Com S 330 HW02

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1. Rosen, Section 1.4: Exercise 10 (a) (b) (d)
   Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$,$D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
   a) A student in your class has a cat, a dog, and a ferret.
   b) All students in your class have a cat, a dog, or a ferret.
   d) No student in your class has a cat, a dog, and a ferret.

2. Rosen, Section 1.4: Exercise 14 (a) (d)
   Determine the truth value of each of these statements if the domain consists of all real numbers.
   a) $\exists x(x^3 = -1)$
   d) $\forall x(2x > x)$

3. Rosen, Section 1.4: Exercise 24 (a) (c)
   Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
   a) Everyone in your class has a cellular phone.
   c) There is a person in your class who cannot swim.

4. Rosen, Section 1.4: Exercise 46 (b)
   Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
   b)$(\exists x P(x)) \vee A \equiv \exists x(P(x) \vee A)$

5. Are $\forall x(P(x) \rightarrow Q(x))and\forall x P(x) \rightarrow Q(x)$logically equivalent?If yes, give a proof. If no, give a counterexample.

6. Rosen, Section 1.5: Exercise 10 (a) (d) Let $F(x, y)$ be the statement x can fool y, where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a) Everybody can fool Fred.
   d) There is no one who can fool everybody.

7. Rosen, Section 1.5: Exercise 12 (i) (n)
   Let $I(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.
   i) Everyone except one student in your class has an Internet connection.
   n) There are at least two students in your class who have not chatted with the same person in your class.

8. Rosen, Section 1.5: Exercise 36 (d)
   Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")
   d) Some student has solved every exercise in this book.

9. Lehman et al. Problem 3.32
   Some students from a large class will be lined up left to right. There will be at least two stduents in the line. Translate each of the following assertions into predicate formulas with the set of students in the class as the domain of discourse. The only predicates you may use are

   - equality and,

- $F(x, y)$, meaning that "x is somewhere to the left of y in the line." For example, in the line "CDA" both $F(C, A)$ and $F(C, D)$ are true. Once you have defined a formula for *a* predicate P you may use the abbreviation "P" in further formulas.

(a) Student x is in the line.

(b) Student x is first in line.

(c) Student x is immediately to the right of student y.

(d) Student x is second.

10. Define predicates and prove the following using the appropriate rules of inference:

    (a) Beth, an ISU student, visited Brazil this summer. Everyone who visited Brazil this summer watched the World Cup. Therefore, an ISU student watched the World Cup.

    (b) Some music majors are also computer science majors. Every music major can play the piano. There is a computer science major who can play the piano.

11. Consider the following argument:
    Every computer science major takes discrete mathematics. Anyone who takes discrete mathematics understands logic. No one who understands logic will lose arguments. Therefore, no computer science majors will lose arguments.

    (a) Prove the argument using the rules of inference in Tables 1 and 2 of the book.

    (b) Prove the **universal transitivity** rule, which states that if $\forall x(P(x) \rightarrow Q(x))$ and $\forall x(P(x) \rightarrow R(x))$

    (c) Now, prove the previous argument using the **universal transitivity** rule.

12. State whether the following arguments are correct. Explain your answer briefly.

    (a) All freshmen live in the dorms. Joe is not a freshman. Therefore, Joe does not live in the dorms.

    (b) Ben likes all comedies. Ben likes Hunger Games. Therefore, Hunger Games is a comedy.