STTP: Deep Learning in Computer Vision and Signal Processing

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Outline of the Lectures

- Introduction to the Theory of Compressed Sensing (CS)
- An example: Practical Compressive Sensing Systems
- Overview of our research in CS

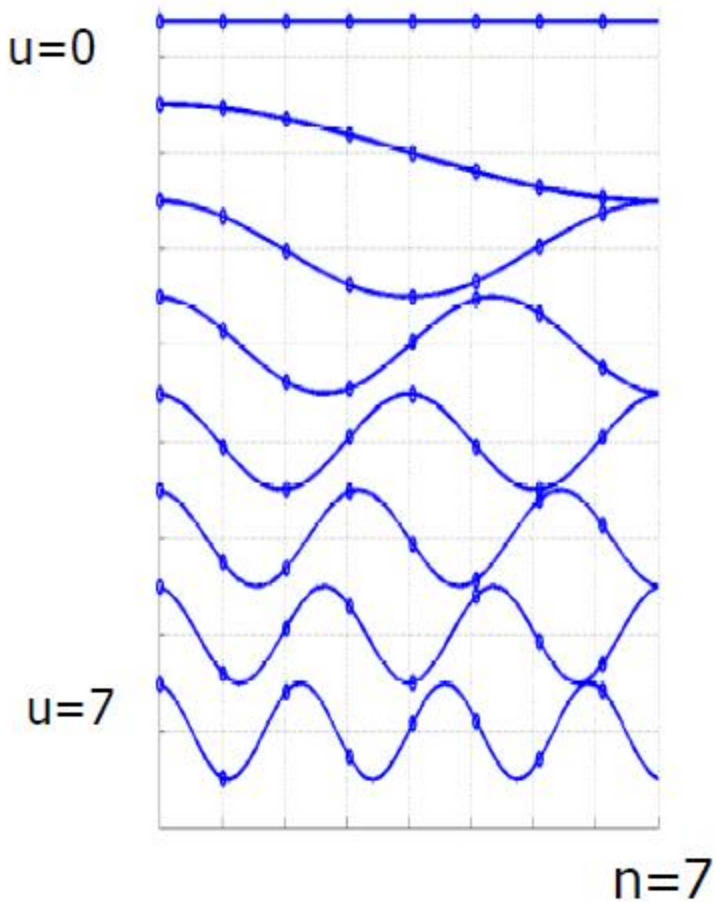
Conventional Sensing and Compression

- Before applying signal/image compression algorithms like MPEG, JPEG, JPEG-2000 etc., the measuring devices typically measure large amounts of data.
- The data is then converted to a transform domain where a majority of the transform coefficients turn out to have near-zero magnitude and can be discarded with small reconstruction error but great savings in storage.
- Example: a digital camera has a detector array where several thousand pixel values are first stored. This 2D array is fed to the JPEG algorithm which computes DCT coefficients of each block in the image, discarding smaller-valued DCT coefficients.

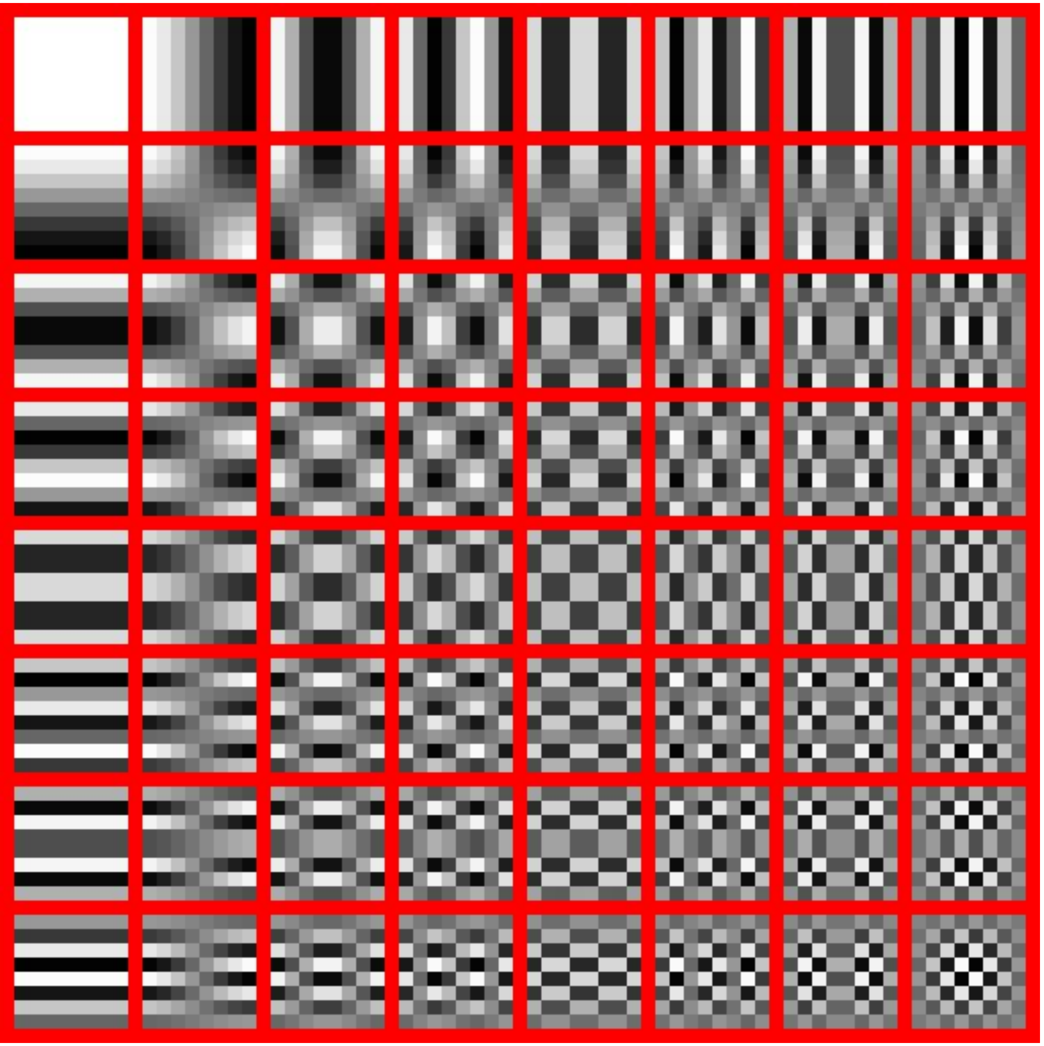
DCT

- Expresses a signal as a linear combination of **cosine** bases (as opposed to the complex exponentials as in the Fourier transform). The coefficients of this linear combination are called **DCT coefficients**.
- Is **real-valued** unlike the Fourier transform!
- Has better compaction properties for signals and images – as compared to DFT.

How do the DCT bases look like? (1D case)



How do the DCT bases look like? (2D-case)



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The DCT transforms an 8×8 block of input values to a [linear combination](http://en.wikipedia.org/wiki/Linear_combination) of these 64 patterns. The patterns are referred to as the two-dimensional DCT *basis functions*, and the output values are referred to as *transform coefficients*.

Each image here is obtained from the 8 x 8 outer product of a pair of DCT basis vectors. Each image is stretched between 0 and 255 – on a common scale.

http://en.wikipedia.org/wiki/JPEG

What is Compressive Sensing?

- Conventional sensing is a "measure and compress+throw" paradigm, which is wasteful!
- Especially for time-consuming acquisitions like MRI, CT, etc.
- Compressive sensing is a new technology where the data are acquired/measured in a compressed format!

What is Compressive Sensing?

- These compressed measurements are then fed to some optimization algorithm (also called inversion algorithm) to produce the complete signal.
- This part is implemented in software.
- Under suitable conditions that the original signal and the measurement system must fulfill, the reconstruction is guaranteed to have very little or even zero error!

Why compressive sensing?

- It has the potential to dramatically improve acquisition speed for MRI, CT, hyper-spectral data and other modalities.
- Potential to dramatically improve video-camera frame rates without sacrificing spatial resolution.

Motivation for CS: Candes' puzzling experiment (Circa 2004)

Ref: Candes, Romberg and Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Transactions on Information Theory, Feb 2006.

$$\min_f \sum_{x=1}^{N-1}\sum_{y=1}^{N-1}\sqrt{f_x^2(x,y)+f_y^2(x,y)}$$

such that

$$\forall (u,v)\in \boldsymbol{C}, F(u,v)=G(u,v)$$

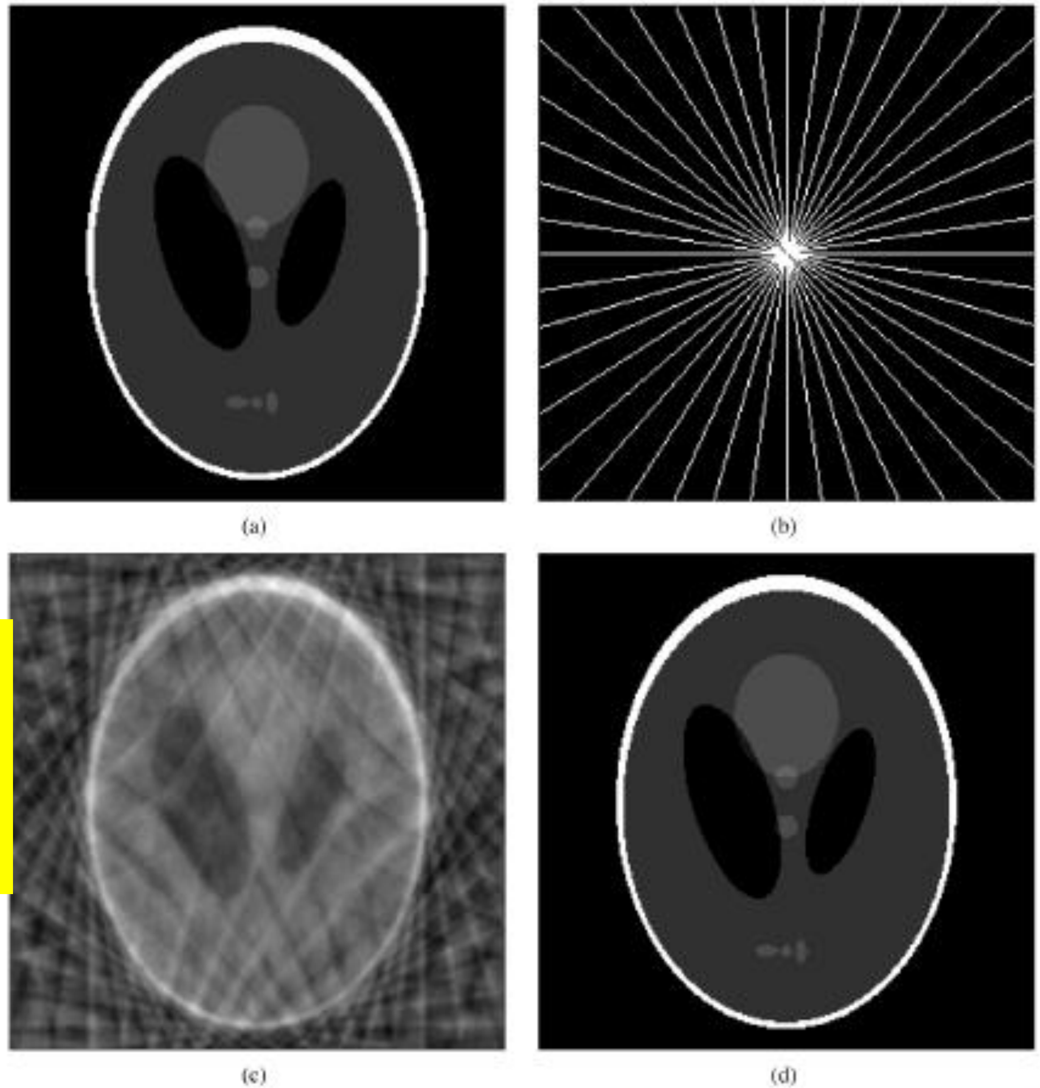

Fig. 1. Example of a simple recovery problem. (a) The Logan–Shepp phantom test image. (b) Sampling domain Ω in the frequency plane; Fourier coefficients are sampled along 2[illegible] radial lines. (c) Minimum energy reconstruction obtained by setting unobserved Fourier coefficients to zero. (d) Reconstruction obtained by minimizing the total variation, as in (1.1). The reconstruction is an exact replica of the image in (a).

Motivation for CS: Candes' puzzling experiment (Circa 2004)

- The top-left image (previous slide) is a standard phantom used in medical imaging - called as Logan-Shepp phantom.

- The top-right image shows 22 radial directions with 512 samples along each, representing those Fourier frequencies (remember we are dealing with 2D frequencies!) which were measured.

- Bottom left: reconstruction obtained using inverse Fourier transform, assuming the rest of the Fourier coefficients were zero.
- Bottom right: image reconstructed by solving the constrained optimization problem in the yellow box on the previous slide. It gives a zero-error result!

THEORY OF COMPRESSIVE SENSING

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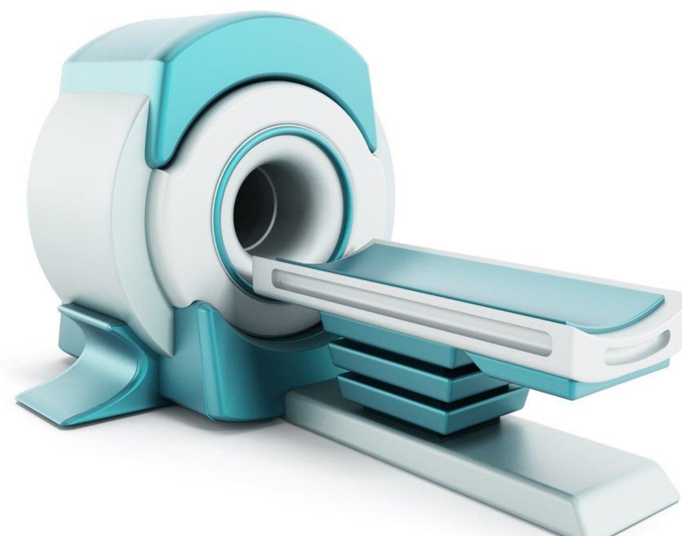
CS: Theory

- Let a occurring naturally signal be denoted **f**.
- Let us denote the measurement of this signal (using some device) as **y**.
- Typically, the mathematical relationship between **f** and **y** can be expressed as a linear equation of the form $\mathbf{y} = \mathbf{\Phi f}$.

CS: Theory

- Typically, the mathematical relationship between **f** and **y** can be expressed as a linear equation of the form $\mathbf{y} = \Phi\mathbf{f}$.
- Φ is called the measurement matrix (or the "sensing matrix" or the "forward model" of the measuring device).
- For standard digital camera, Φ may be approximated by a Gaussian blur.

Measuring devices can be represented as linear systems

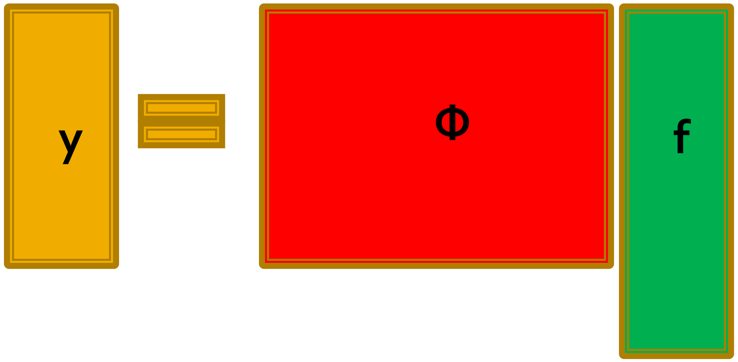


CS: Theory

- Let **f** be a vector with *n* elements.

- Since the measurements need to be compressive, $\boldsymbol{\Phi}$ must have fewer rows (*m*) than columns (*n*) to produce a measurement vector **y** with *m* elements.

- We know **y** and $\boldsymbol{\Phi}$, and we wish to estimate **f**.



CS: Theory

- We know **y** and $\boldsymbol{\Phi}$, and we wish to estimate **f**.
- We know that in *general*, this is an under-determined linear system, and hence there is no unique solution.
- Why?

$$\boldsymbol{\Phi}\tilde{\mathbf{f}} = \boldsymbol{\Phi}(\tilde{\mathbf{f}} + \mathbf{v}), \text{ where } \boldsymbol{\Phi}\mathbf{v} = \mathbf{0}, \text{ i.e. } \mathbf{v} \in \text{Nullspace}(\boldsymbol{\Phi})$$

CS: Theory

- But CS theory states, that in certain cases, this system does have a unique solution.

- Conditions to be satisfied:

1. Vector **f** should be sparse (so not all vectors in R^n are potential solutions).
2. Φ should have the property that no sparse vector lies in its null-space.

CS: Theory

- Wonderful news for signal and image processing.
- Why?

1. Natural signals/images have a sparse representation in some well-known orthonormal basis Ψ such as Fourier, DCT, Haar wavelet etc.
2. The measurement matrix Φ can be designed to have the aforementioned property.

Signal Sparsity

- Many signals have sparse (or compressible) representations in standard orthonormal bases (denoted here as $\mathbf{\Psi}$).
- Example:

$$\mathbf{f} = \mathbf{\Psi}\boldsymbol{\theta} = \sum_{k=1}^{n} \mathbf{\Psi}_{\mathbf{k}} \theta_k,$$

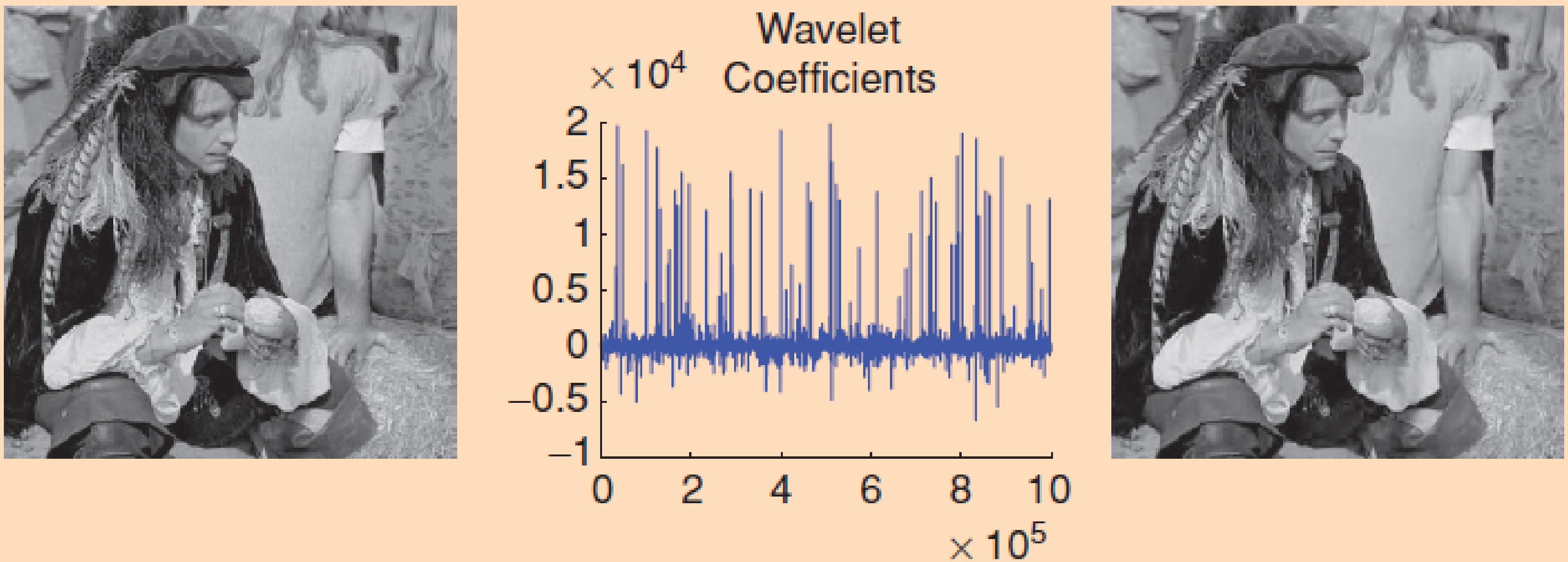
$$\mathbf{f} \in R^n, \boldsymbol{\theta} \in R^n, \mathbf{\Psi} \in R^{n \times n}, \mathbf{\Psi}^T \mathbf{\Psi} = \mathbf{I},$$

$$\|\boldsymbol{\theta}\|_0 << n$$

This is the Lo norm of a vector = the number of non-zero elements in it

Signal Sparsity

Image source: Candes and Wakin, "An introduction to compressive samplig", IEEE Signal processing magazine



(a) (b) (c)

[FIG1] (a) Original megapixel image with pixel values in the range [0,255] and (b) its wavelet transform coefficients (arranged in random order for enhanced visibility). Relatively few wavelet coefficients capture most of the signal energy; many such images are highly compressible. (c) The reconstruction obtained by zeroing out all the coefficients in the wavelet expansion but the 25,000 largest (pixel values are thresholded to the range [0,255]). The difference with the original picture is hardly noticeable. As we describe in "Undersampling and Sparse Signal Recovery," this image can be perfectly recovered from just 96,000 incoherent measurements.

Signal Reconstruction

- Let the measured data be given as:

$$\mathbf{y} = \mathbf{\Phi f} = \mathbf{\Phi \Psi \theta},$$

$$\mathbf{y} \in R^m, \mathbf{\Phi} \in R^{m \times n}, \mathbf{f} \in R^n, m << n$$

- The coefficients of the signal $\mathbf{f}$ in $\mathbf{\Psi}$ – denoted as $\mathbf{\theta}$ – and hence the signal $\mathbf{f}$ itself - can be recovered by solving the following constrained minimization problem:

$$\text{Problem P0}: \min \|\mathbf{\theta}\|_0 \text{ such that } \mathbf{y} = \mathbf{\Phi \Psi \theta}$$

Signal Reconstruction

- Po is a very difficult optimization problem to solve – it is NP-hard.
- Hence, a softer version (known as Basis Pursuit) is solved:

$$\text{Problem BP} : \min \|\boldsymbol{\theta}\|_1 \text{ such that } \mathbf{y} = \boldsymbol{\Phi\Psi\theta}$$

This is the L1 norm of a vector = the sum total of the absolute values of all its elements

- This is a linear programming problem and can be solved with any LP solver (in Matlab, for example) or with packages like L1-magic.

http://users.ece.gatech.edu/~justin/l1magic/

Signal Reconstruction: uniqueness issues

- Is P0 guaranteed to have a unique solution at all? Why? (If answer is no, compressed sensing is not guaranteed to work!)

- Consider the case that any 2S columns of an m x n matrix **A** are linearly independent. Then any S-sparse signal **f** can be uniquely reconstructed from measurements **y** = **Af**. See proof on next slide.

Signal Reconstruction: uniqueness issues

- **Proof by contradiction:**
- Suppose we have *S*-sparse signals **f** and **f'** such that **y** = **Af** = **Af'**.
- Then **A**(**f**-**f'**) = **0** and hence **f**-**f'** lies in the null-space of **A**.
- But **f**-**f'** is a 2*S*-sparse signal. Let *T* be the set of its (2*S*) non-zero elements. Then **A**(**f**-**f'**) = $\mathbf{A}_T(\mathbf{f}-\mathbf{f}')_T = \mathbf{0}$ where $\mathbf{A}_T$ is a sub-matrix with column indices strictly from *T* and $(\mathbf{f}-\mathbf{f}')_T$ is a vector containing values of **f**-**f'** strictly from *T*.
- Hence $(\mathbf{f}-\mathbf{f}')_T$ lies in the null-space of $\mathbf{A}_T$, and hence there exist 2*S* columns of **A** that aren't linearly independent. Hence contradiction!

Signal Reconstruction: uniqueness issues

- But are these conditions on **A** valid?
- The answer is that we need measurement matrices such that **A** will obey this property.
- In fact, we will see something called the "Restricted isometry property".
- If **A** satisfies this property, uniqueness is guaranteed.

New concept: Restricted Isometry Property (RIP)

- For integer $S = 1,2,\ldots,n$, the restricted isometry constant (RIC) δ_S of a matrix $\mathbf{A} = \Phi\Psi$ (*) of size m by n is the smallest number such that for any S-sparse vector $\boldsymbol{\theta}$, i.e.

$$(1-\delta_S)\|\boldsymbol{\theta}\|^2 \leq \|\mathbf{A}\boldsymbol{\theta}\|^2 \leq (1+\delta_s)\|\boldsymbol{\theta}\|^2,$$

- We say that **A** obeys the restricted isometry property (RIP) of order S if δ_S is not too close to 1 (rather as close to 0 as possible).
- If **A** obeys RIP of order S, no S-sparse signal can lie in the null-space of **A** (if it did, then we would have $\mathbf{A}\boldsymbol{\theta} = \mathbf{0}$ and that obviously does not preserve the squared magnitude of the vector $\boldsymbol{\theta}$).

Restricted Isometry Property

- Let's suppose we wanted to sense a signal **x** that is *S*-sparse in some orthonormal basis in which it has representation **θ**, i.e. $\|\boldsymbol{\theta}\|_0 = S << n$
- Then the following is *undesirable* for **A**

$$\mathbf{A}\boldsymbol{\theta}^{(1)} = \mathbf{A}\boldsymbol{\theta}^{(2)}, \text{for } \boldsymbol{\theta}^{(1)} \neq \boldsymbol{\theta}^{(2)}$$

- One way to ensure that this doesn't happen is to design **A** such that:

$$\mathbf{A}\boldsymbol{\theta}^{(1)} \approx \mathbf{A}\boldsymbol{\theta}^{(2)} \leftrightarrow \boldsymbol{\theta}^{(1)} \approx \boldsymbol{\theta}^{(2)}, i.e.$$

$$\left\|\mathbf{A}(\boldsymbol{\theta}^{(1)} - \boldsymbol{\theta}^{(2)})\right\|^2 \approx \left\|(\boldsymbol{\theta}^{(1)} - \boldsymbol{\theta}^{(2)})\right\|^2$$

Restricted Isometry Property (RIP)

- Note that the difference between two *S*-sparse vectors is 2*S*-sparse. Then, if **A** obeys RIP of order 2*S*, we have the following:

$$(1-\delta_{2S})\|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2\|^2 \leq \|\mathbf{A}(\boldsymbol{\theta}_1-\boldsymbol{\theta}_2)\|^2 \leq (1+\delta_{2S})\|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2\|^2,$$

$$\forall \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \text{for which } \|\boldsymbol{\theta}_1\|_0 \leq S, \|\boldsymbol{\theta}_2\|_0 \leq S, \text{ where } \delta_{2S} << 1$$

- Thus **A** should **approximately preserve the squared differences between any two *S*-sparse vectors.**

Theorem 1

Candes, "The restricted isometry property and its implications for CS", CRM, 2008

- Suppose the matrix $\mathbf{A} = \boldsymbol{\Phi\Psi}$ of size *m* by *n* (where sensing matrix $\boldsymbol{\Phi}$ has size *m* by *n*, and basis matrix $\boldsymbol{\Psi}$ has size *n* by *n*) has RIP property of order 2*S* where $\delta_{2S} < 0.41$. Let the solution of the following be denoted as $\boldsymbol{\theta}^*$, (for signal $\mathbf{f} = \boldsymbol{\Psi\theta}$, measurement vector $\mathbf{y} = \boldsymbol{\Phi\Psi\theta}$): $\min \|\boldsymbol{\theta}\|_1 \text{ such that } \mathbf{y} = \boldsymbol{\Phi\Psi\theta}$

  Then we have:

$$\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_2 \le \frac{C_0}{\sqrt{S}} \|\boldsymbol{\theta} - \boldsymbol{\theta}_S\|_1,$$

$$\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_1 \le C_0 \|\boldsymbol{\theta} - \boldsymbol{\theta}_S\|_1$$

$\boldsymbol{\theta}_S$ is created by retaining the *S* largest magnitude elements of $\boldsymbol{\theta}$, and setting the rest to 0.

Comments on Theorem 1

- This theorem says that the reconstruction for *S*-sparse signals is **always** exact if **A** satisfies the RIP.
- For signals that are not *S*-sparse, the reconstruction error is almost as good as what could be provided to us by an **oracle** which knew the *S* largest coefficients of the signal **f**.

Comments on Theorem 1

- Most signals are not sparse but compressible!
- Theorem 1 handles such compressible signals robustly.
- The constant C_o is independent of *n*, and it is an increasing function of just δ_{2S}.

Compressive Sensing under Noise

- So far we assumed that our measurements were exact, i.e. $\mathbf{y} = \mathbf{\Phi f} = \mathbf{\Phi \Psi \theta}$.
- But practical measurements are always noisy so that $\mathbf{y} = \mathbf{\Phi f} + \boldsymbol{\eta} = \mathbf{\Phi \Psi \theta} + \boldsymbol{\eta}$.
- Under the same assumption as before, we can estimate $\boldsymbol{\theta}$ by solving the following problem:

$$\min \|\boldsymbol{\theta}\|_1 \text{ such that } \|\mathbf{y} - \mathbf{\Phi \Psi \theta}\|_2^2 \le \varepsilon$$

Convex problem, called as second-order cone program. Can be solved efficiently with standard packages including MATLAB and L1-MAGIC

Theorem 2

Candes, "The restricted isometry property and its implications for CS", CRM, 2008

- Suppose the matrix $\mathbf{A}=\mathbf{\Phi\Psi}$ of size *m* by *n* (where sensing matrix $\mathbf{\Phi}$ has size *m* by *n*, and basis matrix $\mathbf{\Psi}$ has size *n* by *n*) has RIP property of order 2*S* where $\delta_{2S} < 0.41$. Let the solution of the following be denoted as θ^*, (for signal $\mathbf{f} = \Psi\theta$, measurement vector $\mathbf{y}=\mathbf{\Phi\Psi\theta}$): $\min \|\boldsymbol{\theta}\|_1 \text{ such that } \|\mathbf{y}-\mathbf{\Phi\Psi\theta}\|_2^2 \leq \varepsilon$

  Then we have:

θ_S is created by retaining the S largest magnitude elements of θ, and setting the rest to 0.

$$\|\boldsymbol{\theta}^* - \boldsymbol{\theta}\|_2 \leq \frac{C_0}{\sqrt{S}} \|\boldsymbol{\theta} - \boldsymbol{\theta}_{\mathbf{S}}\|_1 + C_1\varepsilon$$

Comments on Theorem 2

- Theorem 2 is a direct extension of Theorem 1 for the case of noisy measurements.
- Reconstruction error is the sum of two terms:
  - the error of an oracle solution where the oracle told us the *S* largest coefficients of the signal **f**,
  - a term proportional to the noise variance.
- The constants C_0 and C_1 are very small (less than or equal to 5.5 and 6 respectively for $\delta_{2S} = 0.25$) , they are independent of *n*, and they are increasing functions of just δ_{2S}.

Randomness is super-cool!☺

- Consider any fixed orthonormal basis Ψ.
- For a sensing matrix Φ constructed in any of the following ways, the matrix **A** = $\Phi\Psi \in \mathcal{R}^{m\text{x}n}$, will obey the RIP of order *S* with overwhelming probability provided that the number of rows $m \geq CS\log(n/S)$

1. Φ contains entries from zero-mean Gaussian with variance *1/m*.
2. Φ contains entries with values $\pm 1/\sqrt{m}$ with probability 0.5
3. Columns of Φ are sampled uniformly randomly from a unit sphere in *m*-dimensional space
4. Randomly selected rows of a DFT matrix

Randomness is super-super-cool!☺

- These properties of random matrices hold true for any orthonormal basis Ψ with very high probability.

- We do not need to tune Φ for a given Ψ.

Why is BP efficiently solvable?

- Because it can be expressed as a linear program, i.e. an optimization problem with linear objective functions and linear equality/inequality constraints.

- Linear programs can be solved in polynomial time.

Compressive Sensing: Toy Example with images

- We will compare image reconstruction by two different methods.

- We won’t do true compressive sensing. Instead we’ll apply different types of sensing matrices Φ synthetically (in software) to an image loaded into memory, merely for example sake.

Compressive Sensing: Toy Example with images

- In the first method, we compute the DCT coefficients of an image (size N = 256 x 256), and reconstruct the image using the m largest absolute value DCT coefficients (and a simple matrix transpose). m is varied from 1000 to 30000.

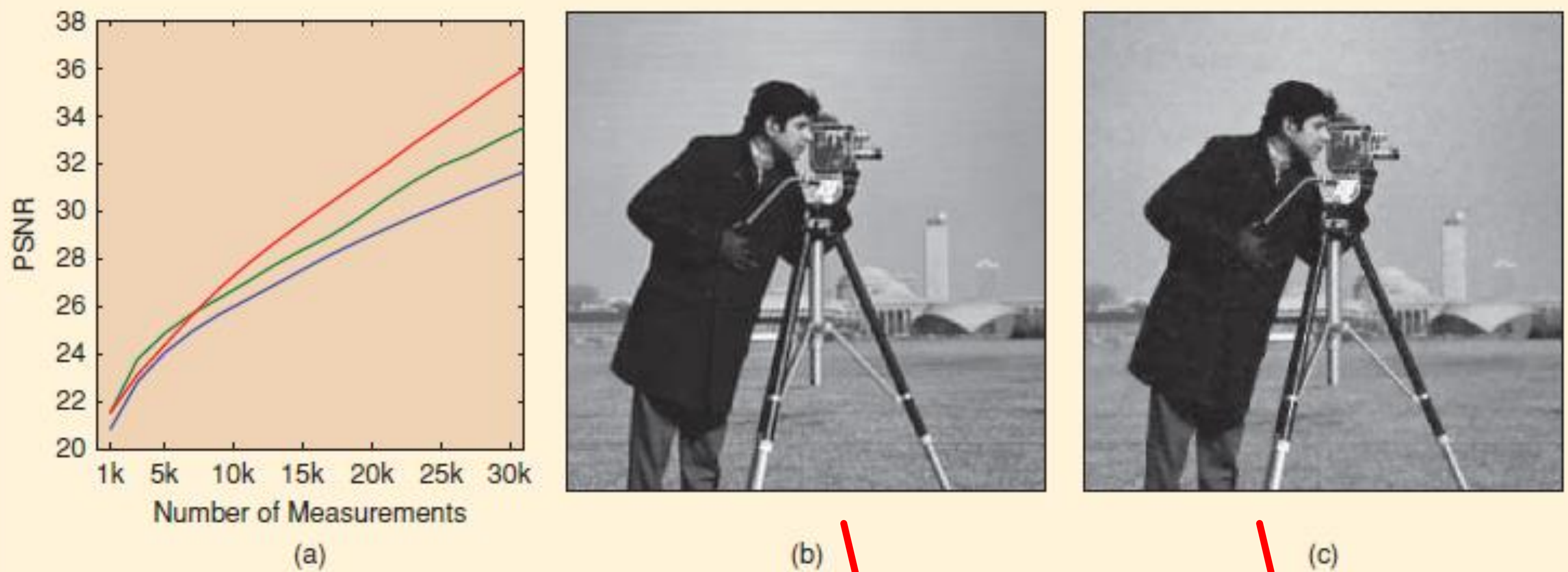
- Note that this is equivalent to using a sensing matrix Φ of size m by N.

Compressive Sensing: Toy Example with images

- In the second method, we reconstruct the image using *m* random linear combinations of elements of **x**, where *m* is again varied from 1000 to 30000.
- The sensing matrix $\boldsymbol{\Phi}$ is appropriately assembled.
- In this case, the reconstruction proceeds by solving the following optimization problem:

$$\min \|\boldsymbol{\theta}\|_1 \text{ such that } \mathbf{y} = \boldsymbol{\Phi\Psi\theta}$$

Compressive Sensing: Toy Example with images



[FIG2] Coded imaging simulation. (a) Recovery error versus number of measurements for linear DCT acquisition (blue), compressive imaging (red), and DCT imaging augmented with total-variation minimization (green). The error is measured using the standard definition of peak signal-to-noise ratio: $\text{PSNR} = 20\log_{10}(255 \cdot 256/\|X - \tilde{X}\|_2)$. (b) Image recovered using linear DCT acquisition with 21,000 measurements. (c) Image recovered using compressive imaging from 1,000 DCT and 20,000 noiselet measurements.

Better edge preservation with the randomly assembled Φ – which actually uses some high frequency information (unlike conventional DCT which discards higher frequencies)

Image source: Romberg, "Imaging via compressive sampling", IEEE Signal Processing Magazine

Summary

- Concept of CS
- Key theorems of CS
- Restricted Isometry Property
- Some example results

A Practical Compressive System: Rice Single Pixel Camera

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Standard Camera

- Consists of an aperture, a lens and a detector array.
- Light from the scene enters camera through aperture and is focussed onto detector array by the lens.
- Number of pixels on detector array = size of the image.

Rice Single Pixel Camera: a CS-based camera

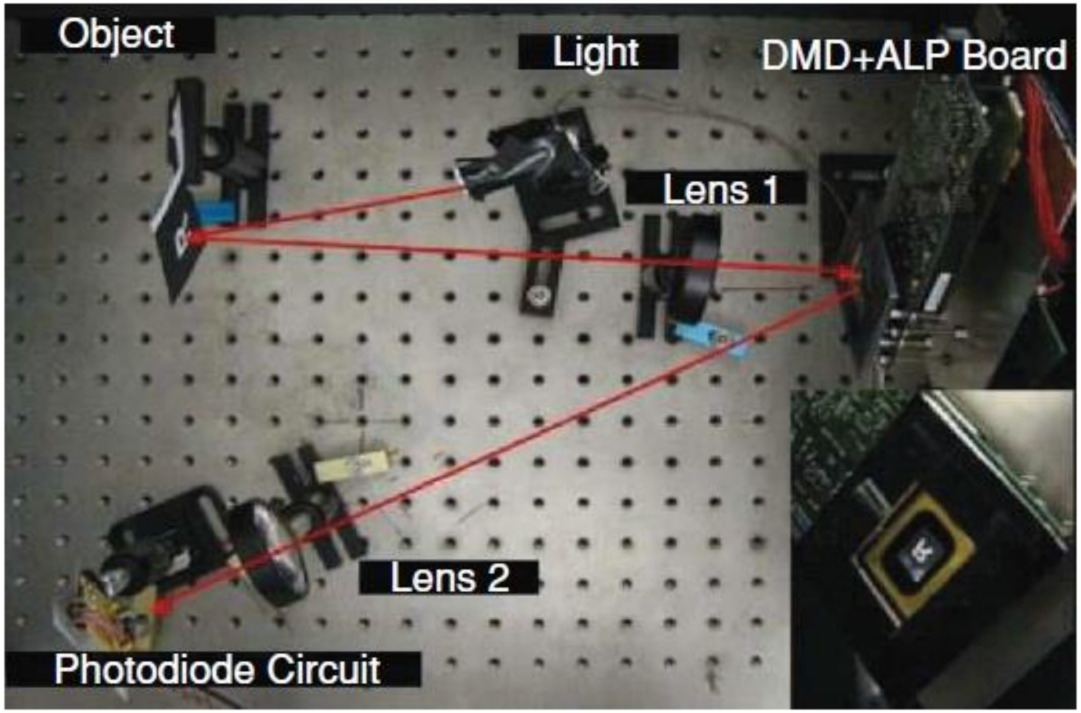
- It does not measure the image 2D array.
- Instead it directly measures the dot-product of the image with a set of random codes. This is mathematically shown below:

$$\forall i, 1 \le i \le m, y_i = \mathbf{f}^T \boldsymbol{\varphi}_\mathbf{i}$$

- We now want to reconstruct **f** given the set of measurements, i.e. $\{y_i\}$ and random codes.

Ref: Duarte et al, "Single pixel imaging via compressive sampling", IEEE Signal Processing Magazine, March 2008.

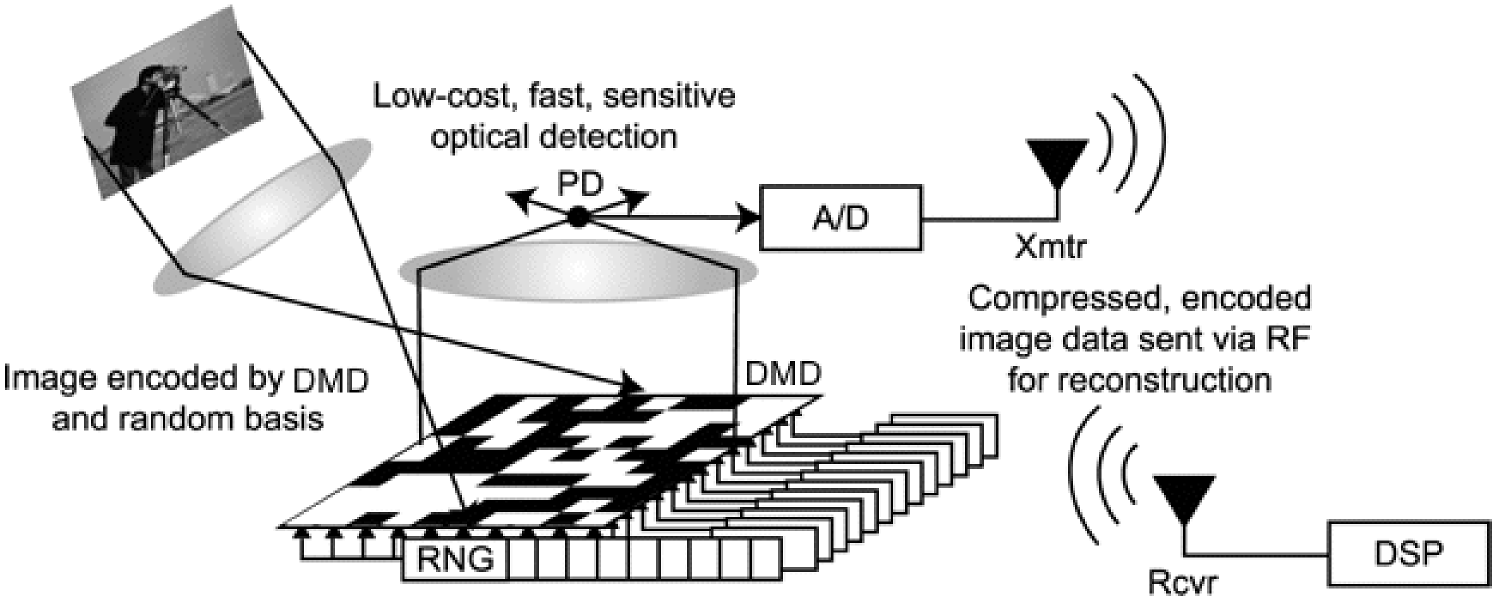
Rice Single Pixel Camera: a CS-based camera



[FIG1] Aerial view of the single-pixel CS camera in the lab [5].

Ref: Duarte et al, "Single pixel imaging via compressive sampling", IEEE Signal Processing Magazine, March 2008.

Rice Single Pixel Camera: a CS-based camera



Ref: Duarte et al, "Single pixel imaging via compressive sampling", IEEE Signal Processing Magazine, March 2008.

Rice Single Pixel Camera: a CS-based camera

- Contains no detector array.
- Light from the scene passes through Lens1 and is focussed on a digital micromirror device (DMD).
- DMD is a 2D array of thousands of very tiny mirrors.
- Light reflected from DMD passes through the second lens and to the photodiode.

Rice Single Pixel Camera: a CS-based camera

- The DMD acts as a random binary array of the same size as image **f**. Each mirror in the DMD corresponds to one pixel in **f**.
- A mirror can be either ON=1 (facing the Lens2) or OFF=0 (facing away from Lens2).
- The photodiode circuit acts as a photon counter – effectively, it measures the dot product between **f** and $\boldsymbol{\phi}_i$, i.e. it measures:

$$y_i = \sum_{j=1}^{n} f_j \phi_{ij}$$

Rice Single Pixel Camera: a CS-based camera

- These values $\{y_i\}$ are output in the form of a voltage which is then digitized by an A/D converter.
- Note that a different binary code vector ϕ_i is used for each y_i, $1 <= i <= m$.
- The random binary code is implemented by setting the orientation of the mirrors (facing toward or away from Lens2) randomly within the hardware.
- But these are codes with 0 and 1 and such a matrix does not obey RIP.
- So instead a matrix with -1 and +1 is "generated" using two measurements:

$$y_1 = \Phi_1 x$$
$$y_2 = \Phi_2 x$$
$$y_1 - y_2 = (\Phi_1 - \Phi_2)x$$

Φ_2 contains a 1 wherever Φ_1 contains a 0, and Φ_2 contains a 1 wherever Φ_1 contains a 0.

Rice Single Pixel Camera: a CS-based camera

- The basic measurement model can be written as follows (in vector notation):

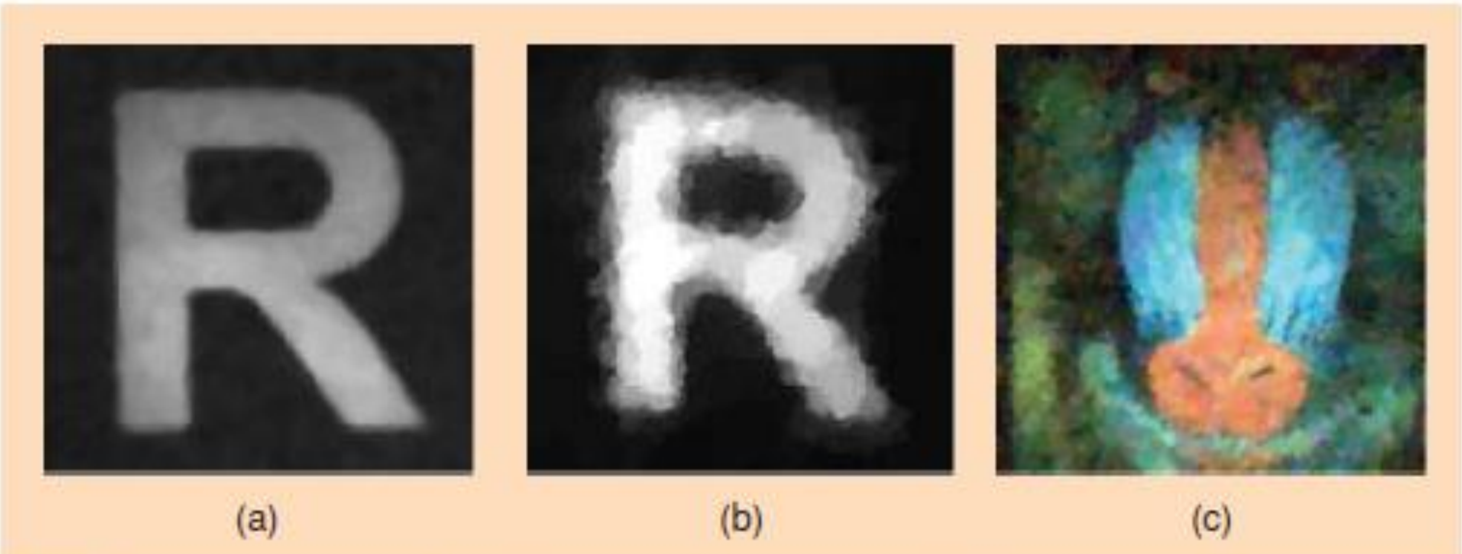
$$\mathbf{y} = \mathbf{\Phi f}, \mathbf{\Phi} = [\boldsymbol{\varphi}_\mathbf{1} \,|\, \boldsymbol{\varphi}_\mathbf{2} \,|\, \ldots \,|\, \boldsymbol{\varphi}_\mathbf{m}]^T,$$

$$\mathbf{y} = (y_1, y_2, \ldots, y_m)$$

- As per CS theory, there are guarantees of good reconstruction if the number of samples obeys (for K-sparse signals):

$$m \geq O(K \log(n/K))$$

Reconstruction Results



[FIG2] Single-pixel photo album. (a) 256×256 conventional image of a black-and-white R. (b) Single-pixel camera reconstructed image from $M = 1,300$ random measurements ($50\times$ sub-Nyquist). (c) 256×256 pixel color reconstruction of a printout of the Mandrill test image imaged in a low-light setting using a single photomultiplier tube sensor, RGB color filters, and $M = 6,500$ random measurements.

Optimization technique used: $\min_f TV(f) \text{ such that } y = \Phi f$

More Results:
http://dsp.rice.edu/cscamera

Original

4096 pixels, 800 measurements, i.e. 20% data

65536 pixels, 6600 measurements, i.e. 10% data

Informal description of Rice Single Pixel Camera:
http://terrytao.wordpress.com/2007/04/13/compressed-sensing-and-single-pixel-cameras/

Compressed sensing on the chip

- Compressive camera developed at Stanford
- Uses the same mathematical model as the Rice SPC.
- The difference is that it calculates all the m dot products **blockwise** on a single CMOS chip and *simultaneously*.
- What dot products? Of a random pattern (with n elements) with a vector of n analog pixel values (from a patch of small size).

Image source: Oike and El-Gamal, "CMOS sensor with programmable compressed sensing", IEEE Journal of Solid State Electronics, January 2013

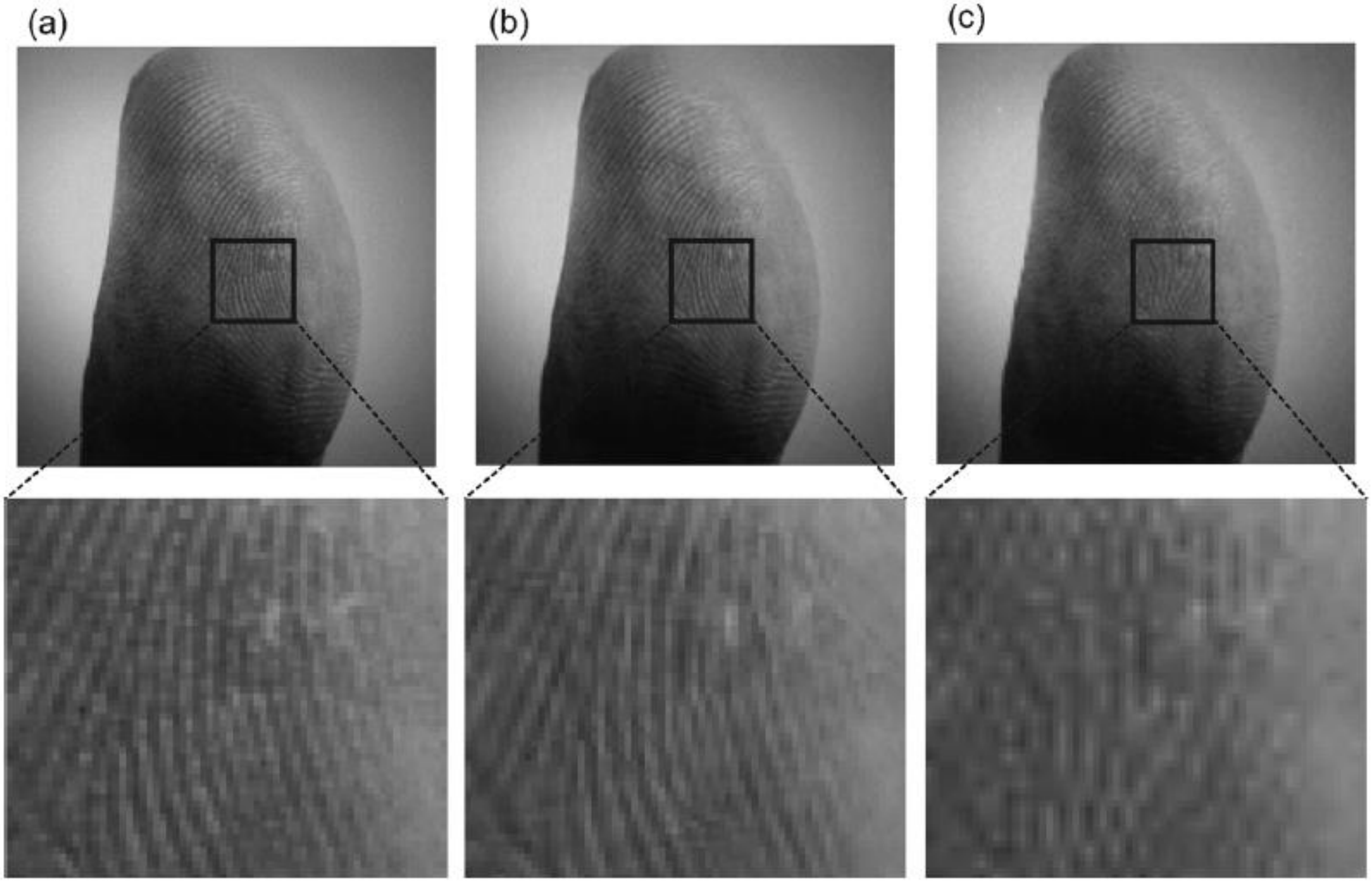


Fig. 16. Sample images captured in: (a) normal mode at 120 fps, (b) compressed sensing at CR = 1/4 and 480 fps, (c) downsampling at 1/4 ratio.

Image source: Oike and El-Gamal, "CMOS sensor with programmable compressed sensing", IEEE Journal of Solid State Electronics, January 2013

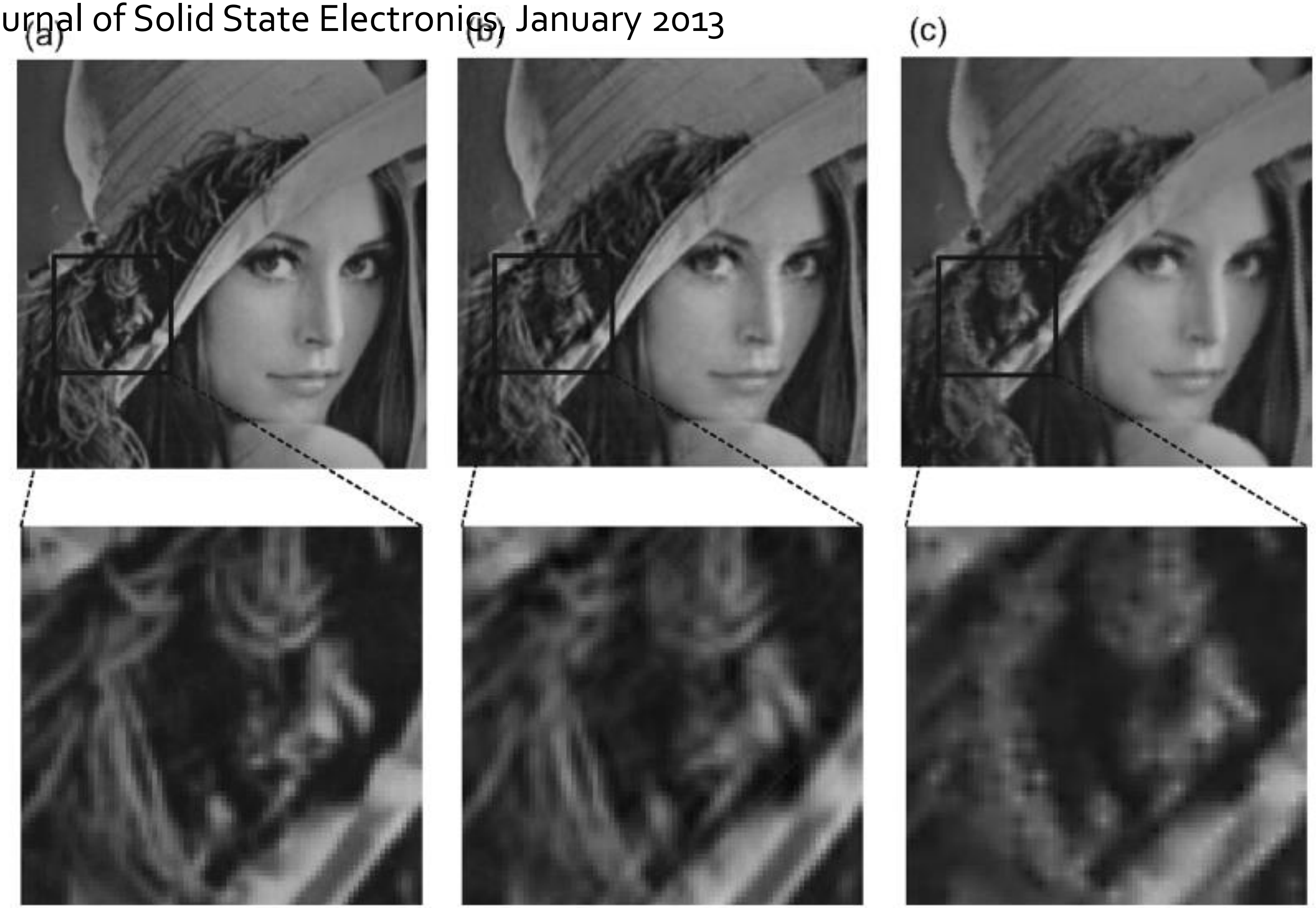


Fig. 17. Sample images captured in: (a) normal mode at 120 fps, (b) compressed sensing at CR = 1/8 and 960 fps, (c) downsampling at 1/8 ratio.