., whether one thinks that the *local* way to understand it is better than the *global*, or the other way around; (2) it does not depend on the special features of the (mixed) inferential/metainferential relations, as this result can be replied for any pair of (mixed) metainferential relations of level $n/n+1$.

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Prior's Dilemma Redux?
Charles Pigden

Consider these inferences:
Tea-Drinker *1*: **A**. Tea–drinking is common in England. Therefore: **B**. Either tea–drinking is common in England or all New Zealanders ought to be shot.
Tea-Drinker 2: **A***. Tea–drinking is NOT common in England. **B**. Either tea–drinking is common in England or all New Zealanders ought to be shot. **C**. All New Zealanders ought to be shot.

According to Prior (1960), 'The Autonomy of Ethics', sentence **B** is either moral or not moral. If it *is* moral then **Tea-Drinker 1** is a valid Is/Ought inference. If it is not then **Tea-Drinker 2** is a valid Is/Ought inference. Either way, No-Ought-From-Is is false.

In my (1989) paper 'Logic and the Autonomy of Ethics' I developed a solution. **B** in **Tea-Drinker 1** is not *substantively* moral since its moral component suffers from *inference-relative vacuity*. So **Tea-Drinker 1** is not an inference from non-moral premises to a *substantively* moral conclusion. Indeed no such inferences are possible.

But Prior might reply as follows: If **B** is *not* substantively moral when it appears as a *conclusion* in **Tea-Drinker 1** doesn't that mean it is not substantively moral when it appears as a *premise* in **Tea-Drinker 2**? In which case **Tea-Drinker 2** is an inference from substantively *non*-moral premises to a substantively *moral* conclusion!

I suggest a response to this (imagined) response, extending the notion of inference relative vacuity from the conclusions to the premises.

Four-valued logics of truth, non-falsity, and exact truth

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The four-valued semantics of Belnap–Dunn logic [1, 2, 3], consisting of the truth values True, False, Neither, and Both, gives rise to several non-classical logics depending on which feature of propositions we wish to preserve: truth, non-falsity, or exact truth (truth and non-falsity). Preserving truth yields the Belnap–Dunn logic (FDE), as does preserving non-falsity. Preserving exact truth (truth and non-falsity) yields the Exactly True Logic introduced by Pietz and Rivieccio [4, 6]. If we interpret equality of truth values in this four-valued semantics as material equivalence of propositions, we can moreover view the equational consequence relation of De Morgan algebras as a logic of material equivalence. The talk is based on the paper [5].

Now there is no reason, other than tradition, to limit ourselves to logical systems which only allow us to express one of these features of propositions. In other words, we may consider the unary predicates True, ExTrue ("exactly true"), and $\mathsf{NonFalse}$ interpreted in the four-valued semantics of Belnap and Dunn by the sets $\{\mathsf{t}, \mathsf{b}\}$, $\{\mathsf{t}\}$, and $\{\mathsf{t}, \mathsf{n}\}$ respectively. (Here t, b, n represent the truth values True, Both, Neither.) We may then consider inferences involving more than one of these predicates. For example, the following inferences are valid in the four-valued semantics of Belnap and Dunn:

$$\mathsf{ExTrue}(x) \;\&\; \mathsf{True}(-x \vee y) \vdash \mathsf{True}(y),$$
$$\mathsf{True}(x) \;\&\; \mathsf{NonFalse}(-x \vee y) \vdash \mathsf{NonFalse}(y).$$

Similarly, if the binary predicate $\approx$ is interpreted by equality of truth values, then the following inference is valid:

$$\mathsf{True}(x) \;\&\; \mathsf{True}(y) \vdash -x \vee y \approx y.$$

We show how to axiomatize such combined systems for any combination of the four predicates.

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Natural Deduction with Alternatives

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In this talk, I will introduce natural deduction with *alternatives*, explaining how this framework provides a simple, well-behaved, single conclusion natural deduction system for a range of logical systems, including classical logic, (classical) linear logic, relevant logic and affine logic, in addition to the familar intuitionistic restrictions of these systems. Each of these proof systems have identical *connective* rules. As we expect in substructural logics, different logical systems are given by varying the structural rules in play. The distinctly classical behaviour of these systems is given by the presence of *alternatives* (formulas in consequent, or *positive* position, other than the conclusion of the proof) in addition to *assumptions* (formulas in antecedent, or *negative* position). Unlike multiple conclusion proof systems, the proof system is single conclusion, since one formula in positive position is singled out as the *conclusion*. The context in which that formula is proved consists, in general, of formulas ruled in (assumptions) and formulas ruled out (alternatives).

In *sequent systems*, and in some natural deduction systems that use labels, the structural rules of contraction and weakening govern an explicitly represented *structure*, such as a set or multiset or sequence of formulas occurring in each sequent. In this natural deduction framework, the structural rules have their force at the point of *discharge*, or more generally, at any point at which it is important to determine whether two occurrences of the same formula (in positive position, or in negative position) are *the same assumption* or *the same alternative*. There is no explicit representation of any *structure* of assumptions or alternatives, other than the structure of the proof itself.

Along the way, this presentation will touch on (1) the connection between normalisation of a natural deduction proof and cut elimination in a corresponding sequent calculus; (2) the separation between the operational rules governing the connectives and the "antecedently given context of deducibility", to borrow a phrase from Nuel Belnap's essay, "Tonk, Plonk and Plink" (1962); (3) the sense in which the operational rules for a connective might be understood as providing a definition of that connective; and (4) the use of the identity or difference of variables in type systems such as the typed λ calculus in keeping track of the sameness or difference of *assumptions* as opposed to the sameness or difference of the *things assumed*.

Nuel Belnap, 1962. "Tonk, Plonk and Plink." *Analysis* 22:30–34.

Hume's Law and *Ought implies Can*

Gillian Russell

2021 AAL

Abstract

A Barrier to Entailment says that you can't get certain kinds of conclusion from certain kinds of premises, for example: you can't get normative conclusions from descriptive premises (the *is/ought* barrier), or you can't get conclusions about the future from premises about the past (the past/future barrier) or universal conclusions from particular premises (the particular/universal barrier.)

The literature on how to prove the *is/ought* barrier is often a response to the counterexamples proposed by A.N. Prior in "The Autonomy of Ethics" (1960) but more recent criticisms (due to e.g. Peter Vranas and Gerhard Schurz) of proofs that avoid Prior's worries in simple deontic logics (due to e.g. Restall and Russell) have emphasised the importance and difficulty of extending such proofs to complex logics which allow for the expression of counterexamples involving e.g. the contrapositive of *ought implies can* ($\neg\Diamond\phi \vDash \neg O\phi$) and principles such as $\Box\phi \vDash O\phi$.

This paper proposes a new way to handle such counterexamples to the *is/ought* barrier and shows how to generalise it to analogous counterexamples to other barrier theses (e.g. $\Box\phi \vDash F\phi$.)

Relevant Logics of Epistemic Update

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Dynamic epistemic logic is a family of extensions of classical epistemic logic formalizing reasoning about change of information accepted by, or available to, cognitive agents [5, 10]. One of the simplest and most well-known dynamic epistemic logics is Public Announcement Logic, PAL, formalizing reasoning about *truthful public announcements*, events where a group of agents is confronted by a true piece of information, each agent in the group accepts the information, each agent knows that each agent accepts the information and so on [6, 10].

PAL suffers from the usual shortcomings of classical epistemic logic; for instance, it takes epistemic states of agents to support all logical validities and to be trivial if inconsistent. One way to avoid these shortcomings is to model epistemic states of agents as sets of *abstract situations* rather than possible worlds. Situation-based semantics give rise to non-classical epistemic logics, and the literature contains studies of PAL in non-classical settings [1, 2, 4, 8, 9].

However, with the exception of the abstract approach of [7], studies of PAL in the context of *relevant logic* do not exist. In this paper we present a formalization of truthful public announcements in the context of relevant modal logic. We note that the standard "model-chopping" formalization of public announcements cannot be applied in the context of relevant logic; instead, we base our semantics on the "arrow-deletion" approach also known as *epistemic update* [3, 11]. Our modelling of epistemic update is based on a more basic notion of *situation update*, the event of merging a situation with a piece of information. In our setting, epistemic update with information expressed by a formula A amounts to taking each situation containing the epistemic state of an agent and merging the situation with A. This general setting can be made more precise in a number of ways, an we show how it can be used to model public vs. private, truthful vs. possibly false, and monotonic vs. non-monotonic epistemic updates. We give an axiomatization for each such special case, using so-called *reduction axioms*, a technique well-known from standard PAL (without common knowledge). Variants of our axiomatization technique work for epistemic update logics based on R, but also other mainstream relevant logics such as T and E.

We will also argue that considerations of epistemic update in the context of situations do not only lead to dynamic epistemic logics where agents are modelled (more) realistically, they also allow us to model updates in a way that relies on the richer structure of situations, including their interactions. For example, the existence of causal interactions between situations plays an important part in the theory, as does the fact that some situations can be proper parts of others. In addition to investigating the properties of updates on situations, we can also investigate the properties of updates *in* situations, where the situations in which the update occurs (as well as those further situations causally or informationally linked to them) themselves affect what information the update carries, and where the action which embodies the update can have affects throughout the collection of situations. One might call these *situated epistemic updates*, and they seem promising as an extension of our work which takes into account the rich *environmental* features of situation semantics. In this talk, we'll most gesture in the direction of an account of situated updates, but will set out some necessary preliminaries for investigating them further.

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A Dilemma for Proof-Theoretic Semantics

Will Stafford (Czech Academy of Sciences)

Any argument that a theory is analytic needs two parts. The first is an explanation of what meanings are in general and what the meanings of the terms that appear in the theory are. The second is an explanation of the tools that can be used to extract what *follows from* the meanings. That second component becomes especially important when considering whether logics? Plural are analytic, as one must avoid the logic simply being included in the second component to avoid begging the question. Proof-theoretic semantics offers, not one, but two arguments for the analyticity of intuitionistic logic. The first can be called the harmony account of definitions and the second, proof-theoretic validity. It will be argued that while the harmony account gives an adequate explanation of what *follows from* meanings, it runs into difficulty in explaining what meanings are in general, whereas proof-theoretic validity avoids the difficulty with explanations of meaning, only to have an inadequate explanation of what *follows from* them. This it is proposed is a dilemma for proof-theoretic semantics; when "following from" is satisfactorily explicated then meanings become problematic, while when meanings are unproblematic the notion of *following from* has no adequate explanation.

The initial idea of proof-theoretic semantics is appealingly simple. The meaning of terms is given by sets of inference rules, and a statement *follows from* the meanings, just in case, one can construct a proof from the rules. This picture is firmly embedded in the meaning-as-use paradigm. A proof shows a statement *follows from* the meanings because it *uses* the meanings. The resources needed to use proof rules are: concatenation, pattern matching, and assumption tracking. As these are prelogical, "following from" does not beg the question. If this simple picture worked, it would offer a plausible story of how a logic is analytic when meanings are given by proof rules.

Of course, this simple picture does not work. Not any set of inference rules can be used to define a term, as is shown by the connective TONK. TONK has the introduction rules for 'or' and the elimination rules for 'and'. As such, TONK is not just inconsistent in settings with at least one theorem, it is explosive. This makes it unacceptable even for those who accept true contradiction, as it trivialises the entire system. If sets of proof rules can be meaning giving, there must be some restriction on them that excludes TONK.

This restriction cannot just be consistency. Why? Because discovering if a set of rules is consistent is a mathematically nontrivial task. Not only must a logic be presumed to check consistency, but a particular reasoner may also find themselves unable to discern if a particular set of rules are consistent. This gap between a definition being admissible and it being known that it is admissible is undesirable.

What is needed is a condition that is easily checked and from which consistency follows. Dummett wanted harmony and stability to be a local constraint, which is a constraint that does not depend on the other rules that are used. He also wanted the local constraint to ensure consistency and conservativity. But well-motivated local conditions that ensure consistency are not readily available. And there may be no local notion that both removes undesirable sets of rules and is not ad hoc (Steinberg). This is the first horn of the dilemma. The point is not novel.

There is another approach within proof-theoretic semantics that does not need an account of harmony. On it, only introduction rules are meaning giving, and the elimination rules, as well as the logic, *follow from* the introduction rules. This approach is called proof-theoretic validity.

To get this extra information out of the meanings, proof-theoretic validity needs a more complex account of what *follows from* the proof rules that are meaning giving. It does this by defining validity over arbitrary potential proofs, rather than the smaller collection of proofs built up from the meaning giving rules (as the harmony account does). An arbitrary potential proof is considered a valid proof if it can slowly be transformed into proofs containing only the introduction rules. This is already a much more complicated story than the harmony accounts. But, for the most part, there is a well-motivated explanation of why the steps extract what *follows from* the meanings. For example, all uses of introduction rules are acceptable because the meanings of the connectives are given by the introduction rules. And, in a potential proof with assumptions, the assumptions are replaced with closed proofs. This step is justified because the meaning of a statement is given by its proofs.

However, there is one step that doesn't fit well in this, the step for proofs that end in non-introduction rules. A proof that ends in a non-introduction rule is valid if there are a series of transformations on proofs that go from it to a proof that does end in an introduction rule. This step is vital for any non-introductory rules to *follow from* the introduction rules. We must explain why a proof rule *follows from* the introduction rules if every proof containing it can be related via transformations to a proof that ends in an introduction rule.

Both Dummett and Prawitz offer stories about why the transformations can be used to explicate "following from". A careful analysis shows that Dummett's proposal is circular. Prawitz changes the definition of proof rules so that a rule is a pair of the usual rule and a set of transformations. On Prawitz's picture, transformations are not a part of the explication of "following from" but something that *follows from* the introduction rules. However, he does not explain how these transformations *follow from* the introduction rules. So, the need to explain why transformations can be used has been moved, not resolved. This is the second horn of our dilemma.

I hope to show that the two approaches, the harmony account of definitions and proof-theoretic validity, while resolving the other's weakness, do so at the expense of the other's strength. The harmony account of definitions does not give a satisfactory account of what meanings can be but provides a simple explication of "following from". Proof-theoretic validity simplifies the conditions on acceptable meanings but does so at the expense of a complicated and unjustified picture of "following from". So, proof-theoretic semantics does not yet offer a complete account of the analyticity of logics.

Subminimal Negation on the Australian Plan

S.Kaan Tabakci

Berto and Restall [2] present and philosophically motivate the (in)compatibility frame semantics for negation, i.e., semantics of the negation on the Australian Plan (for earlier work on (in)compatibility semantics see [6], [7], [8] [13], [1], [12], [14], [15].). They argue that negation on the Australian Plan is based on two ideas: it is modal and it expresses exclusion; and we explain both of these aspects of negation by grounding it in the incompatibility relation.

Although many negations, including the negation of relevant logics, are accommodated by the Australian Plan, subminimal negation –the weakest negation under Curry's negation as refutability account–is not accommodated by the Australian Plan.[1] This can be checked by the frame validity of the following subminimally invalid DeMorgan Law: $\neg A \wedge \neg B \vdash \neg(A \vee B)$. Yet, we have two different reasons for wanting to accommodate subminimal negation on the Australian Plan. First, in the $\wedge$ and $\vee$-free fragment of our language, (in)compatibility frames can accommodate subminimal negation, given that all and only subminimally valid (purely) negation inferences are valid in the class of all (in)compatibility frames. As a consequence of this, we can claim that subminimal negation is a modal exclusion-expressing operator as well because it can also be grounded in incompatibility as the other negations accommodated by the (in)compatibility frames. Since it is also a modal and exclusion-expressing operator, accommodating subminimal negation on the Australian Plan where we also have $\wedge$ and $\vee$ in our language is desirable. Second, a proof system that is sound and complete with respect to the the class of all (in)compatibility frames has to have a rule version of $[\wedge\neg]$, and consequently, it will end up having primitive impure rules [5]. Accommodating subminimal negation will allow us to have a sound and complete logic that has the weakest Australian negation governed by pure primitive rules.

[1] According to the refutability account, A is refutable just in case A implies a counteraxiom [4, p. 255]. We obtain subminimal negation under the refutability account by allowing multiple counteraxioms that are not closed under disjunction. For recent and earlier work on subminimal negation see [3], [6], [7], [9], [10], [11].

We accommodate subminimal negation on the Australian Plan by using multi-relational frames [16], i.e., by allowing multiple incompatibility relations that are not closed under union as we allow multiple countereaxioms that are not closed under disjunction on the refutability conception. One way to interpret having multiple incompatibility relations is that each incompatibility relation is a different way to be incompatible. For instance, the incompatibility between the states where "Sherlock Holmes is a fictional character." is true and where "Sherlock Holmes exists." is true denotes one way to be incompatible, and the incompatibility between a state where "The table is green." is true and where "The table is brown." is true denotes another way to be incompatible given that they are incompatible because of different reasons; and having multiple incompatibility relations can be used to model that difference. Since we still use incompatibility as a relation that defines negation in our semantics, we retain the reasons for both the modal and the exclusion-expressing aspect of the negation on the Australian Plan, and consequently claim that multi-incompatibility frames preserve the philosophical merits of incompatibility frames.

For the remainder of the presentation, we will first give a countermodel to $[\wedge\neg]$ in a multi-incompatibility model and then provide the soundness and completeness proofs of a subminimal logic that only has Local Contraposition and the usual right and left introduction rules for $\wedge$ and $\vee$ in order to show that we accommodate subminimal negation for the full language. Then, we will investigate the new correspondence results of the other negations on Dunn's kite of negations such as minimal, intuitionistic and DeMorgan negations. One interesting consequence of the correspondence results is that we have to add falsum ($\bot$) to our language in order to prove the correspondence theorems for the principles that characterize minimal and intuitionistic negation. On the other hand, we do not need to have the falsum to have subminimal or DeMorgan negation. This indicates that subminimal negation as accommodated by the multi-incompatibility frames can be separated from its cousins in the refutability family.

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What’s So Impossible About Impossible Worlds?

Koji Tanaka (ANU) and Alexander Sandgren (Umeå)

Possible worlds are ubiquitous in contemporary philosophy. Impossible worlds are also gaining currency partly because it is often crucial to make distinctions between necessarily equivalents in so called hyperintensional contexts. Hyperintensionality is commonly handled by adding impossible worlds into more traditional possible worlds based accounts of modal phenomena. As their utility becomes more widely appreciated, we should step back for a moment and ask a question about impossible worlds: what’s so impossible about them? What, exactly, makes an impossible world impossible? In literature, three features have been proposed as the key to understanding what makes impossible worlds impossible: logical violation, logical difference and openness. We will show that the notion of logical violation is key to understanding what makes a world impossible. We will do so in three steps. First, we will present a definition of violation that is distinct from difference and openness. Second, we will argue that difference, by itself, does not make worlds impossible. Third, we will argue that there is nothing about openness that, by itself, serves as the mark of the impossible and if an open world counts as impossible, that is because it contains one or more violations. We will then conclude that it is violations that are the mark of the impossible.

Relational Hypersequent S4 and B are Incomplete

Kai Tanter (Monash)

Parisi (2020) presents relational hypersequent systems intended for standard modal logics K through to S5. These satisfy Došen's principle, in that the systems differ only in their structural rules, keeping the operation rules constant, and are, to my knowledge, the only candidate hypersequent systems in the literature with this feature. Parisi provides an indirect proof of sequent completeness for his systems via a translation to sequent systems for modal logics. However, the proofs for S4 and B require treating Cut as a basic rule. Burns and Zach (2020) have provided direct cut-free proofs of hypersequent completeness for relational hypersequent K, T and D. In this talk I'll give a brief introduction to the landscape of relational hypersequents and known completeness results, before outlining a proof showing that relational hypersequent S4 and B are incomplete relative to standard Kripke frames for S4 and B.

Constructing illoyal algebra-valued models of set theory

Benedikt Löwe, Robert Paßmann, & Sourav Tarafder

The construction of *algebra-valued models of set theory* starts from an algebra $\mathbb{A}$ and a model V of set theory and forms an $\mathbb{A}$-valued model $V^{(\mathbb{A})}$ of set theory that reflects both the set theory of V and the logic of $\mathbb{A}$. This construction is the natural generalisation of Boolean-valued models, Heyting-valued models, lattice-valued models, and orthomodular-valued models [1, 2, 6, 4] and was developed in [3].

Recently, Paßmann introduced the terms "loyalty" and "faithfulness" while studying the precise relationship between the logic of the algebra $\mathbb{A}$ and the logical phenomena witnessed in the $\mathbb{A}$-valued model of set theory in [5]. A model is called *loyal* to its algebra if the propositional logic in the model is the same as the logic of the algebra from which it was constructed and *faithful* if every element of the algebra is the truth value of a sentence in the model. The model constructed in [3] is both loyal and faithful to $\mathbb{PS}_3$, which is a three-valued algebra and can be found in [3] as well.

In this talk, we shall give elementary constructions to produce illoyal models by stretching and twisting Boolean algebras. After we give the basic definitions, we remind the audience of the construction of algebra-valued models of set theory. We then introduce our main technique: a non-trivial automorphisms of an algebra $\mathbb{A}$ excludes values from being truth values of sentences in the $\mathbb{A}$-valued model of set theory. Finally, we apply this technique to produce three classes of models: tail stretches, transposition twists, and maximal twists.

It will be shown that there exist algebras $\mathbb{A}$ which are not Boolean algebras and hence its corresponding propositional logic is non-classical, but any sentence of set theory will get either the value 1 (top) or 0 (bottom) of $\mathbb{A}$ in the algebra-valued model $V^{(\mathbb{A})}$, where the sub algebra of $\mathbb{A}$ having domain $\{0, 1\}$ is same as the two-valued Boolean algebra. This concludes that the base logic for the corresponding set theory is not classical, whereas the set theoretic sentences act classically in this case.

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RELEVANT PROPOSITIONAL DYNAMIC LOGIC

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This talk concerns the extension of relevant logics by the expressive resources of classical propositional dynamic logic (**PDL**). **PDL** is a classical, normal, multimodal logic used to formalise and investigate dynamic processes, such as algorithms, programs, or events, using the machinery of modal logic and the usual relational semantics thereof. In **PDL**, processes are represented by a recursively defined set of terms, starting from a set of basic processes, with more complex terms constructed by operations representing various ways processes can be combined. Each process term α is then associated to a $\Box$-style modality $[\alpha]$, and the formula $[\alpha]A$ is understood, intuitively, to express something like "A holds whenever α operates". This is modeled, in the usual relational semantic framework, by associating with each process a binary accessibility relation on points, which are required to satisfy properties matching the intended meanings of the recursive operations on processes. Complex processes are constructed using such operations as $\beta;\gamma$ (iteration: the result of γ operating after β), $\beta \cup \gamma$ (non-deterministic choice: the result of either β or γ operating), $\beta^\star$ (general iteration: the result of performing β some number of times), and, given a proposition A, $?A$ (test: checking whether A holds).

The resulting logic is well-studied, and has some interesting properties – among the most interesting is that **PDL** is not canonical. One cannot, generally, ensure that the canonical accessibility relation for $\alpha^\star$ is *exactly* the reflexive transitive closure of that for α. As a result, most completeness results have proceeded by *filtrations*. Details on **PDL** are available in [8, 1, 3].

This talk concerns the addition of such a collection of modalities to relevant logics, and relational semantics for the resulting systems which combine, in a more or less standard fashion, the relational semantics for classical **PDL** and the ternary relation semantics most famously developed by Sylvan (née Routley) and Meyer. There has been some investigation of relevant propositional dynamic logics (**RPDL**s) – for instance, by Fuhrmann [2] and Sylvan [7] – but until recent work by Sedlár [5, 6], there have been issues with providing completeness proofs for such systems. In particular, certain frame constraints in the ternary relational semantics, including those corresponding to the axiom forms of prefixing and suffixing, are not preserved by filtrations (see [4, §14.2]). Sedlár has recently shown how to get around this problem by the use of what he calls *partial filtrations*, and thereby has obtained a general completeness proof procedure that applies to a wide range of non-classical versions of **PDL**. This method, however, only applies to logics which include *extensional truth constants* $\top$ and $\bot$, which are required to hold at all and no (respectively) points in any relational model – the reason, in brief,

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for why these must be included is that Sedlár's method requires a language expressive enough to be able to express when two sets of worlds (expressed by formulas) have a null intersection.

In this talk, we aim to do three things. First, we present some reasons, following [7], to investigate **RPDL**s. In particular, we discuss and develop Sylvan's proposed construal of formulas of the form $[\alpha]A$: "It is necessitated through α's operation that A". This construal, as Sylvan describes it, requires that there be some connection between α and A, and rules out some **PDL**-validities, such as $[\alpha](A \vee \neg A)$, and some consequences, expressed in terms of a conditional $\to$, such as $[\alpha]A \to [\alpha](B \to B)$ and $[\alpha](A \wedge \neg A) \to [\alpha]B$. The reason is that there are some processes which fail to necessitate certain logical truths (even when they do necessitate some propositions), and that some processes necessitate contradictory propositions without, thereby, necessitating every proposition. This construal, we argue, is well founded as an interpretation of the **PDL** modalities, and provides a solid justification for the target systems. Furthermore, we argue that the requirement of relevance cooked into relevant logics, and into Sylvan's construal call for versions of **RPDL**s, militates against the inclusion of $\top$ and $\bot$, as the inclusions of these invite in failures of relevance (only to be avoided by *ad hoc* manoeuvres). Thus, we'll argue, **RPDL**s are well motivated by Sylvan's construal, but that the logics motivated this way are not precisely the ones Sedlár has provided completeness arguments for.

The second task we undertake is to show completeness for **RPDL**s, not including the extensional constants, indirectly. To do so, we'll argue that the addition of $\top, \bot$ to the logics not including them is *conservative*, and show how this result can be marshalled to prove completeness with respect to the desired class of models.

Finally, we prove that the systems, without the constants, satisfy the variable sharing property, thus showing that the systems do go some of the way to capturing the target interpretation.

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On non-classical models of ZFC

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Abstract

In this talk we present the recent developments in the study of non-classical models of ZFC. Building on some previous work in this area ([2], [1]), we will show that there are algebras that are neither Boolean, nor Heyting, but that still give rise to models of ZFC. This result is obtained by using an algebra-valued construction similar to that of the Boolean-valued models. Specifically we will show the following theorem.

Theorem 1. *There is an algebra $\mathbb{A}$, whose underlying logic is neither classical, nor intuitionistic such that $\mathbf{V}^{\mathbb{A}} \vDash \mathsf{ZFC}$. Moreover, there are formulas in the pure language of set theory such that $\mathbf{V}^{\mathbb{A}} \vDash \varphi \wedge \neg\varphi$.*

The above result is obtained by a suitable modification of the interpretation of equality and belongingness, which are classical equivalent to the standard ones, used in Boolean-valued constructions.

Towards the end of the talk we will present an application of these constructions, showing the independence of CH from non-classical set theories, together with a general preservation theorem of independence from the classical to the non-classical case.

(This is a joint work with Sourav Tarafder and Santiago Jockwich)

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AAL 2021 Abstract

Title: We will use their own tools against them: The relevance logic of Boolean groups

Speaker: Yale Weiss

Abstract: In this talk, I consider the logic of Boolean groups (i.e., Abelian groups where every non-identity element has order 2), where these are taken as *frames* for an operational semantics à la Urquhart (1972).[1] I call this logic **BG** and focus primarily on its positive fragment (i.e., with the connectives implication, conjunction, and disjunction). **BG** is a fairly exotic system; it is a proper extension of positive **RW** (**R** without contraction), it fails to have the disjunction property but is nevertheless Halldén complete, and satisfies the variable sharing property. I present and discuss a subscripted proof theory for **BG** and sketch soundness and completeness proofs. I show that the logic over the smallest nontrivial **BG** frame, which is obviously closely connected to classical propositional logic (via its algebraic semantics), turns out to be the positive fragment of a (quasi-)relevance logic that was, to the best of my knowledge, first developed by Robles and Méndez (2016) and recently axiomatized by López (2021). I discuss some possible connections between **BG** and a subsystem of **KR** and various other results and problems.

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[1] I should emphasize that the semantic approach I am interested in is distinct from, e.g., that of Abelian logic, where Abelian groups feature prominently in an *algebraic* (not *operational*) semantics. Incidentally, the logic of Boolean groups does not contain Abelian logic. For more on Abelian logic, see, e.g., Meyer and Slaney (1989).