

Expected Value

E[c] = c  
E[cX] = cE[X]  
E[X1 + X2...Xn] = ΣE[Xi]

E[g(X)] = Σg(x) · pmf(x)  
E[g(X)] = ∫ g(x) · pdf(x)dx

E[g(X,Y)] = ΣΣg(x,y) · pmf(x,y)  
E[g(X,Y)] = ∫ ∫ g(x,y) · pdf(x,y)dxdy

Given X1,...,Xn are independent then  
E[(X1)(...)(En)] = (E[X1])(...)(E[Xn])

Marginal

$$(n-1)\frac{S^2}{\sigma^2} \approx \chi^2$$

$$f_x = \int f(X,Y)dy$$

$$f(X|Y) = \frac{f(X,Y)}{f_y(Y)}$$

Given X and Y are independent then

$$\dots f(X,Y) = f_x(X) \cdot f_y(Y)$$

Chebyshev’s inequality

Within k Standard Deviations  
there is at least  $1 - \frac{1}{k^2}$  of the data

Conditional Expectation

E[g(X)|Y = y] = ∫ g(X)f(X|Y)dx  
E[g(X)|Y = y] = Σg(X)f(X|Y)

Sampling Distribution

$$\bar{Y} = \frac{1}{n}\Sigma y_i$$

$$E[\bar{Y}] = \mu \text{ and } V[\bar{Y}] = \frac{\sigma^2}{n}$$

If Y’s are taken from a normal, then  $\bar{Y}$  is normal. Else n must be large to be normal

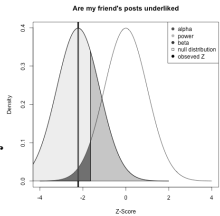
$$Z = \frac{\bar{Y} - \mu_{\bar{Y}}}{\frac{\sigma_{\bar{Y}}}{\sqrt{n}}} = \sqrt{n}(\frac{\bar{Y} - \mu}{\sigma})$$

Likelyhood

$L(\theta|y_1...y_2)$   
 $\Pi pdf(y_i)$

Confidence Interval

$\hat{\theta} \pm Z * StandardError$   
 $SE = \sqrt{Var}$   
Z can sometimes be T



Variance

Var[c] = 0  
Var[cX] = c²Var[X]  
Var[X1 + X2...Xn] = ΣΣCov(Xi, Xj)  
.... = ΣVar[Xi] + 2(ΣΣCov(Xi, Xj))  
If X1,...,Xn are independent then  
Var[X1,...,Xn] = ΣVar[Xi]

Var = σ² = E[X²] - E[X]²  
SE = Standard Error = √Var

CoVariance

cov(X,Y) = E[(X - E[X])(Y - E[Y])]  
.....= E[XY] - E[X]E[Y]  
It follows that cov(X, X)=Var(X)  
And that if X and Y are independent  
cov(X,Y) = 0

Correlation

$\rho = \frac{cov(X,Y)}{\sqrt{var(X) \cdot var(Y)}}$  and  $-1 \leq \rho \leq 1$

Order Statistics

If Y1...Yn iid with CDF F and pdf f then  
 $g^{(k)} = \frac{n!}{(k-1)!(n-k)!} F(y)^{k-1} (1 - F(y))^{n-k} f(y)$   
 $G_{(n)}(y) = P(\dots Y_{n-1} \leq y, Y_n \leq y)$   
 $G_{(1)}(y) = P(y \leq Y_1, y \leq Y_2, \dots)$

Don’t Forget!

$P(Y \leq y) = CDF(y)$   
 $P(y \leq Y) = 1 - CDF(y)$

Bias

$B[\hat{\theta}] = E[\hat{\theta}] - \theta$   
 $MSE[\hat{\theta}] = E[(\hat{\theta} - \theta)^2]$   
 $MSE[\hat{\theta}] = V[\hat{\theta}] + B[\hat{\theta}]^2$

$error = \int_{\theta-b}^{\theta+b} pmf(\hat{\theta})d\hat{\theta}$  which is then bounded using distributions  
Unbiased if  $E[\hat{\theta}] = \theta$

Consistent

Consistent if  $\lim_{n \rightarrow \infty} V[\theta] = 0$

Type	pmf	mean	variance	mgf
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	$np$	$np(1-p)$	$((1-p) + pe^t)^n$
Negative	$\binom{Y-1}{r-1} p^r (1-p)^{Y-r}$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{Pe^t}{1-(1-p)e^t}\right)^r$
Hyper	$\frac{\binom{y}{r} \binom{N-r}{n-y}}{\binom{N}{n}}$	$\frac{nr}{N}$	$\left(\frac{nr}{N}\right) \left(\frac{(N-r)(N-n)}{N(N-1)}\right)$	DNE
Geometric	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Poisson	$e^{-\lambda} \frac{\lambda^y}{y!}$	$\lambda$	$\lambda$	$e^{\lambda(e^t-1)}$

Type	pdf	mean	variance	mgf
Chi-Square	$\frac{y^{\frac{v}{2}-1} e^{-\frac{y}{2}}}{2^{v/2} \Gamma(\frac{v}{2})}$	$v$	$2v$	$(1-2t)^{-\frac{v}{2}}$
Exponential	$\frac{1}{\theta} e^{-\frac{x}{\theta}}$	$\theta$	$\theta^2$	$\frac{1}{1-\theta t}$
Gamma	$\frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^\alpha$
Uniform	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\mu$	$\sigma^2$	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$

Point Estimators				
Parameter	Sample Size	$\hat{\theta}$	$E[\hat{\theta}]$	SE
$\mu$	$n$	$\bar{Y}$	$\mu$	$\frac{\sigma}{\sqrt{n}}$
$p$	$n$	$\hat{p} = \frac{Y}{n}$	$p$	$\sqrt{\frac{pq}{n}}$
$\mu_1 + \mu_2$	$n_1 + n_2$	$\bar{Y}_1 - \bar{Y}_2$	$\mu_1 - \mu_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
$p_1 - p_2$	$n_1 + n_2$	$\hat{p}_1 - \hat{p}_2$	$p_1$	$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

Hypothesis Testing			
Hypothesis	Sample Stat	Distribution	
$H_0 : \mu_0 = \mu_a \quad H_a : \mu_0 \neq \mu_a$	$\bar{x}$	$Z = \frac{\bar{x} - \mu_a}{\frac{s}{\sqrt{n}}}$	
$H_0 : \mu_1 = \mu_2 \quad H_a : \mu_1 \neq \mu_2$	$\bar{x}_1 - \bar{x}_2$	$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	
$H_0 : p_0 = p_a \quad H_a : p_0 \neq p_a$	$\hat{p}$	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	
$H_0 : p_1 = p_2 \quad H_a : p_1 \neq p_2$	$\hat{p}_1 - \hat{p}_2$	$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{p_p(1-p_p) \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	

MLE

Find Likelyhood  
Find the minimum of this  
If it is not obvious take the log  
Set to 0  
Solve for  $\theta$  and call it  $\hat{\theta}$

Neyman–Pearson Lemma

$\frac{L(\theta_0)}{L(\theta_a)} < k$  so  $RR : \{y < k^*\}$   
where  $k^*$  is found in  
 $\alpha p_h \alpha = P(Y < k^* | \theta = \theta_a)$