Assignment 2

Understanding Uncertainty

Due 9/12

1. 'State Bottle Retail' is the price at which the bottle is sold, while 'State Bottle Cost' is the price the state pays for a bottle.

- How do you define the most profitable item? Why? Explain your thinking.
- Use `.groupby` calculations to determine the most profitable liquors sold in Iowa.
- Make a plot of the top liquors.

In [1]:
```
import urllib.request
import os

def download_data(force=False):
    """Download and extract course data from Zenodo."""
    import urllib.request
    import zipfile
    import os

    zip_path = 'data.zip'
    data_dir = 'data'

    if not os.path.exists(zip_path) or force:
        print("Downloading course data...")
        urllib.request.urlretrieve(
            'https://zenodo.org/records/16954427/files/data.zip?download=1',
            zip_path
        )
        print("Download complete")

    if not os.path.exists(data_dir):
        print("Extracting data files...")
        with zipfile.ZipFile(zip_path, 'r') as zip_ref:
            zip_ref.extractall(data_dir)
        print("Data extracted")

    return data_dir

download_data()
```

```
Downloading course data...
Download complete
Extracting data files...
Data extracted
```

Out[1]: 'data'

In [2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [3]:
```
idf = pd.read_csv('./data/iowa.csv')
idf.head()
```

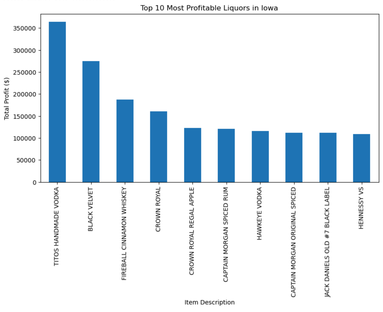
Out[3]:

	Invoice/Item Number	Date	Store Number	Store Name	Zip Code	Category Name	Vendor Name	Item Number	Item Description	Bottle Volume (ml)	State Bottle Cost	State Bottle Retail	Bottles Sold	Sale (Dollars)
0	INV-59108400026	06/06/2023	3723	J D SPIRITS LIQUOR	51040	STRAIGHT RYE WHISKIES	INFINIUM SPIRITS	27102	TEMPLETON RYE 4YR	750	18.09	27.14	2	54.28
1	S16879800057	01/16/2014	3926	LIQUOR DOWNTOWN / IOWA CITY	52240	VODKA 80 PROOF	HEAVEN HILL BRANDS	35416	BURNETT'S VODKA 80 PRF	750	4.84	7.26	12	87.12
2	INV-05301100019	06/05/2017	3829	GARY'S FOODS / MT VERNON	52314	CANADIAN WHISKIES	DIAGEO AMERICAS	11296	CROWN ROYAL	750	15.59	23.39	6	135.66
3	INV-40973500083	10/14/2021	5102	WILKIE LIQUORS	52314	AMERICAN SCHNAPPS	JIM BEAM BRANDS	82787	DEKUYPER BUTTERSHOTS	1000	7.87	11.81	12	141.72
4	INV-17022500013	01/18/2019	2560	HY-VEE FOOD STORE / MARION	52302	WHISKEY LIQUEUR	SAZERAC COMPANY INC	64863	FIREBALL CINNAMON WHISKEY	200	2.50	3.75	12	45.00

In [4]:
```
idf["Profit_per_Bottle"]=idf["State Bottle Retail"]-idf["State Bottle Cost"]
idf["Total_Profit"]=idf["Profit_per_Bottle"]*idf["Bottles Sold"]
profit_summary = idf.groupby("Item Description")["Total_Profit"].sum().sort_values(ascending=False)
```

In [5]:
```
print("Most profitable item:", profit_summary.idxmax())
print("Profit amount:", profit_summary.max())

profit_summary.head(10).plot(kind="bar", figsize=(10,5))
plt.title("Top 10 Most Profitable Liquors in Iowa")
plt.ylabel("Total Profit ($)")
plt.show()
```

```
Most profitable item: TITOS HANDMADE VODKA
Profit amount: 364074.350000000003
```



2. This question is to further explore broadcasting in NumPy.

- In the first code chunk below, there are two vectors, x and y. What happens when you try to subtract them?
- Instead, try `x.reshape(-1,1)-y.reshape(1,-1)`. What do you get?
- Try `x.reshape(1,-1)-y.reshape(-1,1)`. Compare the results to your previous answer.
- Write two double for-loops to replicate the results of the previous two answers.
- Clearly explain how `x.reshape(1,-1)` and `x.reshape(-1,1)` change x

In [6]:
```
import numpy as np

x = np.array([1,3,5])
y = np.array([2,3])
```

In [7]:
```
try:
    print(x - y)
except ValueError as e:
    print("Error:", e)

print("x.reshape(-1,1) - y.reshape(1,-1):\n", x.reshape(-1,1)-y.reshape(1,-1))
print("x.reshape(1,-1) - y.reshape(-1,1):\n", x.reshape(1,-1)-y.reshape(-1,1))

res1 = np.zeros((len(x), len(y)))
res2 = np.zeros((len(y), len(x)))

for i in range(len(x)):
    for j in range(len(y)):
        res1[i,j] = x[i] - y[j]

for i in range(len(y)):
    for j in range(len(x)):
        res2[i,j] = x[j] - y[i]

print("Loop result 1:\n", res1)
print("Loop result 2:\n", res2)
```

```
Error: operands could not be broadcast together with shapes (3,) (2,)
x.reshape(-1,1) - y.reshape(1,-1):
 [[-1 -4]
 [ 1 -2]
 [ 3  0]]
x.reshape(1,-1) - y.reshape(-1,1):
 [[-1  1  3]
 [-4 -2  0]]
Loop result 1:
 [[-1. -4.]
 [ 1. -2.]
 [ 3.  0.]]
Loop result 2:
 [[-1.  1.  3.]
 [-4. -2.  0.]]
```

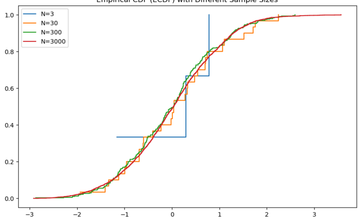
3. The indicator function $\mathbb{I}\{x \leq z\}$ takes the value 0 if $x > z$ and 1 if $x \leq z$ is true, or

$$\mathbb{I}\{x \leq z\} = \begin{cases} 0, & x > z \\ 1, & x \leq z \end{cases}$$

- For $x = 3$, plot the indicator function for any z
- Explain how $1 - \mathbb{I}\{x \leq z\} = \mathbb{I}\{x > z\}$. Drawing a plot might help, or looking at the definition given above.
- The code below generates a random sample for different numbers of observations, $N = 3, 30, 300, 3000$. Write a function to compute the ECDF curve for each sample, and plot your results.
- Describe in your own words what happens to the ECDF qualitatively as the sample size N increases

In [8]:
```
import numpy as np
import seaborn as sns

rng = np.random.default_rng(100)

for N in (3,30,300,3000):
    X = rng.normal(0,1,N)
    # ECDF plot goes here:
```

In [9]:
```
import numpy as np
import matplotlib.pyplot as plt

def indicator(x, z):
    return 1 if x <= z else 0

def ecdf(data):
    x = np.sort(data)
    y = np.arange(1, len(x)+1) / len(x)
    return x, y

rng = np.random.default_rng(100)

plt.figure(figsize=(10,6))
for N in (3,30,300,3000):
    X = rng.normal(0,1,N)
    x, y = ecdf(X)
    plt.step(x, y, where="post", label=f"N={N}")

plt.title("Empirical CDF (ECDF) with Different Sample Sizes")
plt.legend()
plt.show()
```



4. A **quantile** generalizes the concept of the median so that instead of 50% of the mass below and 50% above, the q-quantile is the value for which q proportion of the data are below that value and $(1-q)$ proportion of the data are above that value. This is a concept of order.

- Write a function that computes any quantile without using Numpy's `np.quantile`
- Write a function that computes the **Interquartile Range (IQR)** -- the .25 quantile and the .75 quantile, which brackets 50% of the data -- and the whiskers: $\pm 1.5 \times \text{IQR}$ from the edges of the IQR
- Write a function that computes a five-number summary and values for a boxplot: the minimum, the .25 quantile, the median, the .75 quantile, and the maximum, and the whiskers
- Anything outside the whiskers is an outlier; write a function that returns a Boolean vector indicating if the observations are outliers.
- For a numeric variable of your choice from one of the datasets, apply your functions from above, and compare your answers with `sns.boxplot` and `df.describe`.

In [11]:
```
import numpy as np

def my_quantile(data, q):
    data_sorted = np.sort(data)
    pos = q*(len(data_sorted)-1)
    lower, upper = int(np.floor(pos)), int(np.ceil(pos))
    return (data_sorted[lower] + data_sorted[upper]) / 2

def IQR(data):
    q25, q75 = my_quantile(data, 0.25), my_quantile(data, 0.75)
    return q25, q75, q75-q25

def five_number_summary(data):
    return {
        "min": np.min(data),
        "Q1": my_quantile(data, 0.25),
        "median": my_quantile(data, 0.5),
        "Q3": my_quantile(data, 0.75),
        "max": np.max(data)
    }

def find_outliers(data):
    q25, q75, iqr = IQR(data)
    lower, upper = q25 - 1.5*iqr, q75 + 1.5*iqr
    return (data < lower) | (data > upper)
```

5. The kernel density estimator captures the concentration of the data around various points, and is defined as

$$\hat{f}_{NA}(z) = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{h} k\left(\frac{x_i - z}{h}\right)$$

where h is the bandwidth and $k()$ is one of the kernel functions from the class materials (Gaussian, Epanechnikov, uniform, triangular).

- Write a function that takes the bandwidth h as given, and computes and plots $\hat{f}_{NA}(z)$, for the kernels listed in the slides
- Write a function that computes the plug-in bandwidth
- Use your functions to analyze some numeric variables from a dataset of your choice, using a variety of bandwidths
- Compare bandwidths that are greater or less than the plug-in value with the results for the plug-in value. When does the KDE appear to be overfitting? Underfitting?

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt

def gaussian(u): return (1/np.sqrt(2*np.pi)) * np.exp(-0.5*u**2)
def epanechnikov(u): return 0.75*(1-u**2) if abs(u)<=1 else 0
def uniform(u): return 0.5 if abs(u)<=1 else 0
def triangular(u): return (1-abs(u)) if abs(u)<=1 else 0

def kde(data, z, h, kernel):
    return np.mean([kernel((xi-z)/h) for xi in data]) / h

def plot_kde(data, h, kernels):
    xs = np.linspace(min(data)-2, max(data)+2, 200)
    plt.figure(figsize=(10,6))
    for name, k in kernels.items():
        ys = [kde(data, z, h, k) for z in xs]
        plt.plot(xs, ys, label=name)
    plt.title(f"KDE with bandwidth h={h}")
    plt.legend()
    plt.show()
```