Wave Theory of Light: Verification and Automation

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History of wave theory of light

17th Century:

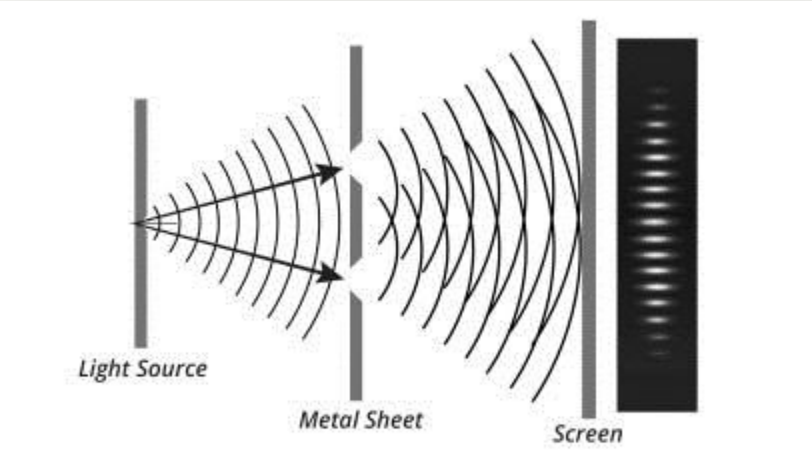
- Debate around nature of light
- Newton's corpuscles and Huygens' wave theories of light conflicted

1801:

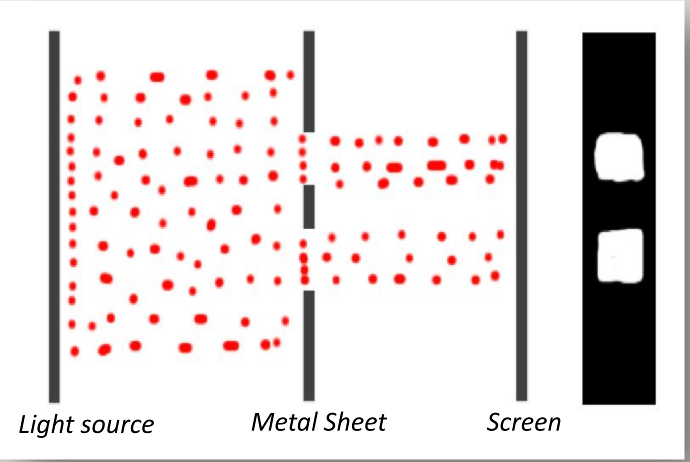
Thomas Young publishes the Double-slit experiment, proving light behaves as a wave

Electron and **X-ray diffraction** can observe the characteristics and structures of certain materials.

Diffraction also leads to implications about 5G



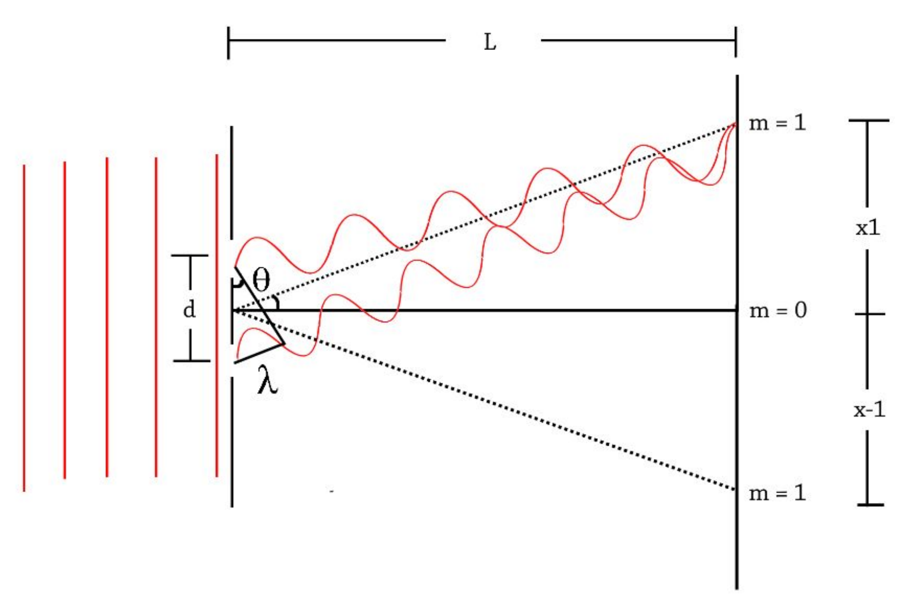
Young's Double Slit Experiment



Particle Behavior

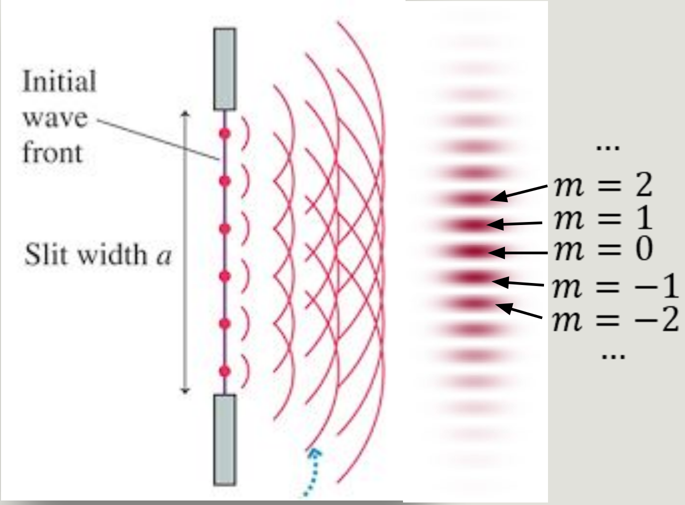
Theory

Waves diffract when they encounter a slit or obstacle. Young hypothesized **constructive** and **deconstructive interference** to predict what occurs when light “waves” interact with slits



Young’s Relation Double-Slit (maxima)

$$m\lambda = d sin\theta \text{ [1]}$$



Young’s Relation Single-slit (minima)

$$m\lambda = a sin\theta \text{ [2]}$$

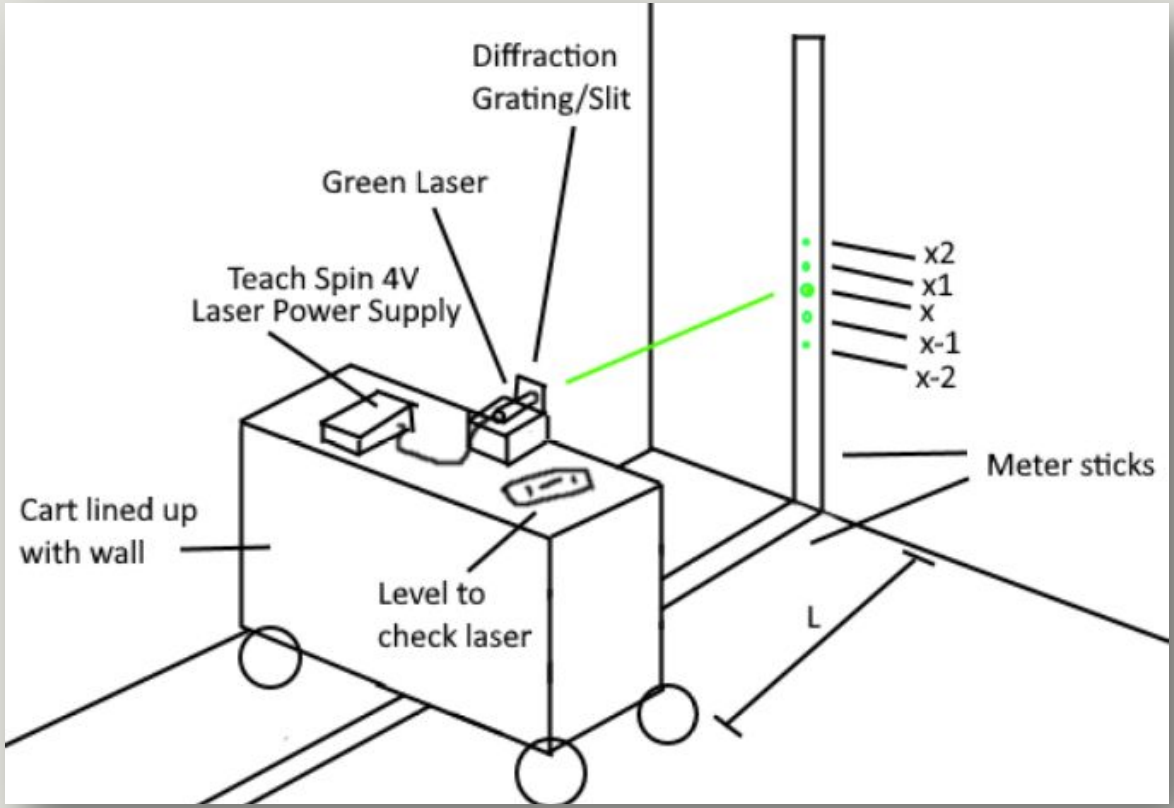
$m = mode$
$\lambda = wavelength$
$d = slit\ spacing(s)$
$a = slit\ widths(s)$

Geometric Method

First found wavelength with known diffraction grating (500 lines/mm)

Using found wavelength, swapped diffraction grating for TeachSpin slits:

Single-slit, Double-slits (14, 16, 18)



Intensity Theory

Integration over the E-field contributions of each point of a wavefront. $Intensity \propto E^2$

Slit width and slit spacing affect the output intensities:

Double-slit:

$$I = I_{max}\frac{sin^2\alpha}{\alpha^2}cos^2\beta \text{ [3]},$$

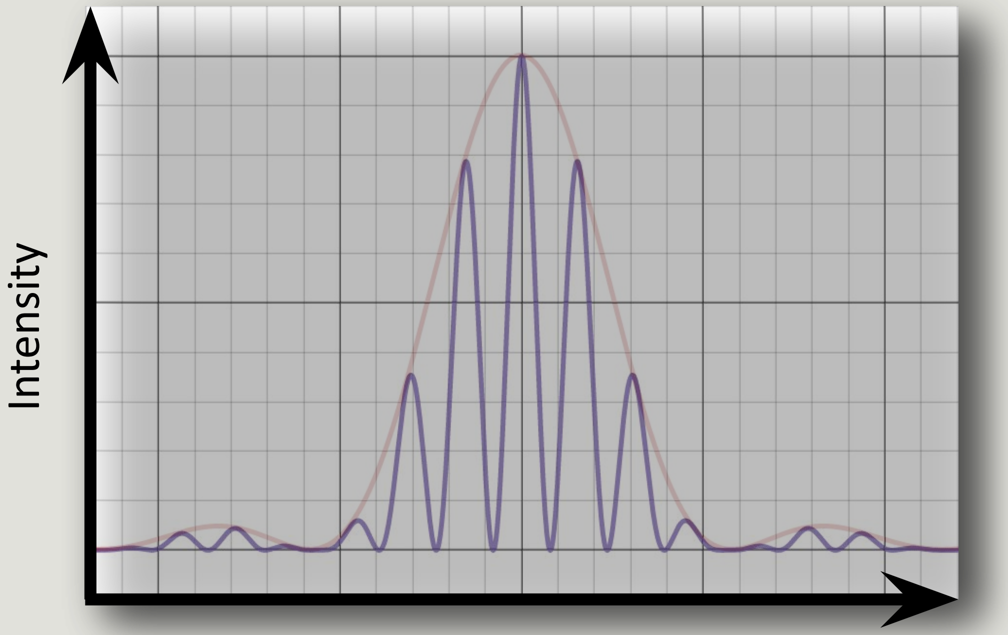
Single-slit ($d = 0$):

$$I = I_{max}\frac{sin^2\alpha}{\alpha^2} \text{ [4]}$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

$$\beta = \frac{\pi d}{\lambda}\sin\theta$$

Red function indicates Single-slit, Purple indicates Double-slit



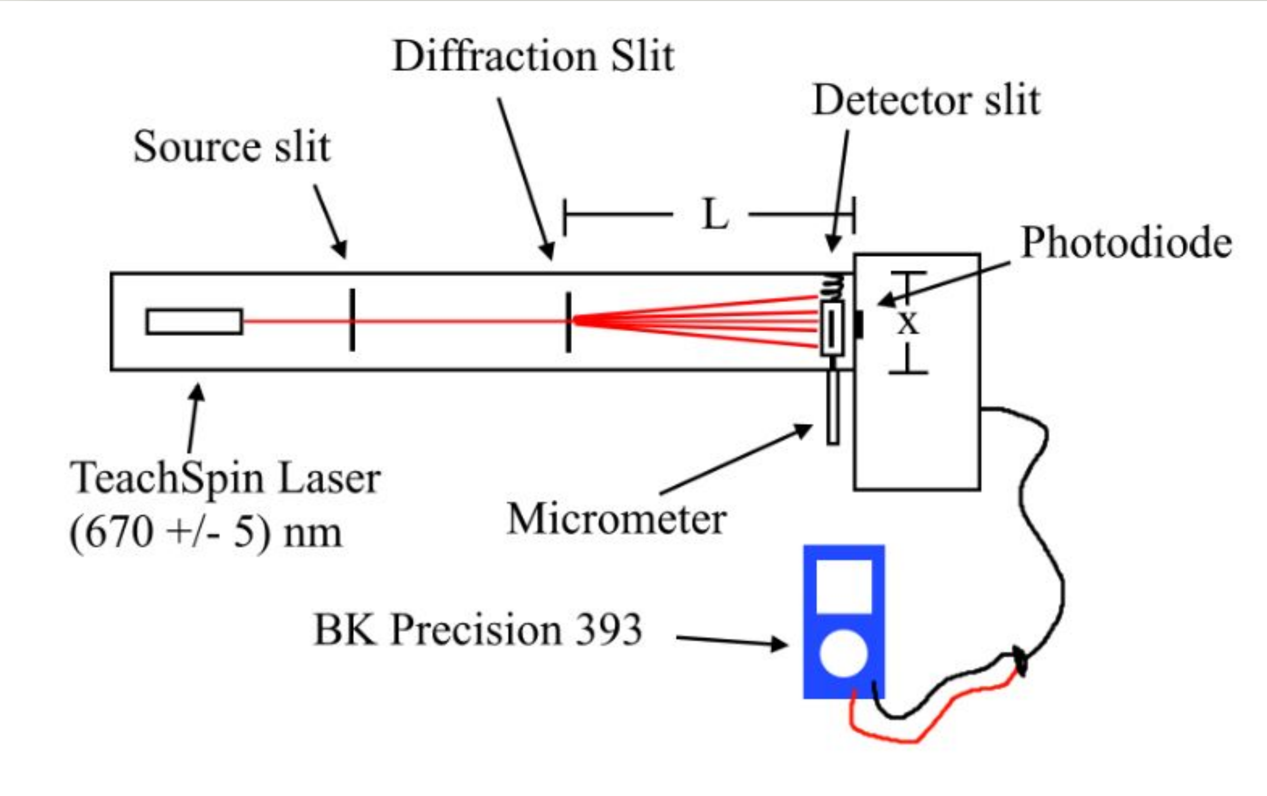
TeachSpin Intensity Method

Output voltage from the photodiode is proportional to intensity (Eqn. [3] and [4])

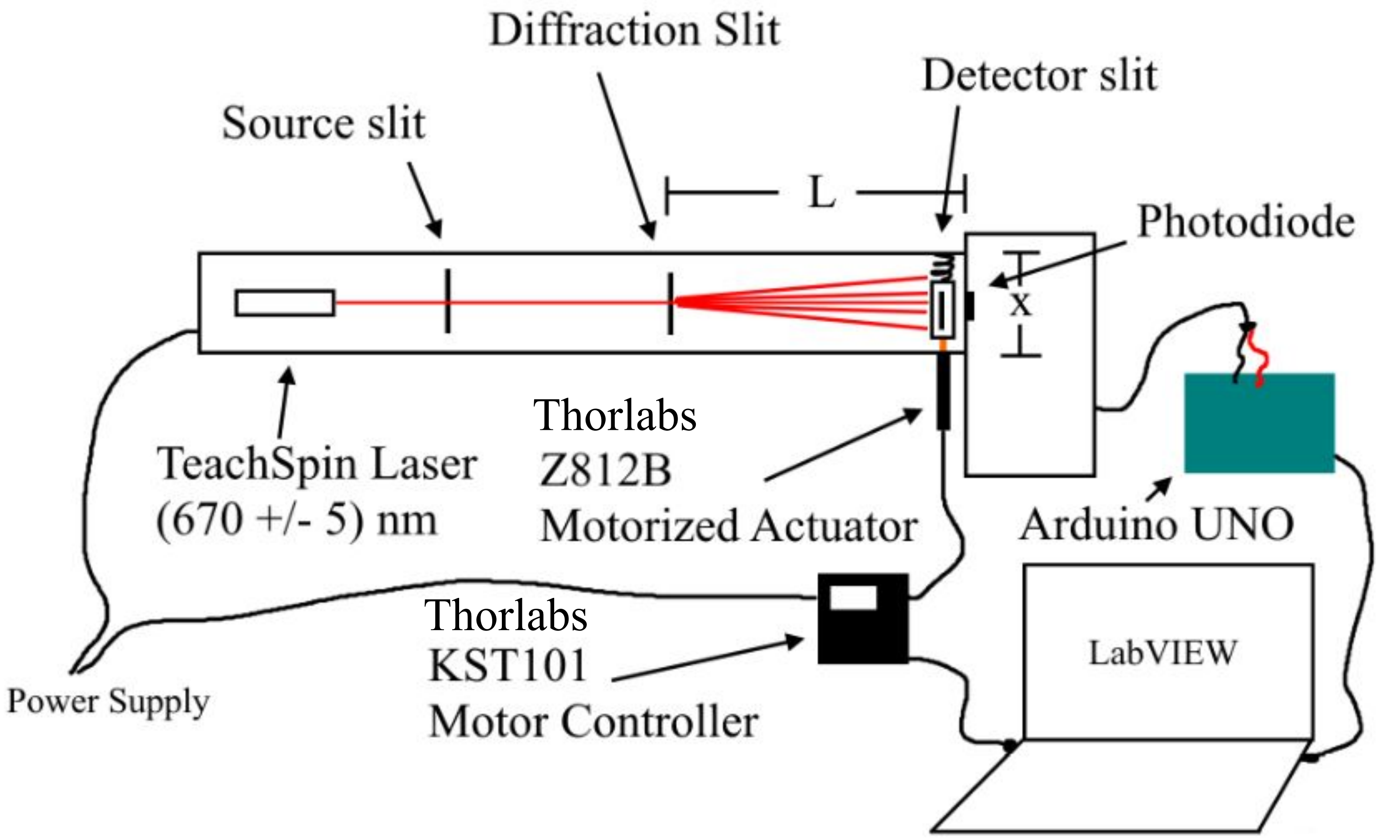
Laser centered with photodiode, validates $\sin\theta$

Data was then centered around the central maxima and normalized with OriginLab

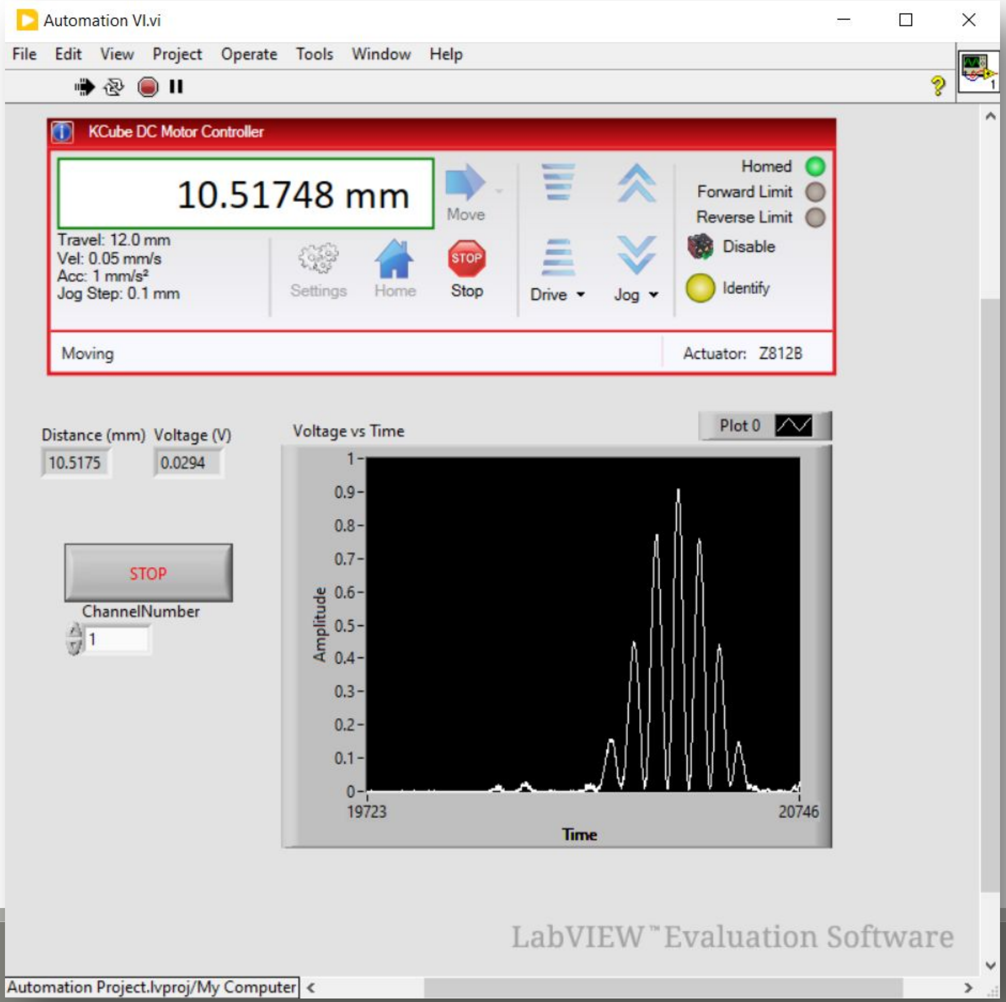
- Extremely useful in obtaining a meaningful curve fit
- Manual methods were tedious
- … automation!

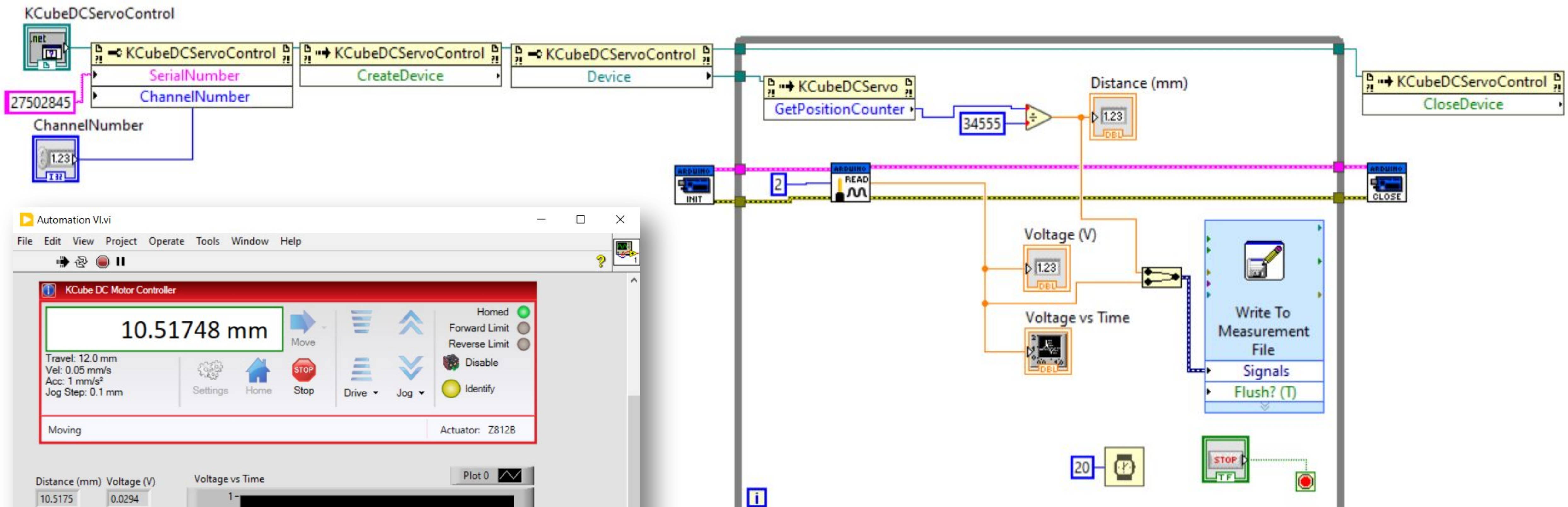


Top-down TeachSpin Apparatus



Automated Setup of TeachSpin Apparatus





LabVIEW

Front Panel / Block Diagram

Intensity Data/Graphing

Equation 3 rearranged for simpler fitting:

$$\frac{V}{V_{max}} = \frac{sin^2 Ax}{A^2x^2} cos^2 Bx, [5]$$

$$A = \frac{\pi a}{\lambda}$$

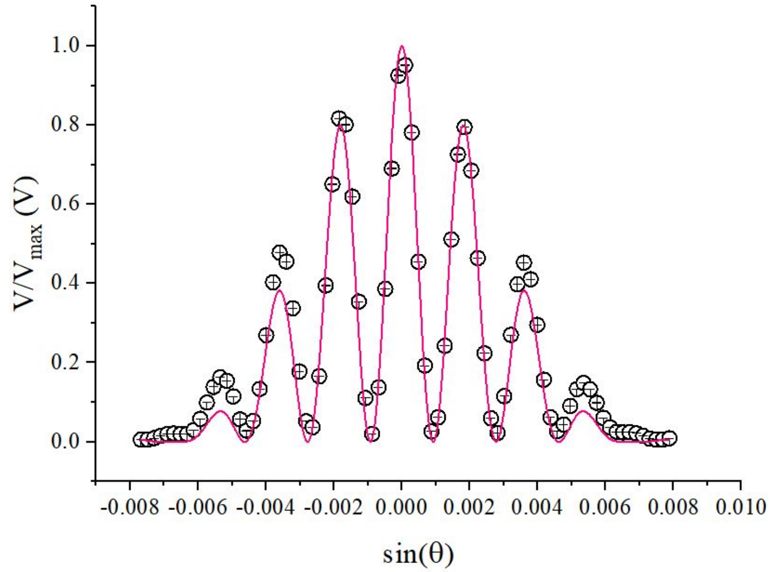
$$B = \frac{\pi d}{\lambda}$$

$$x = \sin\theta$$

The x values were centered for symmetry in $\sin\theta$, which is also calculated in Origin to plot

Fit parameters, A and B, were rearranged to derive values for slit spacing and width

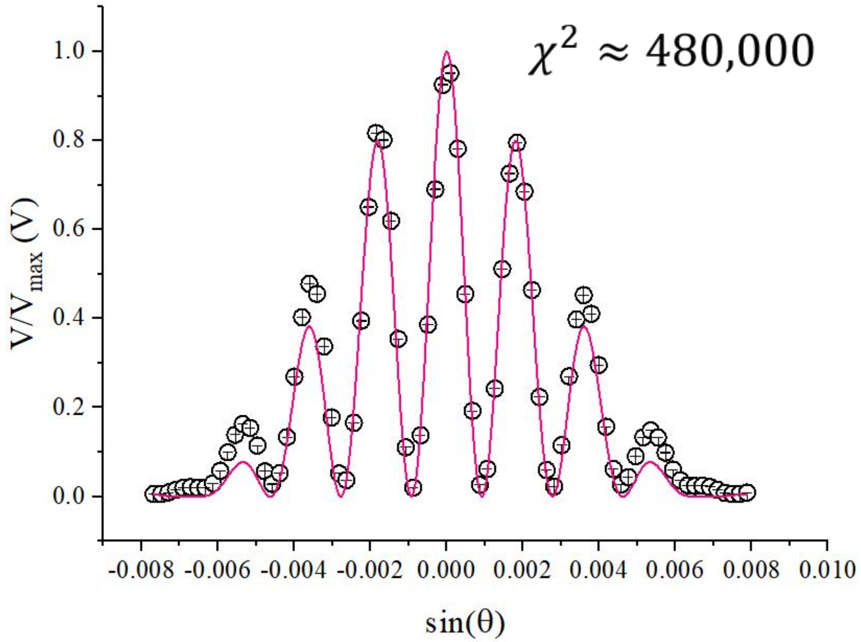
$$\frac{V}{V_{max}} = \frac{sin^2[(442 \pm 6)x]}{(442 \pm 6)^2 x^2} cos^2[(1700 \pm 10)x]$$



Sample graph for manual Double-slit (14) data with OriginLab non-linear curve fitting of the Intensity Equation [5]

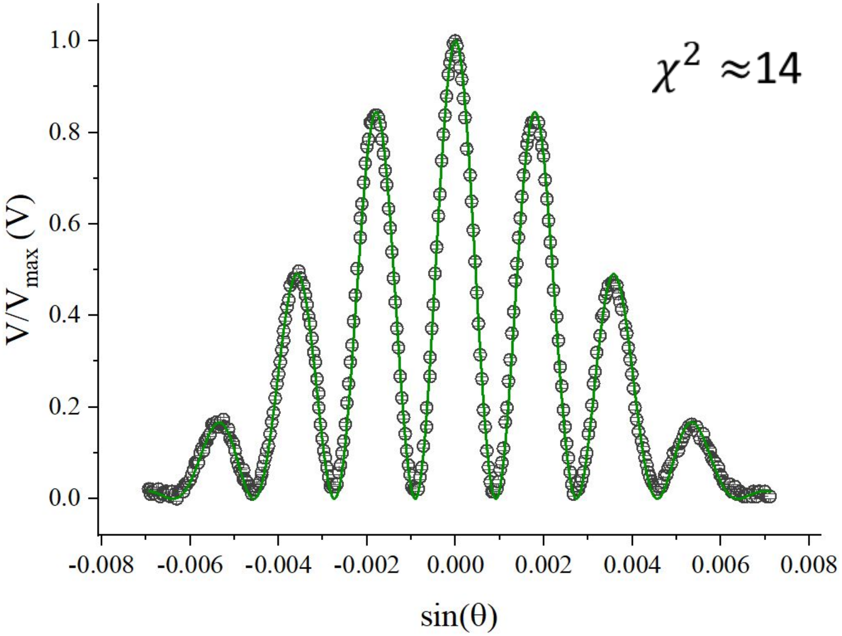
Double-slit 14

MANUAL



$$\frac{V}{V_{max}} = \frac{sin^2[(442 \pm 6)x]}{(442 \pm 6)^2 x^2} cos^2[(1697 \pm 11)x]$$

AUTOMATED



$$\frac{V}{V_{max}} = \frac{sin^2[(389.7 \pm 1.1)x]}{(389.7 \pm 1.1)^2 x^2} cos^2[(1721.3 \pm 1.1)x]$$

Results

The geometric method uses found wavelength of $\lambda_{Green} = (528 \pm 1)\ nm$, whereas TeachSpin suggests the wavelength is $\lambda = (520 \pm 10)\ nm$

	TeachSpin Specs		Geometric Method	Automated Intensity	
Single	85	--	--	--	
14	85	0.353	--		
16	85	0.406	--		
18	85	0.457	--		

Analysis and Improvements

Discrepancies from TeachSpin values

Automated results compliments the initial setup results

Accurate location of central maxima improved percent uncertainties (Automation)

Properly centered data provided meaningful curve fits (χ^2)

	Manual	**Automated**
Time	30 minutes	5 minutes
Data Points	~ 100	~ 300-600

Reducing Uncertainties

$$\delta a = \sqrt{(\frac{\lambda \delta A}{\pi})^2 + (\frac{A \delta \lambda}{\pi})^2} = \sqrt{(0.16 + 2.1) \times 10^{-13}}\, nm$$

TeachSpin Red $\lambda = (670 \pm 5) nm$

Uncertainty could reduce to 1 nm by first method

→ Would lead to reduced uncertainties of a and d, roughly by factor of 2-4x

Curve fit parameters could also be improved with analogReference()

→ Reduce voltage uncertainties by a factor of 2 or more

Conclusions

Automation was very effective and worthwhile

 This provided higher quality data (accuracy and precision), in much less time, with more flexibility to fix alignment

The automated results corroborate the geometric findings:

 Its possible the slits have developed imperfections since being manufactured

Intensity equation verified by Chi-square test, helps support verification of wave theory of light

Acknowledgements

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- Dr. Patricia E. Allen
- Dr. Brooke C. Hester
- Isaac Critcher

Without their help this project would not be possible.

References

- [1] StudySmarter US, *Newton and Huygens' Theories of Light*, (n.d.).
- [2] Massachusetts Institute of Technology, *Interference and Diffraction*, WWW Document, (https://web.mit.edu/8.02t/www/802TEAL3D/visualizations/coursenotes/modules/guide14.pdf)
- [3] J. Li and J. Sun, Acc. Chem. Res., *Application of X-ray Diffraction and Electron Crystallography for Solving Complex Structure Problems* (2017).
- [4] F.L. Pedrotti, L.S. Pedrotti, and L.M. Pedrotti, *Introduction to Optics* (Addison-Wesley, 2007).
- [5] TeachSpin, *Two-Slit Interference, One Photon at a Time, WWW Document, (https://www.teachspin.com/two-slit)*
- [6] Indiana University, *27.3 Young's Double Slit Experiment*, (2016).
- [7] J.R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed (University Science Books, Sausalito, Calif, 1997).

Appendix

Only 3 modes were visible during the experiment so 3 graphs were made for each mode.

To find the wavelength of the laser, Young's equation replaced $\sin\theta = \frac{x}{\sqrt{x^2+L^2}}$.

$$m\lambda = d\frac{x}{\sqrt{x^2+L^2}}$$

Then $\frac{x}{L} = slope$ was substituted and the equation was simplified into:

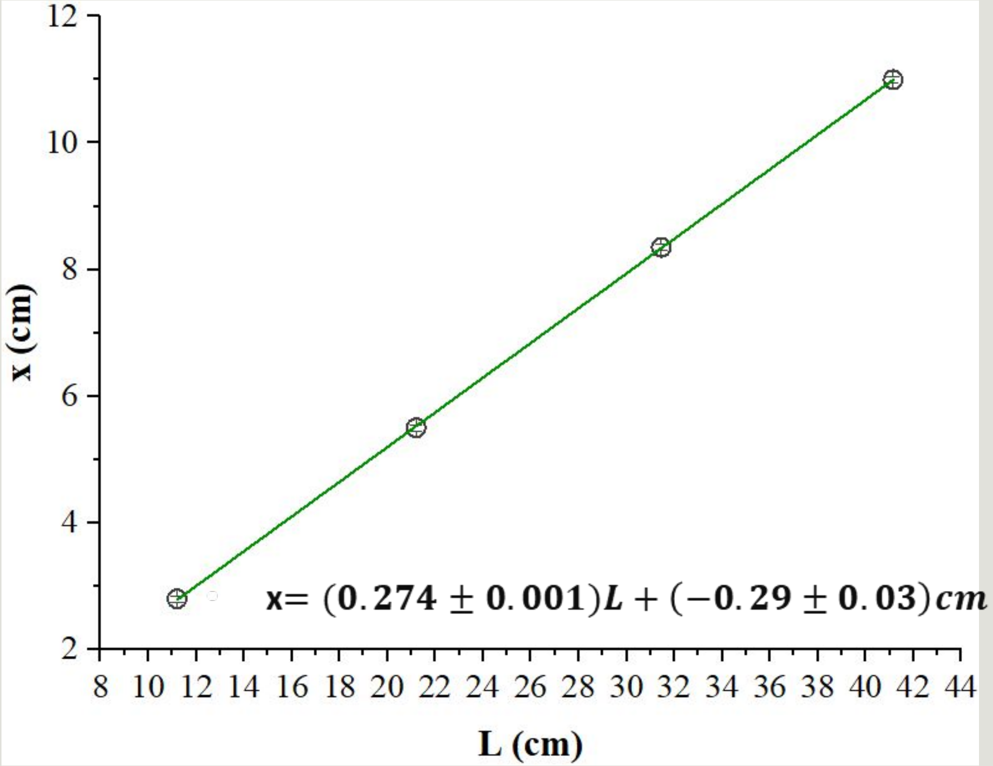
$$\lambda = \frac{d}{m}\left(\frac{1}{slope^2}+1\right)^{-1/2}$$

With uncertainty:

$$\delta\lambda = \frac{d\delta slope}{m(1+slope^2)}^{3/2}$$

Sample Graph

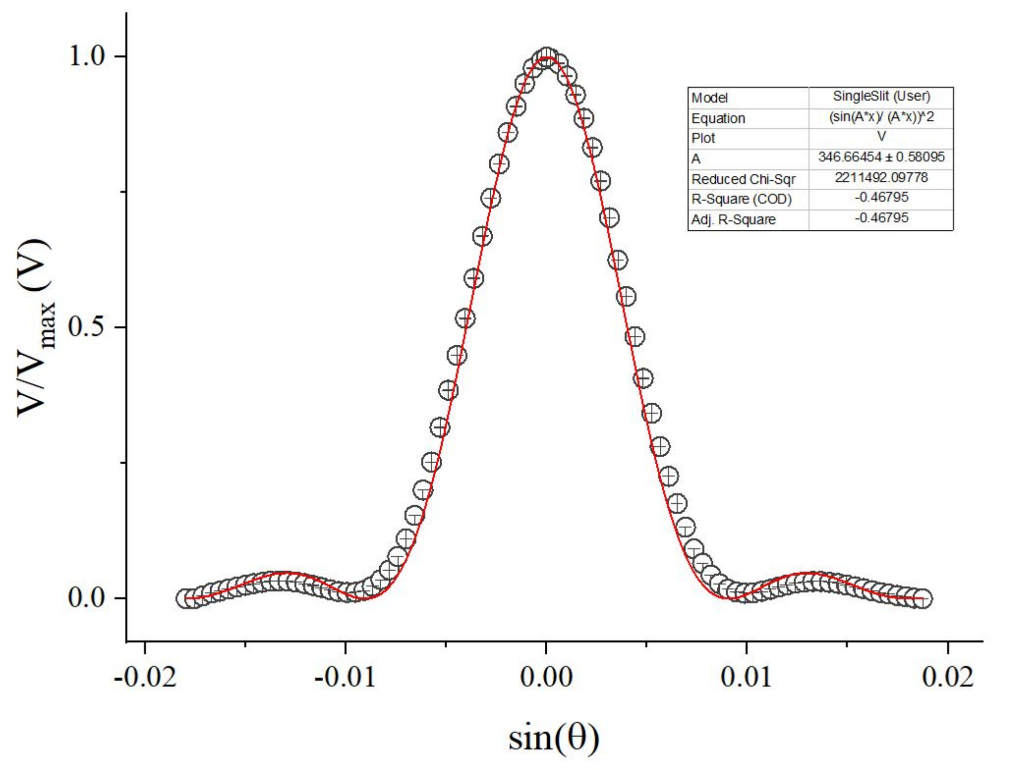
Mode 1 (500 lines/mm)



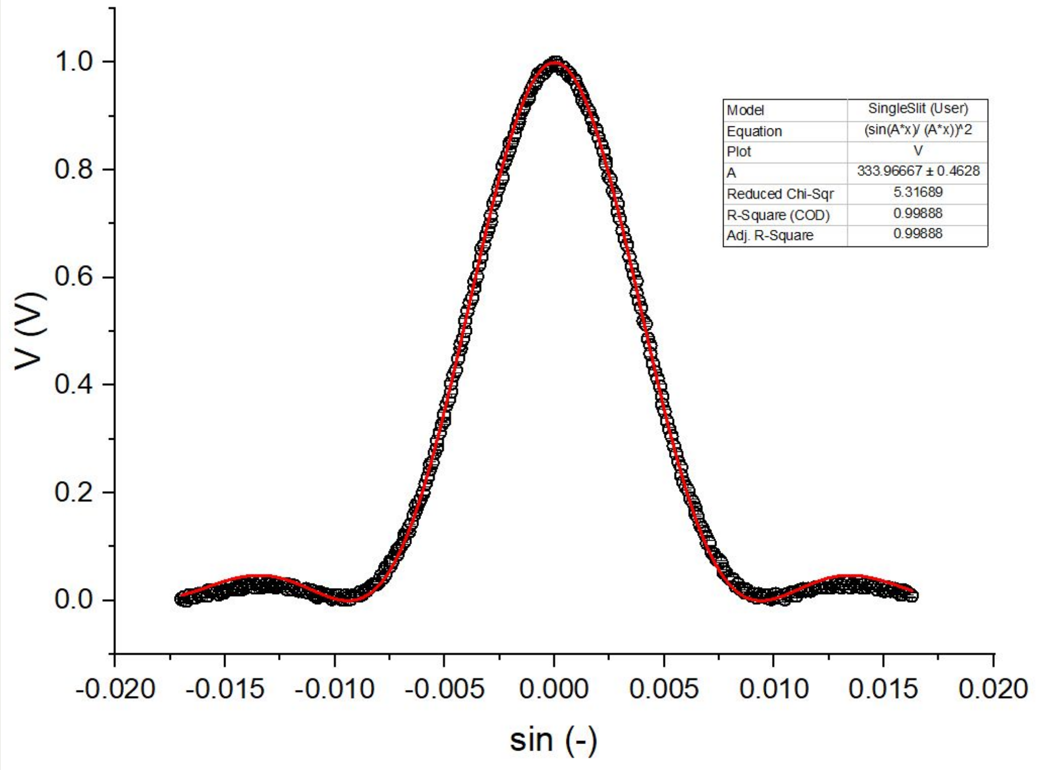
All Experimental Values

	Geometric Method	Manual Intensity Method	Automated Intensity
Single	--	--	--
14 --			
16 --			
18 --			

Single slit



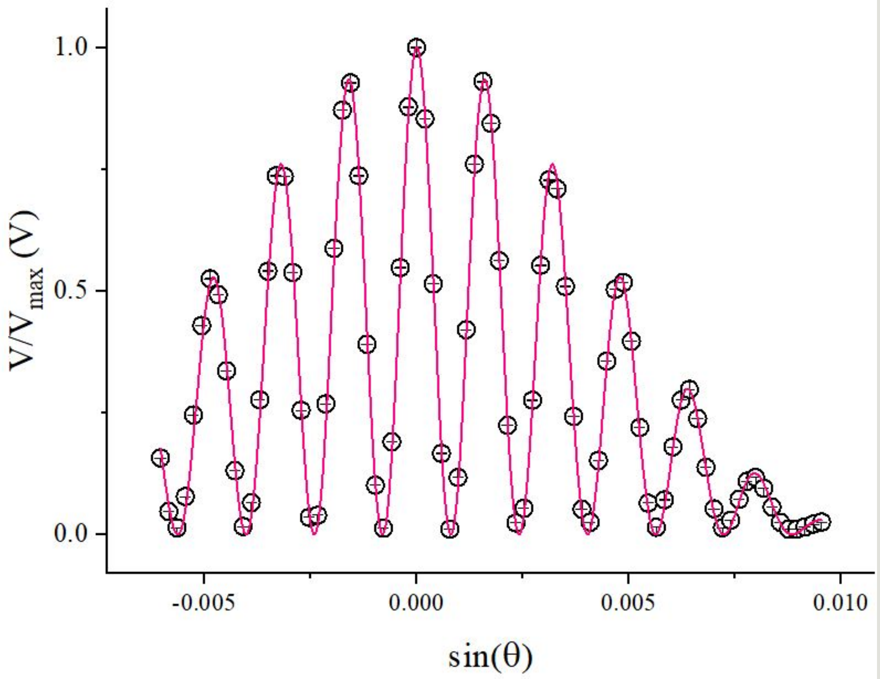
Model
SingleSlit (User)
Equation
(sin(A*x)/ (A*x))^2
Plot
V
A
346.66454 ± 0.58095
Reduced Chi-Sqr
2211492.09778
R-Square (COD)
-0.46795
Adj. R-Square
-0.46795
V/V$_{max}$ (V)
sin(θ)



Model
SingleSlit (User)
Equation
(sin(A*x)/ (A*x))^2
Plot
V
A
333.96667 ± 0.4628
Reduced Chi-Sqr
5.31689
R-Square (COD)
0.99888
Adj. R-Square
0.99888
V (V)
sin (-)

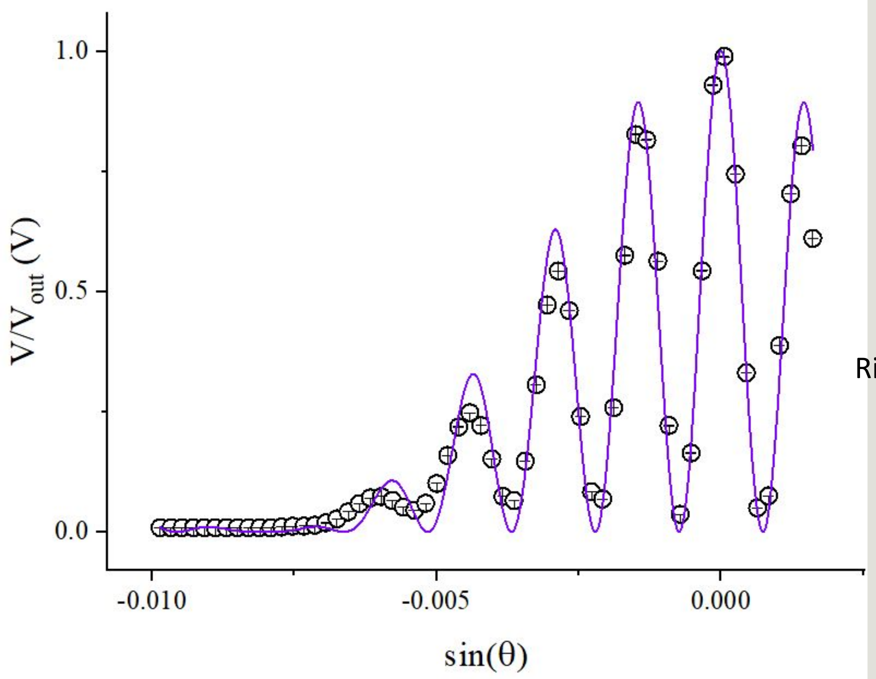
Appendix

Additional Graph for Double-slit (16)



$$I = \frac{sin^2[(279 \pm 3)x]}{(279 \pm 3)x} cos^2[(1954 \pm 3)x]$$

Additional Graph for Double-slit (18)

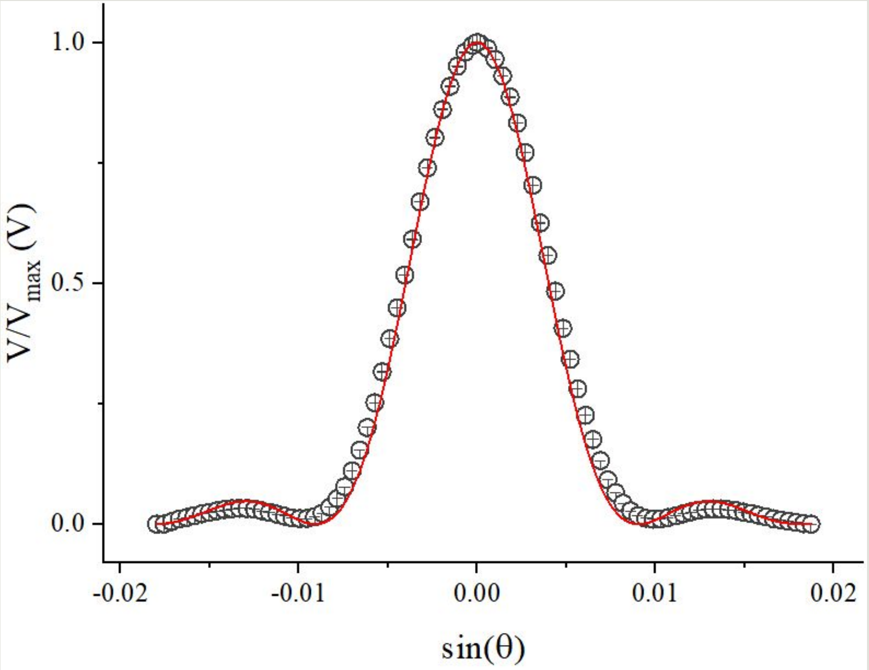


Right side of data cut due to asymmetry (Bad data)

$$I = \frac{sin^2[(392 \pm 5)x]}{(392 \pm 5)x} cos^2[(2132 \pm 9)x]$$

Appendix

Repeated Graph for Single-slit with curve fitting of intensity function



$$I = \frac{sin^2[(346.6 \pm 0.6)x]}{(346.6 \pm 0.6)x}$$

Following the substitution made in the data/graphing section we can solve for a and d trivially:

$$a = \frac{A\lambda}{\pi}$$

$$\delta a = \sqrt{\frac{\lambda \delta A}{\pi}^2 + \frac{A\delta\lambda}{\pi}^2}$$

$$d = \frac{B\lambda}{\pi}$$

$$\delta d = \sqrt{\frac{\lambda \delta B}{\pi}^2 + \frac{B\delta\lambda}{\pi}^2}$$