

# **Spatial Discrete Choice Models**

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## 1 Introduction

Municipal governments borrow money for crucial municipal services, for ex, for supporting building of new infrastructure or economic development. An important role of the a municipal government investment is “*to help sustain a high quality of local infrastructure*“ (Preston, 1981), which is sometimes financed by borrowing money. However, increasing dependence on municipal debt for development and infrastructure purposes can become a burden on the municipality, since, it will impose constraints on municipality’s operating budget as the creditors must be paid back. Hence, we should study about the determinants of municipal debts so as to know why certain municipalities have high debt levels and if there is any spatial dependence between municipalities regarding the debt.

## 2 Data

The data used in this study is mainly taken from two sources - INSEE and DGCL. The data is for the *Seine-et-Marne* department in France (Fig 1). We use three independent variables to explain municipal debt, namely:-

- PMUN - municipal population
- DIPLMIN - % of the out-of-school population aged 15 and over with no diploma or at most a BEPC, college certificate or DNB
- CFT\_COM - municipal current expenditure

Since, we want to know why some municipalities in the region have high debt levels, our dependent variable will be DETTE\_COM, which will take the following form:-

# Study Region: Seine-et-Marne

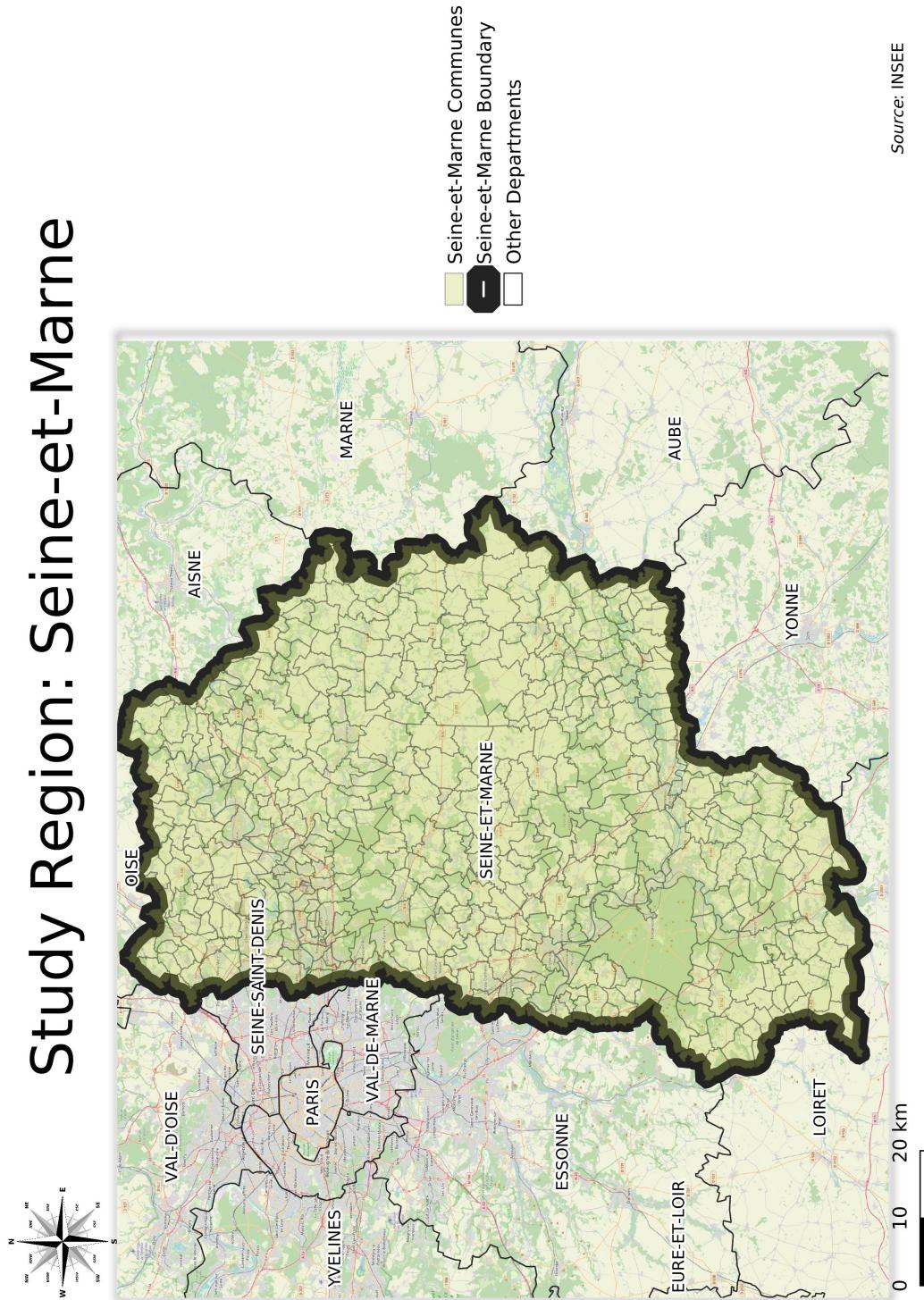


Figure 1: Study Region

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$$DETTE\_COM = \begin{cases} 1, & DETTE\_COM^* > \overline{DETTE\_COM^*} \\ 0, & DETTE\_COM^* \leq \overline{DETTE\_COM^*} \end{cases}$$

where  $DETTE\_COM$  is the latent variable if the municipal debt in euros per capita is higher than its median,

$\overline{DETTE\_COM^*}$  is the median municipal debt in euros per capita in the department

Hence, if value of municipal debt is higher than the median municipal debt in the department, it has a value of 1, otherwise, 0 (Fig 2 and 3).

The department of Seine-et-Marne consists of 514 municipalities. Due to unavailability of the data, we are going to use only 507 of these municipalities. The data selected is for the year of 2014. Below are the maps for Seine-et-Marne department regarding the distribution of different variables (Fig 4, 5 and 6).

### 3 Model

To study the effects of the independent variables on our dependent variables and to study if there is any spatial dependence present in our data we are going to use spatial discrete choice models, more specifically the SAR Probit model. Below is the structural form of the SAR Probit model:-

$$DETTE\_COM^* = \rho \mathbf{W} DETTE\_COM^* + X\beta + \varepsilon, \\ \varepsilon \sim N(0, I_n)$$

where  $X$  contains the independent variables  $PMUN$ ,  $DIPLMIN$  and  $CFT\_COM$ .

The reduced form of the equation is as following:-

$$DETTE\_COM^* = (I_n - \rho \mathbf{W})^{-1} X\beta + (I_n - \rho \mathbf{W})^{-1} \varepsilon$$

## Municipal Debt (euros per capita)

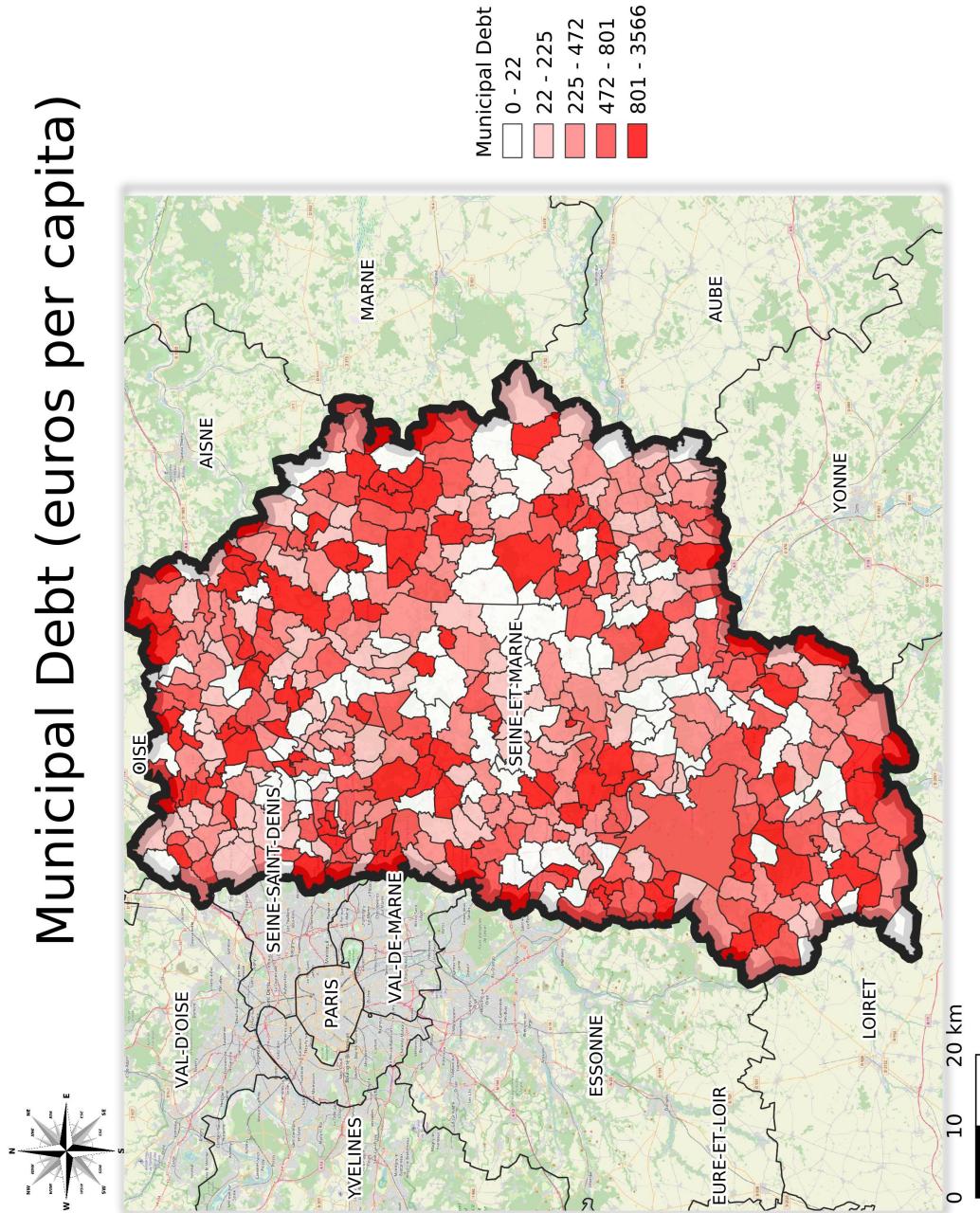


Figure 2: Municipal Debt (euros per capita)

## Municipal Debt (above median or below)

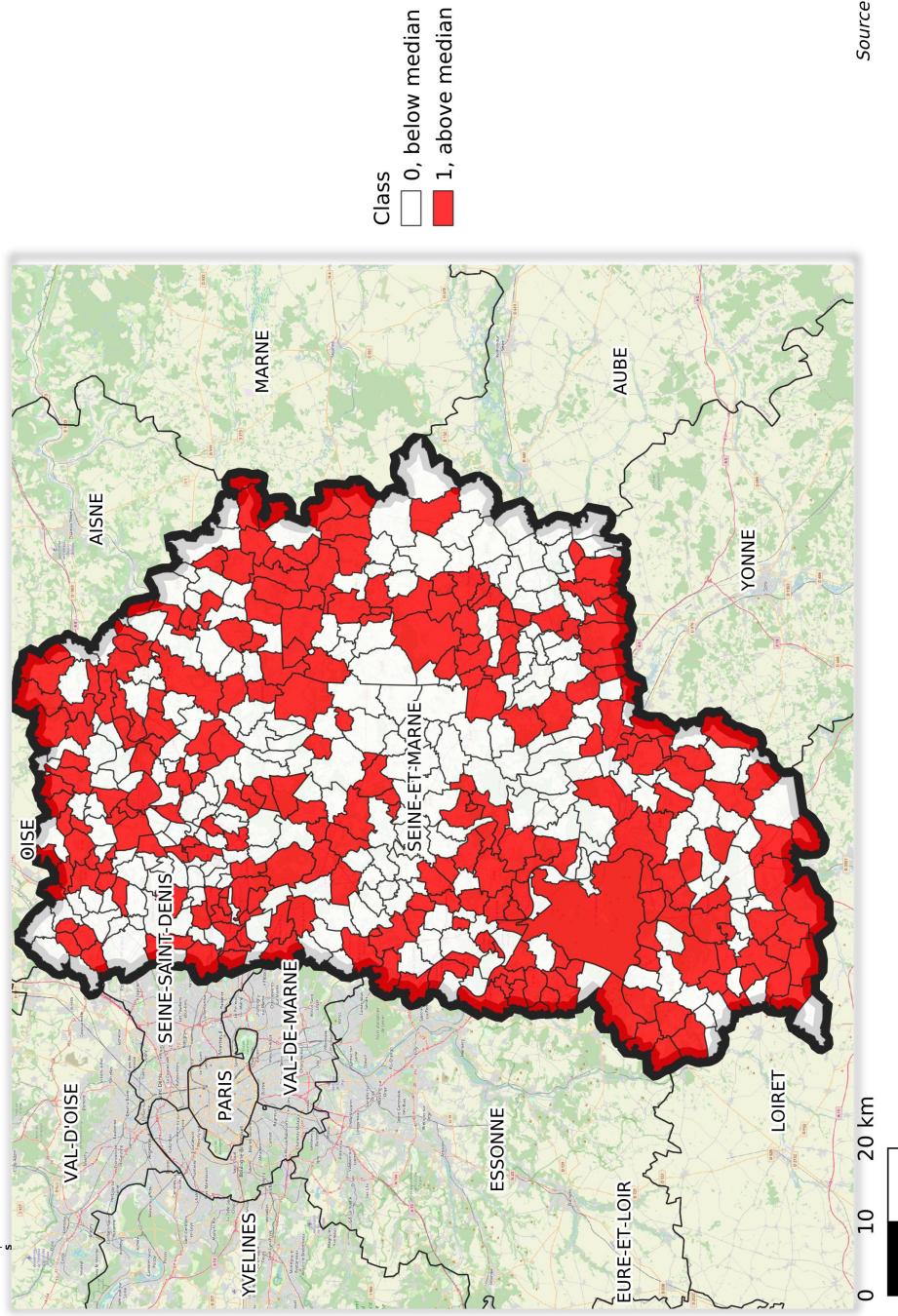


Figure 3: Municipal debt (above or below median)

# Municipal Population

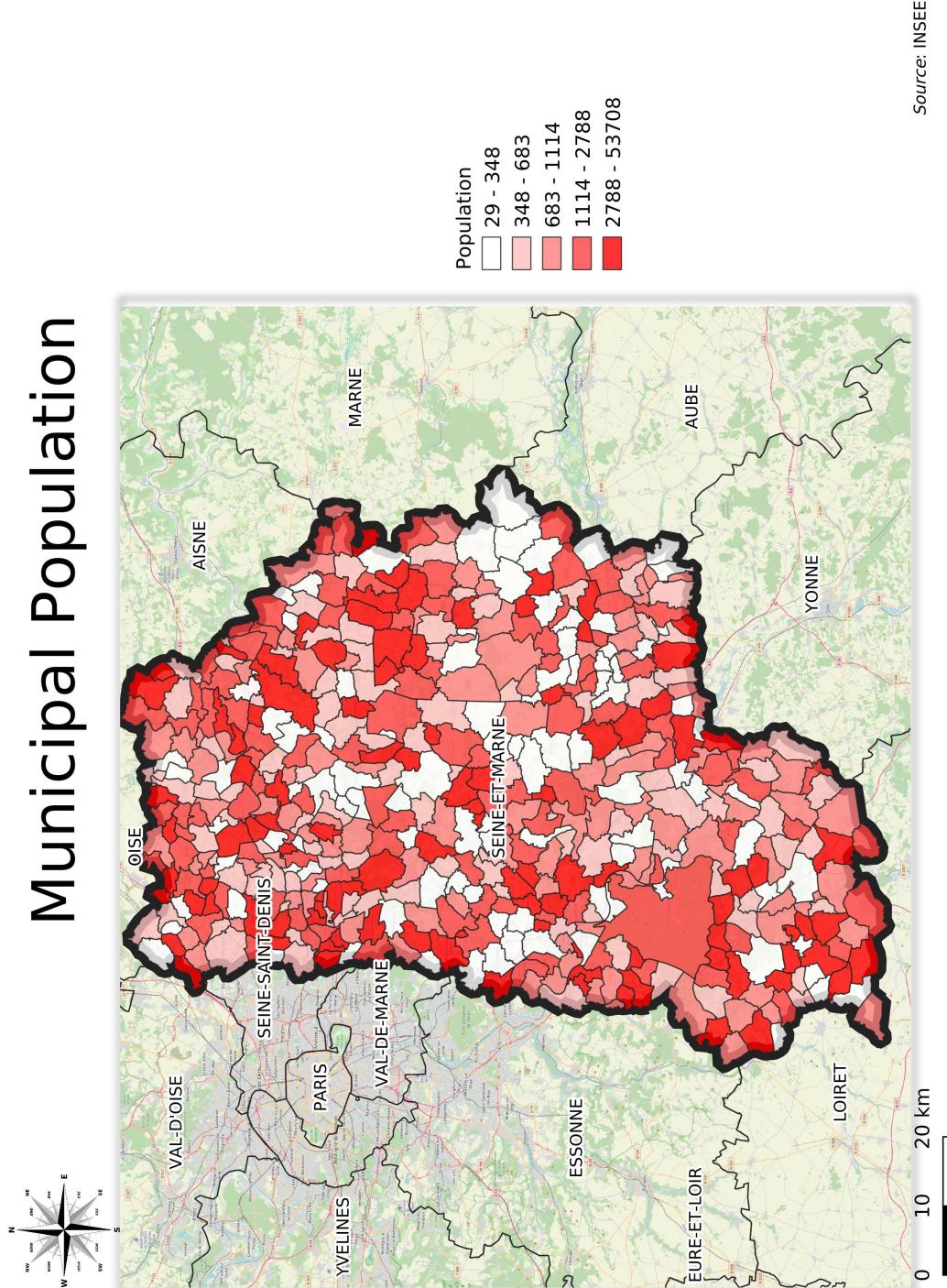


Figure 4: Municipal Population

# % of People > 15 years with no Diploma

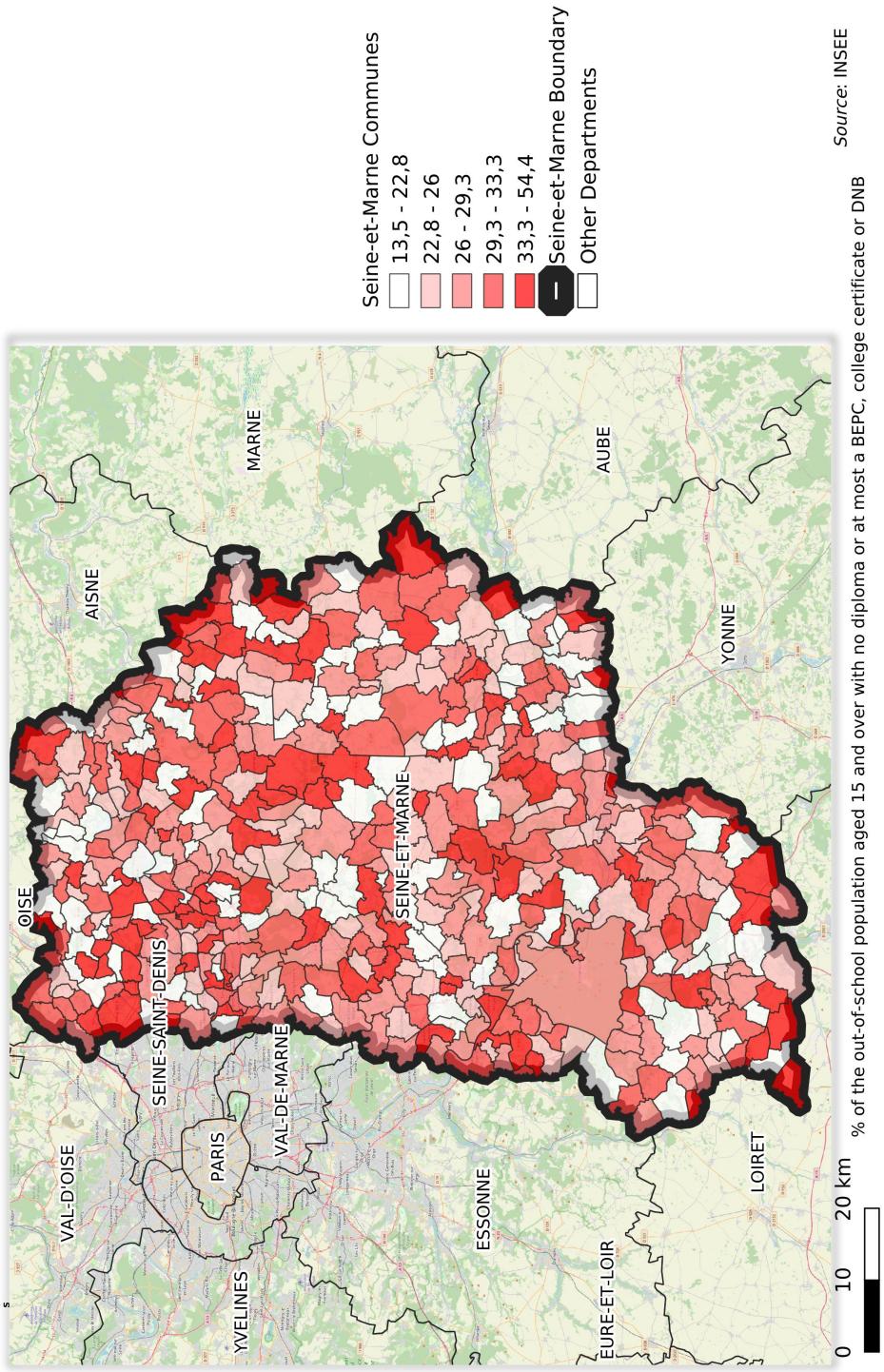


Figure 5: % of People  $\geq 15$  years with no Diploma

# Capital Expenditure (euros per capita)

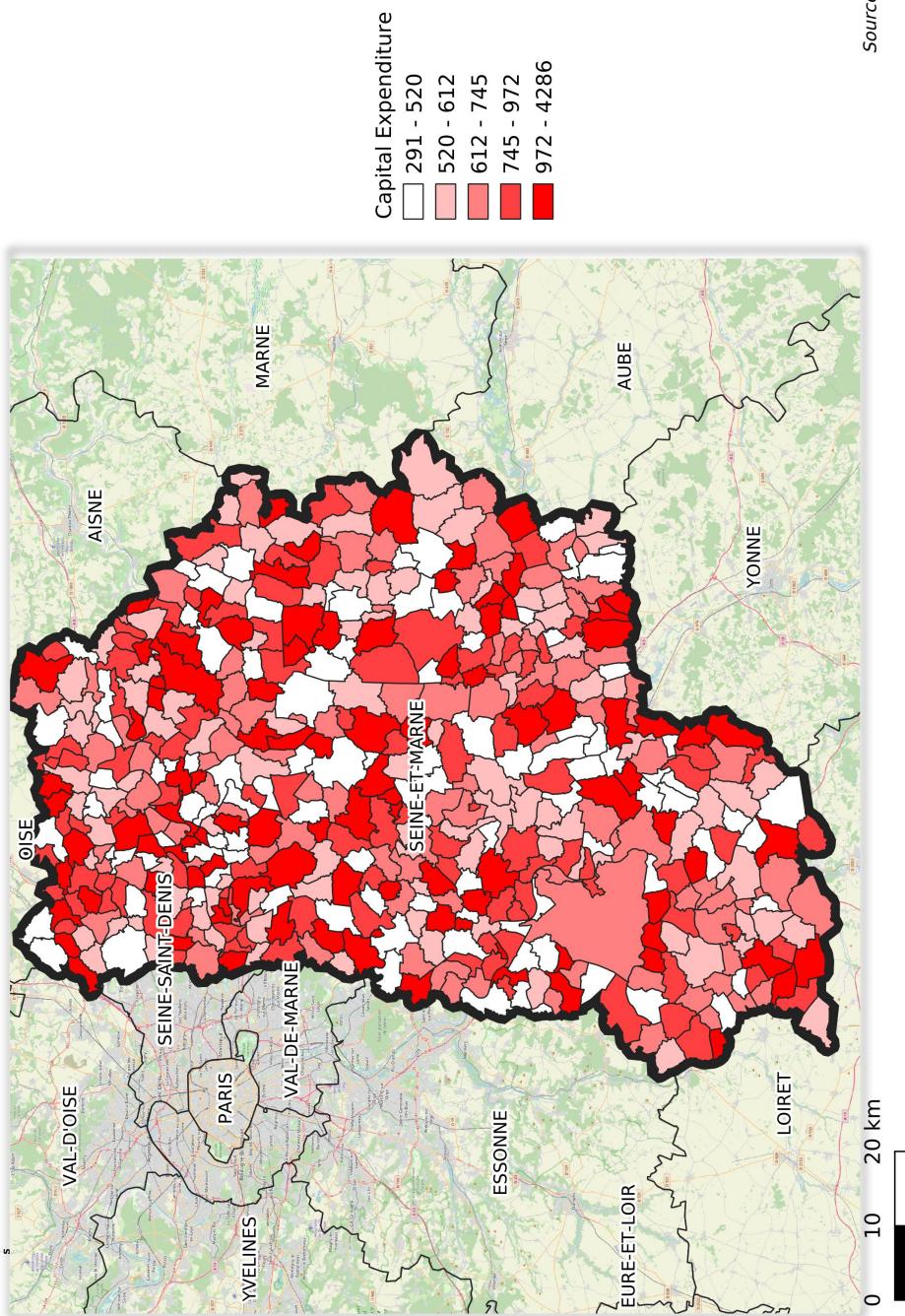


Figure 6: Capital Expenditures

Source: INSEE

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Estimation of the model is done through using Bayesian Markov Chain Monte Carlo method. We try to sample from a posterior distribution of the model parameters  $p(DETTE\_COM^*, \beta, \rho | DETTE\_COM)$  given the data  $DETTE\_COM$  and prior distributions  $p(\rho)$ ,  $p(\beta)$  and  $p(DETTE\_COM^*)$ .

## 4 Results

Table 1 shows the different models that we have estimated in our study. The first model (Model 1) is a simple Spatial Lag Probit Model which is also estimated using Bayesian MCMC method. Rest others are SAR Probit Models. Also the *burn-in* value for all the models is **15000**. The (*Intercept*) in most of the models (Model 1, 2, 4 and 8) is insignificant. It is only significant in Models 6 and 7. *CFT\\_COM* (Capital Expenditures) are also insignificant in the two models (Model 6 and 8). However, *PUMN* (Municipal Population) is highly significant in many of the models (Model 2, 3, 4, 5 and 7) and has roughly the same values all over. *DIPLMIN* (% of the out-of-school population aged 15 and over with no diploma) seem to be significant only at 95% confidence level in Model 2 and 4. However, it is quite significant in Models 3 and 5, i.e., models which do not have an intercept term. According to the analysis of these coefficients in the many models, it does seems that sometimes *DIPLMIN* and *CFT\\_COM* are significant and sometimes they are not so. But their signs never change. *DIPLMIN* is always negative and *CFT\\_COM* is always positive. Hence, there is some evidence that these variables do affect our dependent variable. Since, this is a discrete choice model, we will have to look at the marginal effects to see the extent to which these variables affect our dependent variable (Table 2).

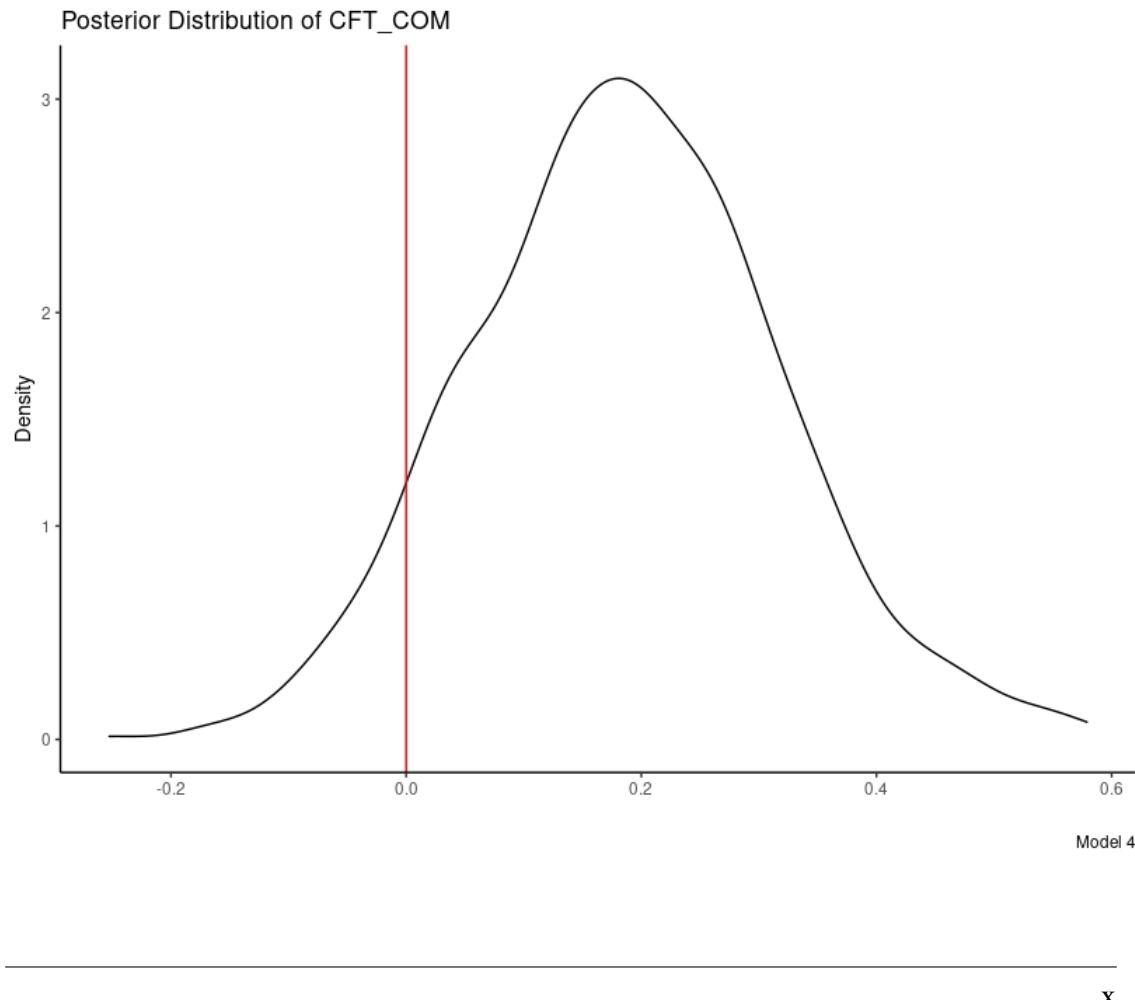
Now, when we look at the  $\rho$  coefficient, which is our spatial dependence parameter, it is always and only significant at the

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95% confidence level and always positive (as well as less than 1). Hence, we are also able to find evidence that the decisions of our neighbours do affect us. Hence, if a location's neighbours tend to have high municipal debt (low municipal debt), the probability that the location has high municipal debt (low municipal debt) also increases.

Since, all the variables do seem to have an impact on our dependent variable, hence, we will present the marginal effects for Model 4 which takes into account all the variables in the Table 2. For the variable *CFT\_COM* we cannot not be really sure of its impact since the signs of the marginal effects change between Lower 5% and Upper 95%.

Figure 7: Model 4: Posterior Distribution of *CFT\_COM*



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In Figure 7, we can see the posterior distribution of *CFT\_COM* and we can see that the distribution does consists of 0 not in the middle but not far away from it. For the variable *PMUN*, all the effects are positive and, hence, we can conclude that the coefficient has converged. Therefore, there is an increase in the probability of a municipality having debt more than the departmental median if there is an increase in the *PMUN* (municipal population) of that municipality (direct effect). And there is increase in this probability even if there is an increase in *PMUN* of a neighbouring region (indirect effect), although this is quite small compared to direct effect.

Also, for the variable *DIPLMIN*, increasing it leads to an increase in the probability that a municipality will have municipal debt higher than departmental median. But its indirect effect cannot be gauged since, it changes sign in the upper 95% of the distribution. Hence, we cannot be sure about its indirect effect. However, its total effect seems to be negative.

## 5 Conclusions

In this project, we studied the spatial dependence of if a municipality has high municipal debt using the data for the department of Seine-et-Marne through the application of SAR Probit models. Although, there was not a decisive evidence on the significance of some coefficients (*DIPLMIN* and *CFT\_COM*), looking at the Impacts, we were able to see that the effect of municipal current expenditure on whether a municipality has higher debt or not was not very significant. We were, although, able to see that there is spatial dependence in our data.

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Table 1: SAR Probit MCMC Models

	Model 1	Model 2	Model 3
(Intercept)	-0.0067 (0.0483)	0.06807 (0.2659)	
$\rho$	0.2366* (0.0956)	0.1499* (0.0740)	0.1554* (0.0768)
<i>PMUN</i>		0.0002*** (0.00003)	0.0002*** (0.00003)
<i>DIPLMIN</i>		-0.01660 . (0.0091)	-0.01427*** (0.0027)
	Model 4	Model 5	Model 6
(Intercept)	0.0222 (0.2863)		-0.27789*** 0.07599
$\rho$	0.1597* (0.0770)	0.1644* (0.0757)	0.23605** 0.09052
<i>PMUN</i>	0.0002*** (0.000036)	0.0002*** (0.00003)	
<i>DIPLMIN</i>	-0.01672 . (0.0096)	-0.0165*** (0.0028)	
<i>CFT_COM</i>	0.1864 (0.1299)	0.1828 (0.1186)	0.55538*** 0.11271
	Model 7	Model 8	
(Intercept)	-0.4618*** (0.0845)	-0.0202 (0.2621)	
$\rho$	0.1543* (0.0764)	0.2348* (0.1037)	
<i>PMUN</i>	0.0002*** (0.00003)		
<i>DIPLMIN</i>		-0.0090 (0.0090)	
<i>CFT_COM</i>	0.1931 (0.1220)	0.5426*** (0.1095)	

Signif codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

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<i>Direct Effects</i>			
	Lower 5%	Posterior Mean	Upper 95%
<i>PMUN</i>	0.00004	0.00006	0.00008
<i>DIPLMIN</i>	-0.0113	-0.0058	-0.0005
<i>CFT_COM</i>	-0.0082	0.0649	0.1388

<i>Indirect Effects</i>			
	Lower 5%	Posterior Mean	Upper 95%
<i>PMUN</i>	0.000003	0.000012	0.000020
<i>DIPLMIN</i>	-0.00310	-0.00120	0.00002
<i>CFT_COM</i>	-0.0013	0.0133	0.0375

<i>Total Effects</i>			
	Lower 5%	Posterior Mean	Upper 95%
<i>PMUN</i>	0.00005	0.00007	0.00010
<i>DIPLMIN</i>	-0.0139	-0.0070	-0.0006
<i>CFT_COM</i>	-0.0089	0.0781	0.1714

Table 2: Impacts