Com S 330 HW01

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Recitation Section 1
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1. Rosen, Section 1.1: Exercise 10 (b) (d) (h)
Solution:
p: The election is decided
q: The votes have been counted
(b) The election is decided or the votes have been counted.
(d) The votes have counted implies that the election is decided.
(h) The votes have not been counted, or the election is not decided and the votes have been counted.

2. Rosen, Section 1.1: Exercise 12 (c) (e)
Solution:
p: You have the flu.
q: You miss the final exam.
r: You pass the course.
(c) You miss the final exam implies that you don't pass the course.
(e) You have the flu implies that you don't pass the course, or you miss the exam implies that you dont pass the course.

3. Rosen, Section 1.1: Exercise 24 (b) (h)
Solution:
(b) If you were born in the United States, you are to be a citizen of this country.
(h) If you don't begin your climb too late, you will reach the summit.

4. Rosen, Section 1.1: Exercise 28 (b)
Solution:
Converse: If I go to the beach, then it is a sunny summer day.
Inverse: If it is not a sunny summer day, then I do not go to the beach.
Contrapositive: If I do not go to the beach, then it is not a summer sunny day.

5. Rosen, Section 1.1: Exercise 34 (f)
Solution:
Truth Table:

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
F	F	T	T	T
F	T	F	F	F
T	F	F	F	F
T	T	T	T	T

By the truth table, $p \oplus \neg q$ and $(p \oplus q) \wedge (p \oplus \neg q)$ are logically equivalent.

6. Rosen, Section 1.2: Exercise 18
Solution:

1. p: Jasmine attends the party.
2. q: Samir attends the party.
3. r: Kanti attends the party.
4. $(p \wedge q) \rightarrow$ Jasmine will not be happy. If Jasmine and Samir attend the party, Jasmine will become unhappy.
5. $q \rightarrow r$: If Samir will attend, then Kanti will be there.
6. $\neg p \rightarrow \neg r$: If Jasmine does not attend, then Kanti will not attend.

By 4 we have known that if 1 and 2 are both true, then Jasmine will not be happy. So if one of them is false, then Jasmine will not be unhappy. If 1 is false, then from 6, we know that 3 is false. So we can only invite Samir. But by 5, since Kanti will not be there, Samir will not be there either, i.e., 2 is false. The result is none of three will attend. If Samir does not attend the party, i.e., 2 is false, we can make 1 or 3 true. By 6, we know that if 3 is true, then 1 is true. And no one is unhappy. So we can invite one of Jasmine and Kanti or both of them.

7. Rosen, Section 1.2: Exercise 24
Solution:

1. p: A tells the truth.
2. q: B tells the truth.
3. r: C tells the truth.
4. Only one of them is the knight, one of them is the knave and one of them is the spy.
5. By 4, $p \to \neg r$: C is the knave.
6. By 4, $q \to p$: A is the knight.
7. By 4, $r \to ((p \wedge \neg q) \vee (q \wedge \neg p))$: C is the spy.

By 4, we know that at least one of them tells the truth, so we can assume that one of 5, 6 and 7 is true.

Assume that 5 is true, we know that 3 is false, and A is either the knight or the spy and C is the knave. And by 4, we know that A is not the spy. Because if A is the spy, then B is the knight, 2 and 6 are true; B and A are knights contradicts with 4. The result is A is the knight, B is the spy and C is the knave.

Assume that 6 is true, we know that A is the knight and C is the knave. Based on 4, we can conclude that B is the spy. The result does not contradict with 4.

Assume that 7 is true, we know that C is the spy. So 5 is false. By 4, we know that A is the knave and B is the knight, so 6 is true, which contradicts with the fact that A is the knave.

So we know that A is the knight, B is the spy and C is the knave.

8. Rosen, Section 1.2: Exercise 34
Solution:

1. p: Kevin is chatting.
2. q: Heather is chatting.
3. r: Randy is chatting.
4. s: Vijay is chatting.
5. t: Abby is chatting.
6. $p \vee q$
7. $(r \wedge \neg s) \vee (\neg r \wedge s)$
8. $t \rightarrow r$
9. $(p \wedge s) \vee (\neg p \wedge \not s)$
10. $q \rightarrow t \wedge p$

$$t \rightarrow r \equiv r \vee \neg t \qquad \text{[Logical equivalences Involving Implications]} \tag{1}$$

$$q \rightarrow t \wedge p \equiv \neg q \vee (p \wedge t) \quad \text{[Logical equivalences Involving Implications]} \tag{2}$$

By (1), we know that t is false or r is true.

So we assume that t is false. Here we get $t \wedge p$ is false. By (2), we know that q is false. By 6, p is true, so $\neg p \wedge \neg s$ is false. By 9, we know that s is true, so $r \wedge \neg s$ is false. By 7, $\neg r$ is true. So we have Kevin is chatting, Heather is not chatting, Randy is not chatting, Vijay is chatting and Abby is not chatting.

We can also assume that r is true. The $\neg r \wedge s$ is false. By 7, $\neg s$ is true. So $p \wedge s$ is false. By 9, $\neg p$ is true, so p is false. As a result, $t \wedge p$ is false, and by (2), q is false. So p and q are false, which contradicts with 6. The assumption is wrong.

According to the above inference, we have that Kevin is chatting, Heather is not chatting, Randy is not chatting, Vijay is chatting and Abby is not chatting.

9.
Solution:

1. p: the direction out of Paradox is North.
2. q: Bill claims what you told him is true.

$N \to L \oplus N \equiv \neg N \vee [L \oplus N]$ [Logical equivalences Involving Implications]

$L \oplus N \to N \equiv \neg[L \oplus N] \vee N$ [Logical equivalences Involving Implications]

L	N	$L \oplus N$	$N \to L \oplus N$	$L \oplus N \to N$
T	T	F	F	T
T	F	T	T	F
F	T	T	T	T
F	F	F	T	T

We prove p implies q and vice versa.

First, we show p implies q.

Assume that L is true, i.e., Bill is the liar. p is logically equivalent to N, so when p is true, N is true. By the truth table, we have $L \oplus N$ false. q is Bill claims $L \oplus N$ is true. A liar would definetely not tell the truth. Because we have assumed Bill is a liar and $L \oplus N$ is false, q is true.

Assume that L is false, i,e, Bill tells the truth. So when p is true, by the truth table we have $L \oplus N$ true. Because Bill tells the truth and what he claims, $L \oplus N$, is true, q is true.

Now, we show q implies p.

Assume that L is true, i.e., Bill does not tell the truth. Because q is true and Bill is a liar, what I told him should be false. $L \oplus N$ is false. By the true table, N is true.

Assume that L is false. Because q is true, what I told him should be true. $L \oplus N$ is true. By the truth table, N is true.

10. Rosen, Section 1.3: Exercise 26 Use truth tables
Solution:
Truth table

p	q	r	$\neg p \to (q \to r)$	$q \to (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

By the truth table, $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

11.
Solution:
(a) Truth table

p	q	r	$(p \to q) \wedge (p \to r)$	$p \to (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

(b) Deduction

$$[p \to q] \wedge [p \to r] \equiv [\neg p \vee q] \wedge [\neg p \vee r] \quad \text{[Logical equivalences Involving Implications]} \tag{3}$$
$$\equiv \neg p \vee [q \wedge r] \quad \text{[Distributive law]} \tag{4}$$

$$p \to [q \wedge r] \equiv \neg p \vee [q \wedge r] \quad \text{[Logical equivalences Involving Implications]} \tag{5}$$

By 3 and 5, we know that they are logically equivalent.

12.
Solution:
Truth table:

p	q	p NOR q	$\neg(p \vee q)$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

We have known that AND, OR, and NOT are functionally complete. If we want to prove that NOR is functionally complete, we can express AND, OR, or NOT using NOR.

$$
\begin{aligned}
\neg p &\equiv \neg[p \vee p] && \text{[Idempotent laws]} && (6)\\
&\equiv p \text{ NOR } p && \text{[Definition of NOR]} && (7)
\end{aligned}
$$

$$
\begin{aligned}
p \vee q &\equiv \neg[p \text{ NOR } q] && \text{[Definition of NOR]} && (8)\\
&\equiv [p \text{ NOR } q] \text{ NOR } [p \text{ NOR } q] && \text{By 7} && (9)
\end{aligned}
$$

$$
\begin{aligned}
p \wedge q &\equiv \neg[\neg p \vee \neg q] && \text{[De Morgan's Law]} && (10)\\
&\equiv \neg p \text{ NOR } \neg q && \text{[Definition of NOR]} && (11)\\
&\equiv [p \text{ NOR } p] \text{ NOR } [q \text{ NOR } q] && \text{By 7} && (12)
\end{aligned}
$$

So we know how to express AND using NOR. Because AND is functionally complete, NOR is functionally complete as well.