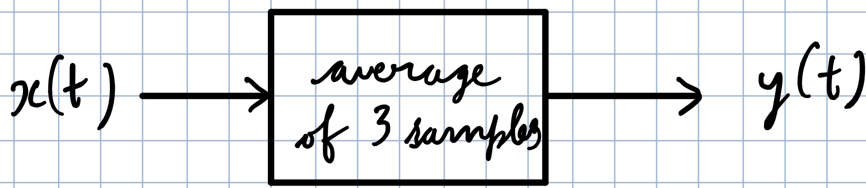
Convolution: Consider a signal averaging operation

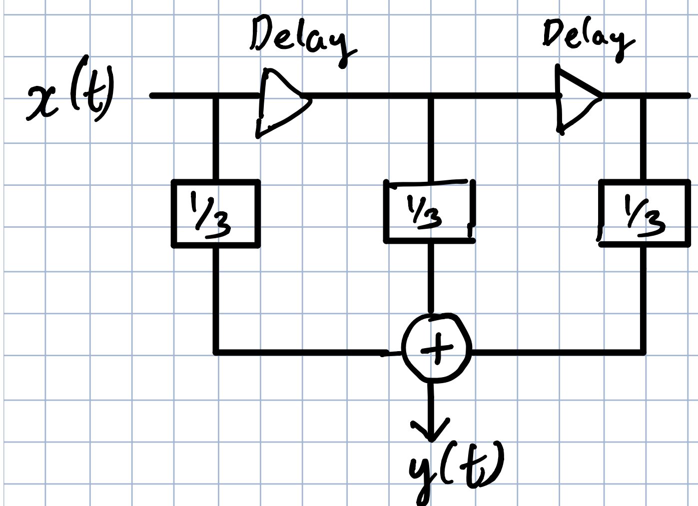


$$y_0 = \frac{x_0}{3}$$

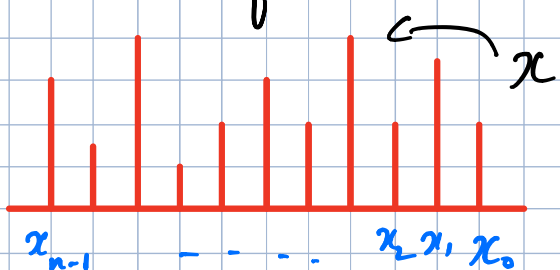
$$y_1 = \frac{x_0 + x_1}{3}$$

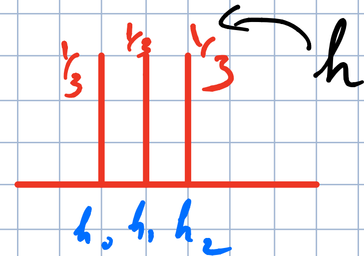
$$y_2 = \frac{x_0 + x_1 + x_2}{3}$$

$$y_3 = \frac{x_1 + x_2 + x_3}{3}$$



Think of this as an operation on two signals





given by

$$y = h * x \;, \quad y(n) = \sum_{s=0}^{2} h(s)\, x(n-s)$$

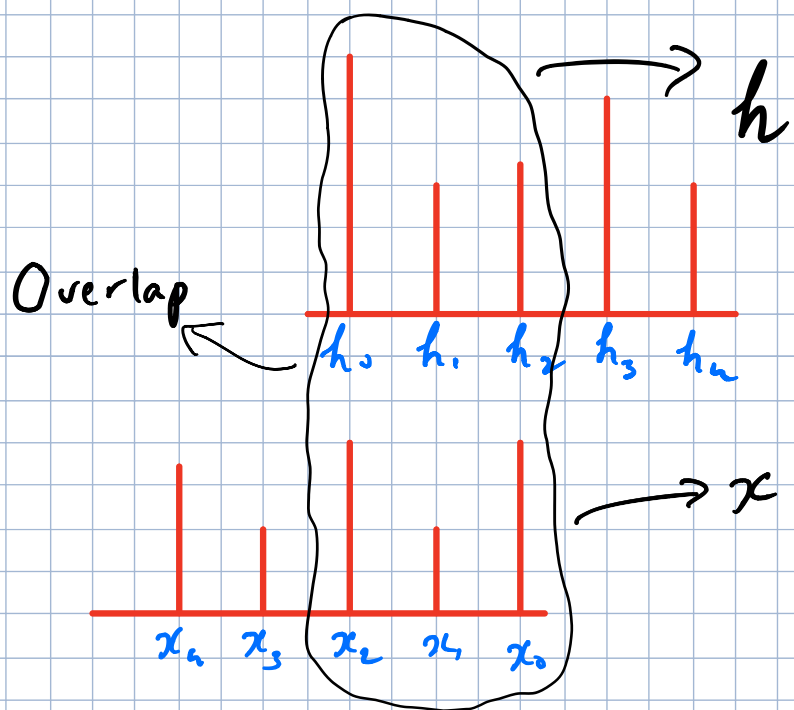
This is convolution denoted by $*$

Steps in convolution: $y = h * x$

① Take signal x and flip it by x-axis

② Multiply overlapping values of x and h and add them up

③ Shift signal x one step to right and repeat step 2 (from $-\infty$ to $-\infty$)



$$y_2 = x_0 h_2 + x_1 h_1 + x_2 h_0$$

In general,

$$y_n = x_0 h_n + x_1 h_{n-1} + \ldots\ldots x_{n-1} h_1 + x_n h_0$$

$$y_n = \sum_{\xi=0}^{n-1} h(\xi)\, x(n-\xi)$$

Properties of convolution

Consider two signals x and h

$$y = h * x$$

$$X = fft(x), \quad H = fft(h), \quad Y = fft(y)$$

① If $y = h * x$, then $Y(m) = H(m) \cdot X(m)$

A convolution in time domain is multiplication in frequency domain

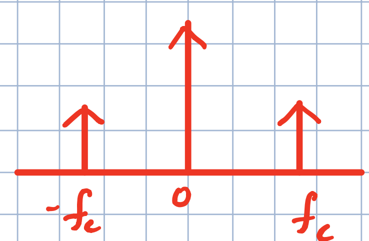
② If $Y = H * X$ (Convolution in frequency domain),

then $y(n) = h(n) \cdot x(n)$

A convolution in frequency domain is multiplication in time domain

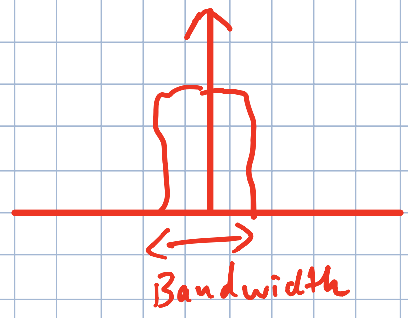
→ Example:

$$FFT(\cos 2\pi f_c t) =$$



Suppose you have a message signal

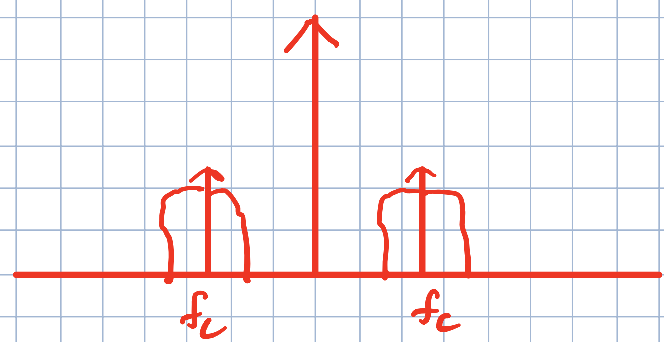
$m(t) =$



$y(t) = m(t) \cdot \cos 2\pi f_c t$

(time domain mult. is freq domain conv.)

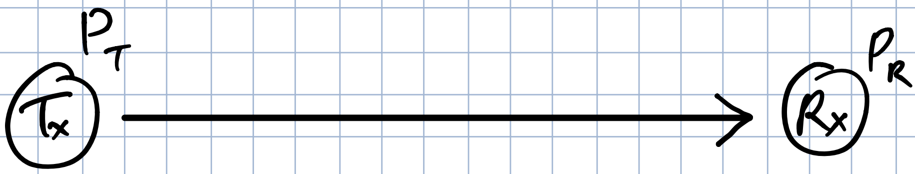
Thus, $y(t) =$



The message signal is shifted in frequency to occupy a new center frequency f_c

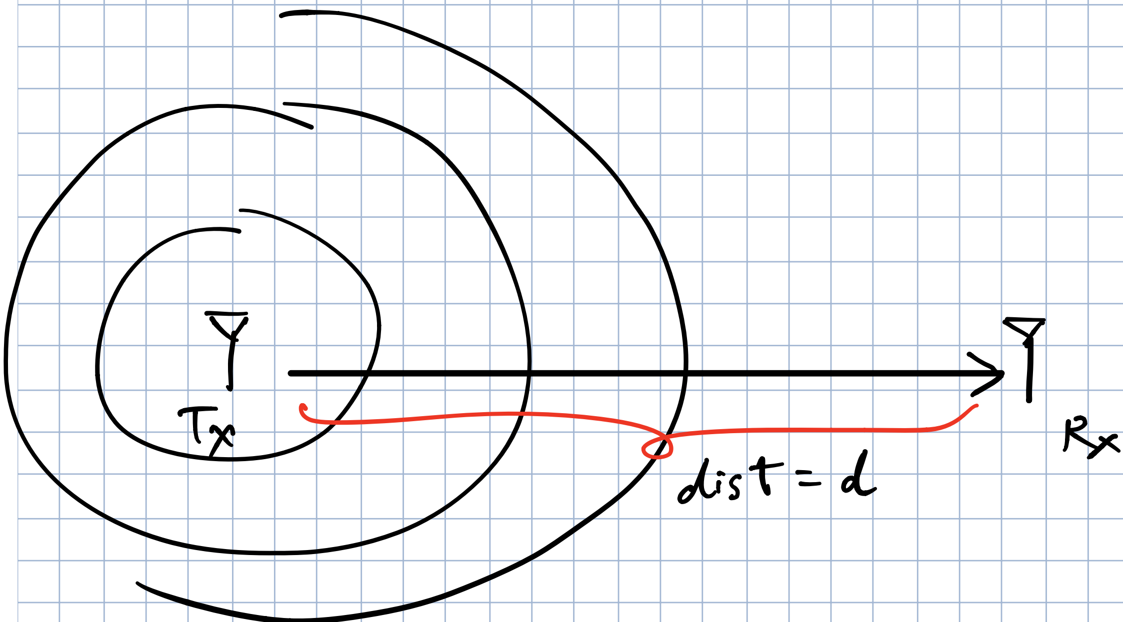
Different users can choose separate f_c to enable frequency multiplexing

Wireless Channel



$$P_R = \frac{P_T}{r^{\alpha}} \qquad \alpha = \text{path loss index}$$

If T_x and R_x are in free space

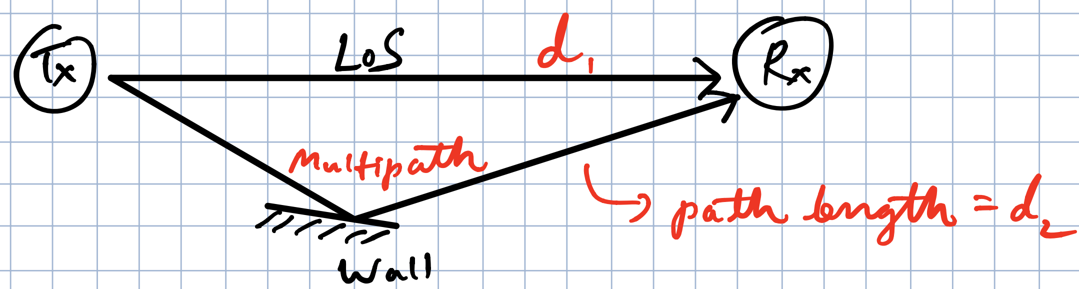


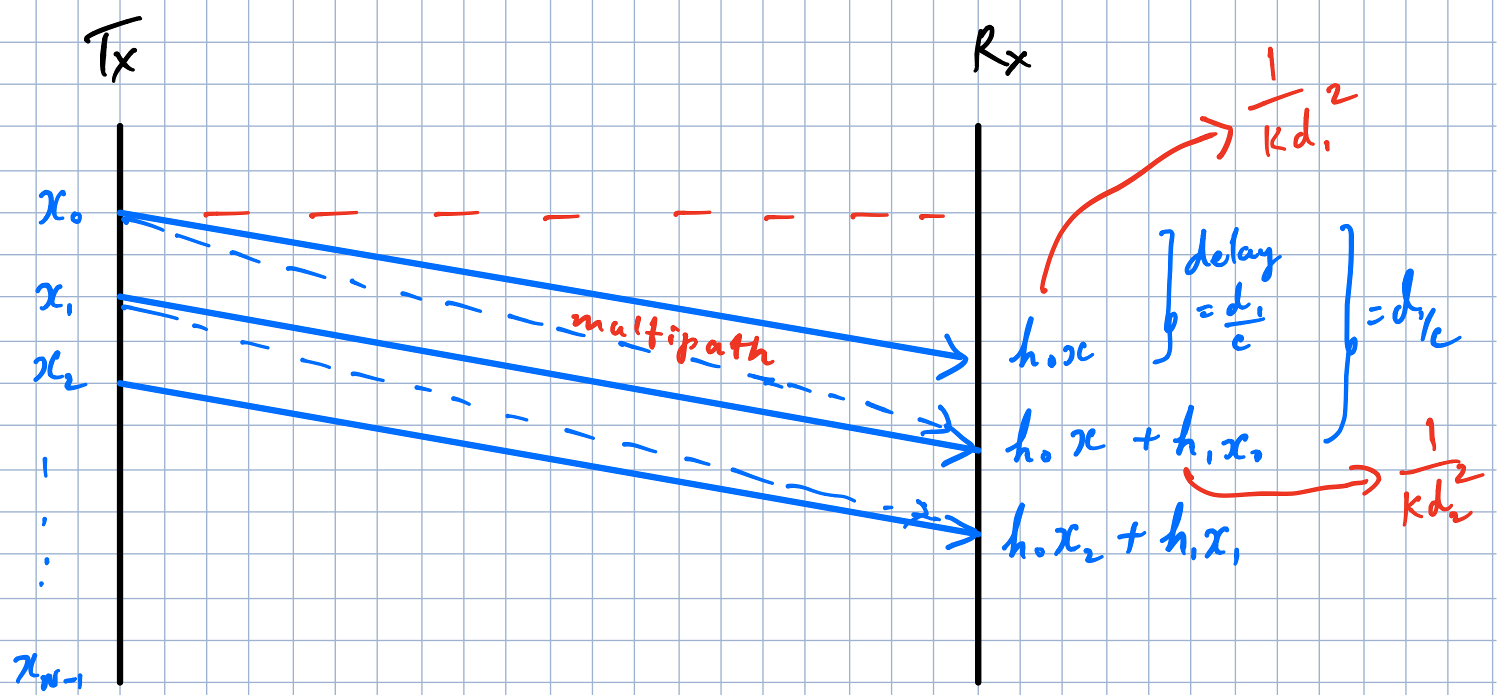
T_x signal reaches R_x after a delay of $\frac{d}{c}$

$$P_R = \frac{P_T}{4\pi d^2}$$, since the power is spread uniformly around a sphere (area = $4\pi d^2$)

This is called line of sight (LoS) channel

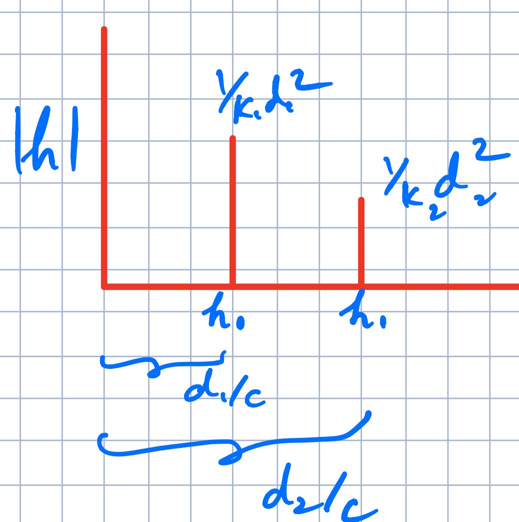
→ Multipath channel



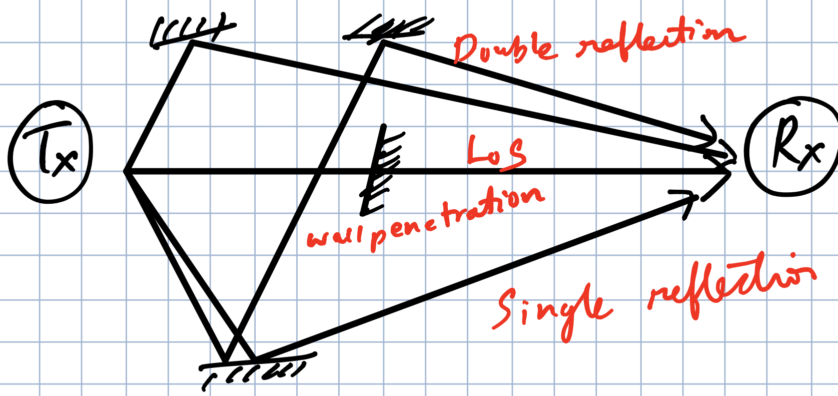


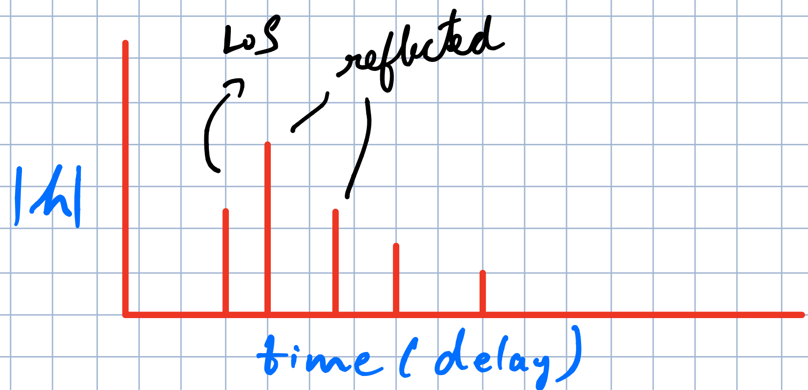
Received signal (y) can be expressed as

$$y = h * x$$ → Wireless channel



→ Realistic multipath channel





The channel provides information about

1. Number of multipath components
2. Their arrival time (delay)
3. Their attenuation

LoS may not be strongest path (Ex: LoS may be degraded due to wall penetration)

The channel "h" above is called
Channel Impulse Response (CIR)

if $x = \delta$ (Impulse), then $y = \delta * h = h$,
Thus h is the response to an impulse.

Hence h is called Channel Impulse Response

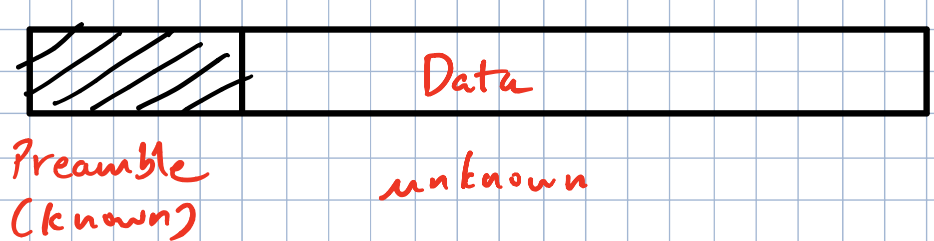
$H = \mathrm{fft}(h)$

H is the Channel Frequency Response (CFR)

→ Channel estimation

A WiFi receiver has to estimate CIR to be able to decode transmitted data (x)

Every packet is prepended with a known sequence called Preamble



Preamble serves as a training data to learn the wireless channel

$$Y_f = H_f \cdot X_f$$

Y_f → known (received)

H_f → unknown

X_f → known (preamble)

Y_f is also corrupted due to hardware noise

$$Y_f = H_f X_f + n_f$$

n_f → noise

$$\hat{H}_f = \frac{Y_f}{X_f}$$

$\hat{H}_f$ (estimated)

(channel estimation at frequency f)

$$\hat{H}_f = H_f + \frac{n_f}{X_f}$$

$\frac{n_f}{X_f}$ → error in estimation

Channel is estimated at different frequencies to obtain CFR

$$\hat{H}_{f_1} = \frac{Y_{f_1}}{X_{f_1}}$$

$$\hat{H}_{f_2} = \frac{Y_{f_2}}{X_{f_2}}$$

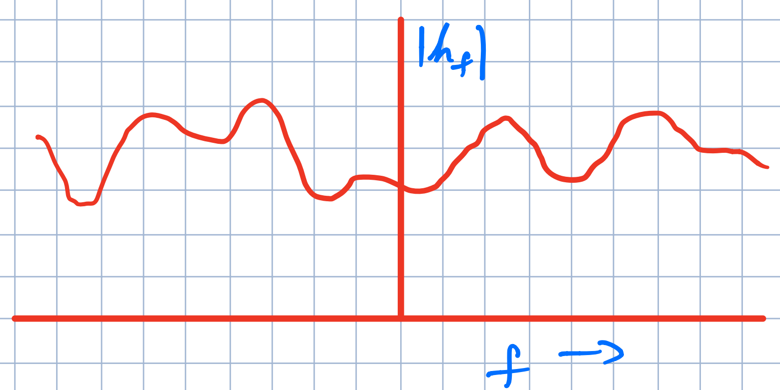
$$\vdots$$

$$\hat{H}_{f_n} = \frac{Y_{f_n}}{X_{f_n}}$$

CIR is obtained by computing IFFT on CFR

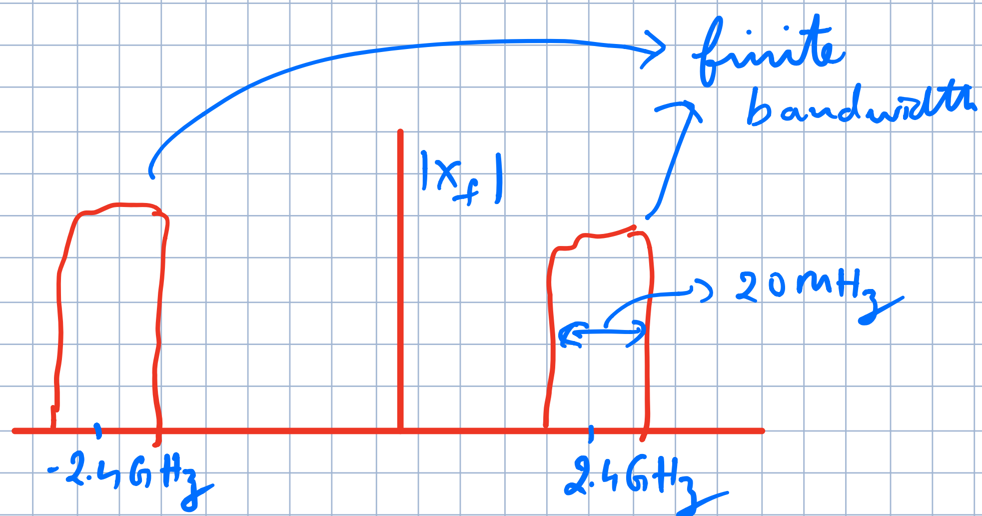
It is impractical to perform CFR estimation at all frequencies

Suppose H_f =

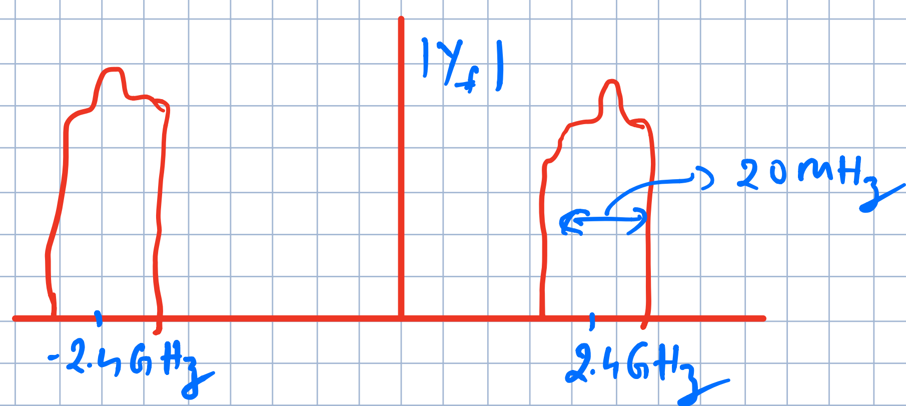


Real world CFR can have infinite frequencies
However, practical wireless systems can
only generate finite frequencies
(Ex: WiFi typically has a bandwidth of
20 MHz

X_f from WiFi

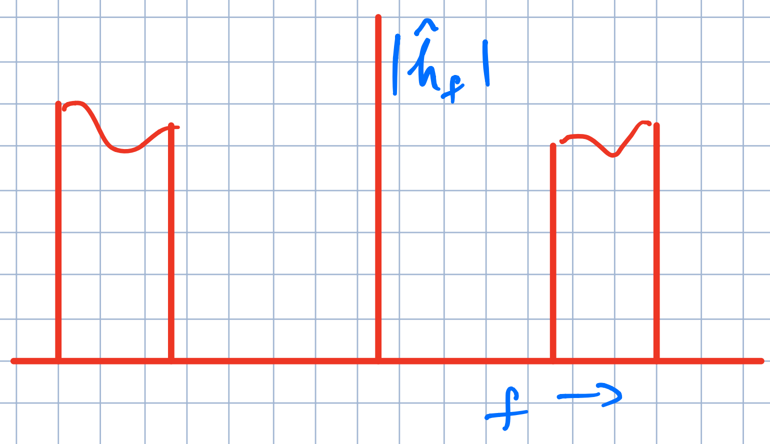


$$Y_f = X_f \cdot H_f$$



Y_f only exists where X_f is non-zero

$$\hat{H}_f = \frac{Y_f}{X_f}$$



$\hat{H}_f$ can only be estimated where X_f is non-zero

Thus $\hat{h} = \mathrm{IFFT}(\hat{H}_f)$ may not represent real world CIR. However it provides valuable hints of immense value for several sensing applications
(Ex: gesture tracking, localization, material identification, imaging, security etc)