# Com S 330 HW01

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September 2, 2014

1. Rosen, Section 1.1: Exercise 10 (b) (d) (h)
(b) $p \vee q$ (d) $q \rightarrow p$ (h) $\neg q \vee (\neg p \wedge q)$

Solution:
$p$: The election is decided
$q$: The votes have been counted
(b) The election is decided or the votes have been counted.
(d) The votes have counted implies that the election is decided.
(h) The votes have not been counted, or the election is not decided and the votes have been counted.

2. Rosen, Section 1.1: Exercise 12 (c) (e)
(c) $q \rightarrow \neg r$ (e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

Solution:
$p$: You have the flu.
$q$: You miss the final exam.
$r$: You pass the course.
(c) You miss the final exam implies that you don't pass the course.
(e) You have the flu implies that you don't pass the course, or you miss the exam implies that you don't pass the course.

3. Rosen, Section 1.1: Exercise 24 (b) (h)
(b) To be a citizen of this country, it is sufficient that you were born in the United States.
(h) You will reach the summit unless you begin your climb too late.

Solution:
(b) If you were born in the United States, you are to be a citizen of this country.
(h) If you don't begin your climb too late, you will reach the summit.

4. Rosen, Section 1.1: Exercise 28 (b)
(b) I go to the beach whenever it is a sunny summer day.

Solution:
Converse: If I go to the beach, then it is a sunny summer day.

Inverse: If it is not a sunny summer day, then I don't go to the beach.
Contrapositive: If I don't go to the beach, then it is not a summer sunny day.

5. Rosen, Section 1.1: Exercise 34 (f)
(f) $(p \oplus q) \wedge (p \oplus \neg q)$

Solution:
Truth table

| $p$ | $q$ | $p \oplus q$ | $p \oplus \neg q$ | $(p \oplus q) \wedge (p \oplus \neg q)$ |
|---|---|---|---|---|
| F | F | T | T | T |
| F | T | F | F | F |
| T | F | F | F | F |
| T | T | T | T | T |

6. Rosen, Section 1.2: Exercise 18
18. When planning a party you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attend only if Kanti will be there, and Kanti will not attend unless Jasmine also does. Which combinations of these three friends can you invite so as not to make someone unhappy?

Solution:

1. $p$: Jasmine attends the party.
2. $q$: Samir attends the party.
3. $r$: Kanti attends the party.
4. $p \wedge q \rightarrow$ Jasmine will not be happy. If Jasmine and Samir attend the party, Jasmine will become unhappy.
5. $q \rightarrow r$: If Samir will attend, then Kanti will be there.
6. $\neg p \rightarrow \neg r$: If Jasmine does not attend, then Kanti will not attend.

By 4 we have known that if 1 and 2 are both true, then Jasmine will not be happy. So if one of them is false, Jasmine will be happy. If 1 is false, then from 6, we know that 3 is false. So we can only invite Samir. But by 5, since Kanti will not be there, Samir will not be there either, i.e, 2 is false. The result is none of three will attend. If Samir does not attend the party, i.e, 2 is false, we can make 1 or 3 true. By 6, we know that if 3 is true, then 1 is true. And no one is unhappy. So we can invite Jasmine, Kanti or both of them.

7. Rosen, Section 1.2: Exercise 24
A says C is the knave, B says, A is the knight, and C says I am the spy.
Solution:

1. Only one of them is the knight, one of them is the knave and one of them is the spy.
2. p$\rightarrow \neg$r: C is the knave.
3. q$\rightarrow$p: A is the knight.
4. r$\rightarrow$ ((p$\wedge\neg$q) $\vee$ (q$\wedge\neg$p)): C is the spy.
5. $p$: A tells the truth.
6. $q$: B tells the truth.
7. $r$: C tells the truth.

By 1, we know that at least of them tells the true, so we can assume that one of 2, 3 and 4 is true.

Assume that 2 is true, we know that 7 is false, and A is either the knight or the spy and C is the knave. And by 1, we know that A is not the spy. Because if A is the spy, then B is the knight and 6 is true, which contradicts with 3. The result is A is the knight, B is the spy and C is the knave.

Assume that 3 is true, we know that A is the knight and B is the spy. So C is the knave. The result does not contradict with 1.

Assume that 4 is true, we know that C is the spy. So 2 is false. By 1, we know that A is the knave and B is the knight, so 3 is true, which contradicts with the fact that A is the knave.

So we know that A is the knight, B is the spy and C is the knave.

8. Rosen, Section 1.2: Exercise 34

Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

Solution:

1. $p$: Kevin is chatting.
2. $q$: Heather is chatting.
3. $r$: Randy is chatting.
4. $s$: Vijay is chatting.
5. $t$: Abby is chatting.
6. $p \vee q$
7. $(r \wedge \neg s) \vee (\neg r \wedge s)$
8. $t \rightarrow r$
9. $(p \wedge s) \vee (\neg p \wedge \neg s)$
10. $q \rightarrow t \wedge p$

By 8, we have $r \vee \neg t$. By 10, we have $\neg q \vee (p \wedge t)$.

By 8, we know that $t$ is false or $r$ is true, so we assume that $t$ is false. Here we get $t \wedge p$ is false. By 10, we know that $q$ is false. By 6, $p$ is true, so $\neg p \wedge \neg s$ is false. By 9, we know that $s$ is true, so $r \wedge \neg s$ is false. By 7, $\neg r$ is true. So we have Kevin is chatting, Heather is not chatting, Randy is not chatting, Vijay is chatting and Abby is not chatting.

We can also assume that $r$ is true. The $\neg r \wedge s$ is false. By 7, $\neg s$ is true. So $p \wedge s$ is false. By 9, $\neg p$ is true, so $p$ is false. As a result, $t \wedge p$ is false, and by 10, $q$ is false. So $p$ and $q$ are false, which contradicts with 6. The assumption is wrong.

According to the above inference, we have that Kevin is chatting, Heather is not chatting, Randy is not chatting, Vijay is chatting and Abby is not chatting.

9. There are two kinds of people in the land of Paradox: knights and knaves. Knights always tell the truth and knaves always lie.There is only one direction to go to get out of the land of Paradox, either North or South. Bill is a resident of Paradox and you say to him $L \oplus N$ where $L$ means "you, Bill, are a liar" and $N$ means "the direction out of Paradox is North." Draw a truth table to show that North is the direction out of Paradox if and only if Bill claims what you told him is true. (Remember, you start out with no idea whether Bill is a liar or truth-teller.)

Solution:

$$N \to L \oplus N \equiv \neg N \vee [L \oplus N]$$

$$L \oplus N \to N \equiv \neg[L \oplus N] \vee N$$

Truth table

| $L$ | $N$ | $L \oplus N$ | $N \to L \oplus N$ | $L \oplus N \to N$ |
|---|---|---|---|---|
| T | T | F | F | T |
| T | F | T | T | F |
| F | T | T | T | T |
| F | F | F | T | T |

By the truth table, we know that when $L$ is false and $N$ is true the direction out of Paradox if and only if $L \oplus N$ is true. So $L$ is false, which means that Bill is not a liar.

10. Rosen, Section 1.3: Exercise 26 Use truth tables

26. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent

Solution:

Truth table

| $p$ | $q$ | $r$ | $\neg p \to (q \to r)$ | $q \to (p \vee r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | T | F | F | F |
| F | F | T | T | T |
| F | F | F | T | T |

11. Prove that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent, (a) by truth tables, and (b) by deduction using the logical equavalences studied in class.

Solution:

(a) Truth table

| $p$ | $q$ | $r$ | $(p \to q) \wedge (p \to r)$ | $p \to (q \wedge r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | T | F | T | T |
| F | F | T | T | T |
| F | F | F | T | T |

(b) Deduction

$$[p \to q] \wedge [p \to r] \equiv [\neg p \vee q] \wedge [\neg p \vee r] \equiv \neg p \vee [q \wedge r] \tag{1}$$

$$p \to [q \wedge r] \equiv \neg p \vee [q \wedge r] \tag{2}$$

By 1 and 2, we know that they are logically equivalent.

12. The proposition $p$ NOR $q$ is true when both $p$ and $q$ are false, and false otherwise. Draw a truth table for NOR based on the definition given. Now, show that $p$ NOR $q$ is logically equivalent to $\neg(p \vee q)$. Then prove that {NOR} is functionally complete.

Solution:

Truth table

| $p$ | $q$ | $p$ NOR $q$ | $\neg(p \vee q)$ |
|---|---|---|---|
| T | T | F | F |
| T | F | F | F |
| F | T | F | F |
| F | F | T | T |