Application of Matrix Exponential Spatial Specification in Hedonic Price Models

Sudhakar BRAR
Université de Bourgogne
M2 DASEE

April 15, 2022

Contents

1 Introduction

In the pursuit to model housing prices, there are two approaches which are quite popular. First is a monocentric model approach, where the housing price is a function of one central business district (Chin, Chau (2003)). And the other hedonic price model, where goods are sold as a package of inherent attributes (Rosen (1974)). This methodology has been widely used to examine the relationship between inherent attributes and the house prices (Yang (2001), Chau; Yiu; Wong; Lai (2002), Maurer; Pitzer (2008), Páez; Long; Farber (2008), Monson (2009), Lazrak; Nijkamp; Rietveld; Rouwendal (2014), Zhang; Dong (2018), Gokmenoglu; Hesami (2019), Hussain; Abbas; Wei; Nurunnabi (2019)). Therefore, the differences in prices of houses is due to the differences in inherent attributes.

Estimation of a hedonic price model is usually done through regression analysis. Usually in this model is used to determine either the rent or the house value aganist its characteristics assuming that they are already known. A basic hedonic model equation (Herath, Maier (2010)) is as following:-

$$R = f(P,N,L,C,t)$$

where R is either the rent of the house value, P is attributes realting to property, N is characteristics of the neighbourhood, L is locational variables, C is contract conditions and t is time indicator.

However, the hedonic pricing models tend to fail where there is spatial data, since, they do not take into account spatial autocorrelation. Not taking it into account can lead to biased and/or inconsistent model estimates. Hence, the hedonic model was extended to take into account spatial autocorrelation. Anselin (1998) specified two main models - namely spatial lag model and spatial error model.

Although these models are quite widely used in spatial econometrics literature, they are not short of their own problems. For ex. to solve these models, one needs to compute an inverse of a matrix which might not exist always. Also, these models require a lot of computational resources if the data is quite large. Hence, Lesage and Pace (2003), have created a Matrix Exponential Spatial Specification (MESS) which uses an exponential pattern of decay. This results in theoretical simplicity and takes less computational resources. Chiu, Leonard and Tsui (1996) have discussed the following advantages of an exponential specification:-

- Using this specification always leads to positive definite covariance matrices and, hence, one does not needs to restrict the parameter space for coefficient spatial autocorrelation.
- Inversing an exponential matrix is quite easy to implement since it has a simple mathematical form.
- Also, in the log-determinant, there is a troubling term of log-determinant of *nxn* covariance matrix which vanishes.

Hence, in this study we perform a hedonic price model which takes into account the spatial autocorrelation and is specified as a MESS model. We also, estimate the direct, indirect and total effects for this model which can help us determine which variables have the most impact on the sale price of the houses.

In section 2, we talk about the data used for this study and provide some summary statistics for the data. In section 3, we introduce what is a spatial weights matrix and why it is required. We also specify the model equation and form. In section 4, we present the coefficient estimates and the direct and indirect effects. In section, 5 we present the conclusion for the study. And in 6 we present the bibliography for the paper.

2 Data

The data for this study was made available by Robin Durin, Weatherhead School of Management, Case Western Research University, Cleveland, OH. The data consists of house sales prices and the characteristics of the said houses. The data is available for 211 houses and 13 variables (6 continuous, 7 dummy). Below, Table 1, shows the variables, their description and their class (whether a continuous variable (C) or a dummy variable (D)). Table 2 shows the summary statistics of the data only for the continuous variables.

3 Spatial Weights Matrix and Specification

Spatial Weights Matrix W_{ij} helps us to define the relationship (w_{ij}) between the houses i and j. These relationships could be defined according to distance, interaction, contiguity, nearest neighbours, etc. In this study we define them using the 10 nearest neighbours. This gives us a binary matrix which tells us which are the 10 nearest neighbours of each house. We then divide this matrix with the maximum eigen value of this matrix. And hence, we get the following:-

Table 1: Variables

Variable Name	Descripition	Class
PRICE	Sales price of house in $1,000 (MLS)	C
NROOM	Number of rooms	C
DWELL	1 if detached unit, 0 otherwise	D
NBATH	Number of bathrooms	C
PATIO	1 if patio, 0 otherwise	D
FIREPL	1 if fireplace, 0 otherwise	D
AC	1 if air conditioning, 0 otherwise	D
BMENT	1 if basement, 0 otherwise	D
NSTOR	Number of stores	C
GAR	Number of car spaces in garage (0 = no garage)	D
AGE	Age of dwelling in years	C
CITCOU	1 if dwelling is in Baltimore County, 0 otherwise	D
SQFT	Interior living space in hundreds of square feet	C

$$w_{ij}^{*} = \begin{cases} w_{ij}/max(\lambda), & if\, w_{ij} = 1 \\ 0, & otherwise \end{cases}$$

where, λ is the vector of eigenvalues for the spatial weights matrix W_{ij}.

For a hedonic price model, there exist several function forms such - linear, semi-log and log-log. An incorrect choice of functional form can lead to inconsistent estimates (Bloomquist; Worley (1981), Goodman (1978)). However, there is little to no guidance on the proper choice of functional form. Therefore, we use a linear function form. Next, we specify the equation for the MESS model. It is as following:-

$$e^{\alpha W} PRICE = X\beta + \varepsilon \tag{1}$$

$$PRICE = e^{-\alpha W}(X\beta + \varepsilon) \tag{2}$$

where, X is the matrix of the variables NROOM, DWELL, NBATH, PATIO, FIREPL, AC, BMENT, NSTOR, GAR, AGE, CITCOU and SQFT. And eq (1) is the structural form and eq (2) is the reduced form. Using eq (2), we can also derive the impacts matrix, which is as following:-

$$\frac{\partial(PRICE|X)}{\partial X_k} = e^{-\alpha W}\beta_k \tag{3}$$

Table 2: Data Summary Statistics

	PRICE	NROOM	NBATH	NSTOR	AGE	SQFT
Min.	3.50	3.000	1.000	1.000	0.0	5.76
1st Qu.	30.95	5.000	1.000	2.000	20.0	11.02
Median	40.00	5.000	1.500	2.000	25.0	13.44
Mean	44.31	5.199	1.573	1.905	30.1	16.43
3rd Qu.	53.75	6.000	2.000	2.000	40.0	19.94
Max.	165.00	10.000	5.000	3.000	148.0	47.61

where,

- diagnol elements are the contain direct effects
- off diagnol elements represent the indirect effects
- the sum of the above two are the total effects

As discussed in the introduction that there are some advantages of using a MESS model rather than a SAR. They are as following:-

- The filter of the model is always invertible, i.e., $(e^{\alpha W})^{-1} = e^{-\alpha W}$. Hence, variance-covariance matrix is positive definite, i.e., $VC(y) = e^{-\alpha_0 W}E(\varepsilon\varepsilon')e^{-\alpha_0 W}$, is positive definite ($\alpha_0$ is the true value of α (Lesage, Pace (2003).
- Also, the determinant of this specification is easy to compute: $|e^{\alpha W}| = e^{trace(\alpha_0 W)}$. Since, W has 0 as diagnol elements, hence, $ln|e^{\alpha W}| = 0$ (Lesage, Pace (2003).

4 Results

Using our data, we first need to know if there is any presence of spatial autocorrelation in our dependent variable PRICE. As if there isn't, doing a spatial analysis will not be very helpful. To test for spatial autocorrelation, we conduct a Monte-Carlo Simulations of Moran's I on our dependent variable. The null hypothesis is that there is no presence of spatial autocorrelation. In Table 3, we present the result of this test.

We find that we are able to reject the null hypothesis. And hence, there is presence of spatial autocorrelation in our data.

Table 3: Monte-Carlo simulation of Moran I on PRICE

	Value	
Statistic	0.40129	***

In Table 4, we present the results of our MESS model. Here, we can see that the α is negative and significant. And hence, there is evidence of presence of positive spatial autocorrelation in our data. Also, variables such as NROOM and AGE do not seem to significantly impact the sale price of a house. This might be due to the fact that a house with smaller occupied area but with higher number of rooms might sell for lower that a house with smaller number of rooms but a large occupied area. And the house might be refurbished after some years, therefore, the AGE of a house might not impact its price, atleast for this data.

Most of the variables have a positive effect on the PRICE, other than, NSTOR which seems to have a negative coefficient and a significant one. This is an unexpected sign for this variable as one would expect if the number of stores in the neighbourhood increases, the price of the house should increase as well.

Since, this is a spatial model, hence, the model coefficients are not directly interpretable. Therefore, we have calculated the effects for this model. The direct effects represent the expected change in the PRICE of a house due to a change of one unit in a specific explanatory variable of the same house. The indirect effects represents the expected change in the PRICE of a house due to a change of one unit in the neighbouring houses. And the total effect represents the sum of direct and indirect effect.

For the MESS model, the various effects are calculated in the following manner:-

$$f_{k,direct} = \frac{1}{N} tr(e^{-\alpha W})\beta_k \tag{4}$$

$$f_{k,indirect} = \frac{1}{N} i'_N (e^{-\alpha W})\beta_k i_N - \frac{1}{N} tr(e^{-\alpha W})\beta_k \tag{5}$$

$$f_{k,total} = \frac{1}{n} i'_N (e^{-\alpha W})\beta_k i_N \tag{6}$$

They are presented in the Table 5. Here, we can again see that the various effects of NROOM and AGE seems to be insignificant on price. The impacts of other variables seems to be significant on PRICE. The variable with the highest impacts is CITCOU (if the dwelling is in Baltimore County or not) which might suggest that having the house in the Baltimore county leads to a high increase in price. Houses with patio and fireplaces (PATIO, FIREPL) also seem to have higher impacts on the prices of the houses.

5 Conclusion

In this data, we were able to find positive spatial autocorrelation in the data. And hence, we were able to pursue a spatial analysis. We also estimated a spatial hedonic model but with MESS specification. And found out that the variables CITCOU, PATIO FIREPL have the highest impact on the sale prices of the houses, i.e., the houses which are located in the Baltimore County, have a patio and a fireplace tend to have higher sales prices. We also provide a figure below (fig 1) which graphically shows the effects. This can be helpful in seeing the effects graphically and deciding which variables are more important.

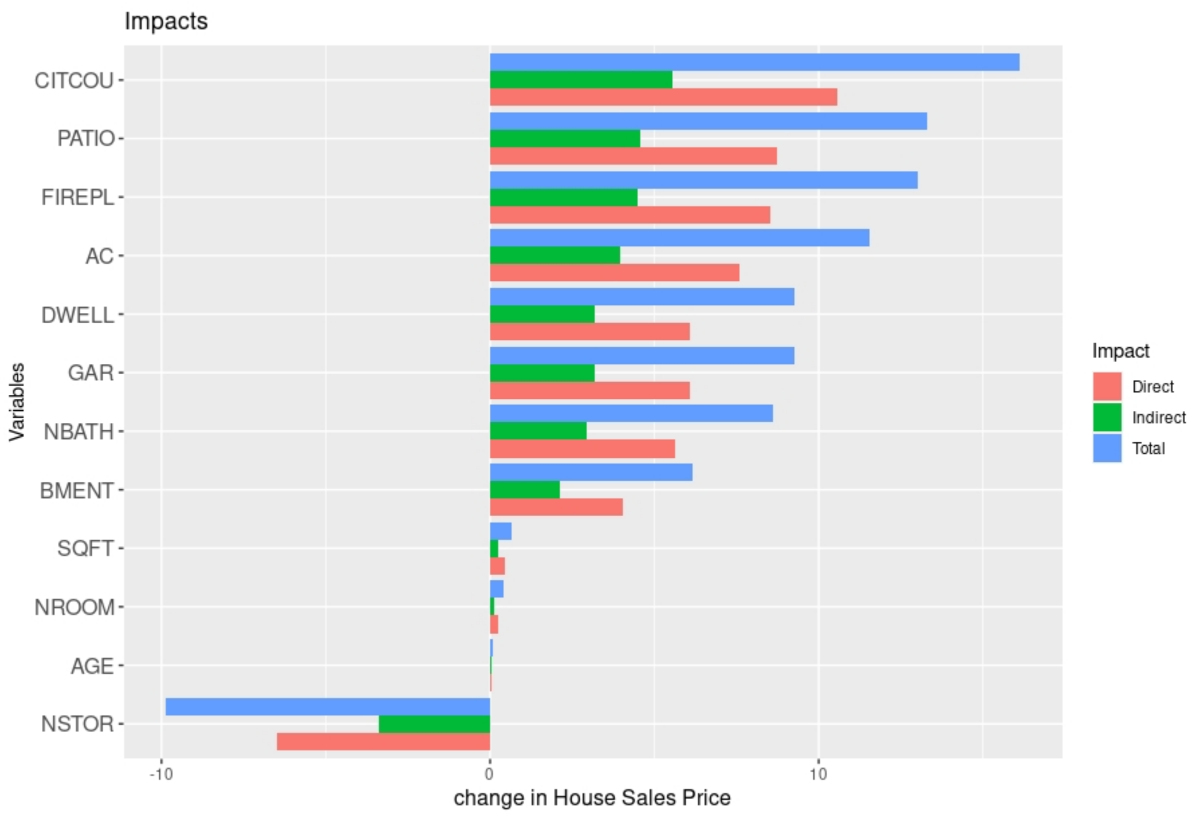

Figure 1: Impacts

6 Bibliography

- Anselin, L. (1988), 'Spatial Econometrics: Methods and Models', Kluwer Academic Publishers, Dordrecht, The Netherlands
- Chau, K. W., Yiu, E. C. Y., Wong, S. K., & Lai, L. W. C. (2002). Hedonic price modelling of environmental attributes: a review of the literature and a Hong Kong case study. Chau, KW, Yiu, CY, Wong, SK, & Lai, LWC (2003). Hedonic price modelling of environmental attributes: a review of the literature and a Hong Kong case study. Understanding and implementing sustainable development, 87-110.
- Bloomquist, G. & Worley, L. (1981). Hedonic prices, demands for urban housing attributes and benefit estimates, Journal of Urban Economics, vol. 9, pp. 212-221.
- Chin, T. L. and Chau, K. W. (2003). A critical review of literature on the hedonic price model, International Journal for Housing and Its Applications 27 (2), 145-165.
- Debarsy, N., Jin, F., & Lee, L. F. (2015). Large sample properties of the matrix exponential spatial specification with an application to FDI. Journal of Econometrics, 188(1), 1-21.
- Lazrak, F., Nijkamp, P., Rietveld, P., & Rouwendal, J. (2014). The market value of cultural heritage in urban areas: an application of spatial hedonic pricing. Journal of Geographical Systems, 16(1), 89-114.
- LeSage, J. P., & Pace, R. K. (2007). A matrix exponential spatial specification. Journal of Econometrics, 140(1), 190-214.
- Goodman, A. C. (1978). Hedonic prices, price indices and housing markets, Journal of Urban Economics, vol. 5, pp. 471-482.
- Gokmenoglu, K., & Hesami, S. (2019). Real estate prices and stock market in Germany: analysis based on hedonic price index. International Journal of Housing Markets and Analysis.
- Golgher, A. B., & Voss, P. R. (2015). How to Interpret the Coefficients of Spatial Models: Spillovers, Direct and Indirect Effects. Spatial Demography, 4(3), 175–205. doi:10.1007/s40980-015-0016-y
- Hussain, T., Abbas, J., Wei, Z., & Nurunnabi, M. (2019). The effect of sustainable urban planning and slum disamenity on the value of neighboring residential property: Application of the hedonic pricing model in rent price appraisal. Sustainability, 11(4), 1144.
- Maurer, R., & Pitzer, M. (2004). Hedonic price indices for the Paris housing market. Allgemeines Statistisches Archiv, 88(3), 303-326.

- Monson, M. (2009). Valuation using hedonic pricing models.
- Páez, A., Long, F., & Farber, S. (2008). Moving window approaches for hedonic price estimation: an empirical comparison of modelling techniques. Urban Studies, 45(8), 1565-1581.
- Dubin, Robin A. (1992). Spatial autocorrelation and neighborhood quality. Regional Science and Urban Economics 22(3), 433-452.
- Rosen, S. (1974). Hedonic prices and implicit markets: Product differentiation in pure competition, Journal of Political Economy, vol. 82, no. 1, pp. 35-55.
- Yang, Z. (2001). An application of the hedonic price model with uncertain attribute-The case of the People's Republic of China. Property management.
- Zhang, Y., & Dong, R. (2018). Impacts of street-visible greenery on housing prices: Evidence from a hedonic price model and a massive street view image dataset in Beijing. ISPRS International Journal of Geo-Information, 7(3), 104.

Table 4: MESS Coefficient Estimates

	Estimates	
(Intercept)	-1.58	
	(6.43)	
NROOM	0.25	
	(1.01)	
DWELL	6.04	***
	(2.34)	
NBATH	5.59	***
	(1.78)	
PATIO	8.66	***
	(2.55)	
FIREPL	8.47	***
	(2.30)	
AC	7.51	***
	(2.24)	
BMENT	4.01	***
	(0.94)	
NSTOR	-6.41	***
	(2.62)	
GAR	6.03	***
	(1.54)	
AGE	0.05	
	(0.05)	
CITCOU	10.49	***
	(2.30)	
SQFT	0.43	***
	(0.19)	
α	-0.43	***
	(0.09)	

Table 5: Effects of MESS Model

	Direct		Indirect		Total	
NROOM	0.255		0.133		0.389	
	(1.022)		(0.535)		(1.014)	
DWELL	6.091	***	3.188	***	9.279	***
	(2.358)		(1.218)		(2.508)	
NBATH	5.643	***	2.954	***	8.598	***
	(1.794)		(0.949)		(1.915)	
PATIO	8.734	***	4.572	***	13.306	***
	(2.577)		(1.385)		(2.741)	
FIREPL	8.544	***	4.472	***	13.017	***
	(2.318)		(1.226)		(2.546)	
AC	7.578	***	3.967	***	11.545	***
	(2.261)		(1.206)		(2.419)	
BMENT	4.051	***	2.121	***	6.173	***
	(0.953)		(0.540)		(1.011)	
NSTOR	-6.468	***	-3.386	***	-9.855	***
	(2.646)		(1.444)		(2.644)	
GAR	6.085	***	3.185	***	9.271	***
	(1.552)		(0.855)		(1.661)	
AGE	0.057		0.029		0.087	
	(0.055)		(0.029)		(0.054)	
CITCOU	10.585	***	5.541	***	16.126	***
	(2.310)		(1.203)		(2.708)	
SQFT	0.436	***	0.228	***	0.665	***
	(0.201)		(0.107)		(0.202)	