Lab 4: Phase-Plane Analysis and Design
By: Dr. Randy C. Hoover
Date Assigned: November 1, 2019
Date Due: Friday November 8, 2019
You may team up with another student to work on this particular lab.
Submit a single report with both partners names on it.

1 Introduction

In this lab we will investigate the construction of a phase-plane analysis for systems of differential equations. Recall that in class we discussed that a system of differential equations for the form

$$\begin{aligned}\dot{x}_1 &= f_1(\mathbf{x}, \mathbf{u}) \\ \dot{x}_2 &= f_2(\mathbf{x}, \mathbf{u}),\end{aligned}$$

for a given pair $(x_1,\ x_2)$, the pair $(\dot{x}_1,\ \dot{x}_2)$ is a vector originating from $(x_1,\ x_2)$. We can use this information to gain qualitative insight about the entire state space of the system without the need to solve the differential equations (either numerically or analytically if possible). An example of this is illustrated in Figure 1) for the system

$$\begin{aligned}\dot{x}_1 &= x_1 + x_1 x_2 \\ \dot{x}_2 &= -x_2 + x_2^2 + x_1 x_2 + x_1^3,\end{aligned}$$

where T_s determines the "resolution" of the grid we're evaluating the system over.

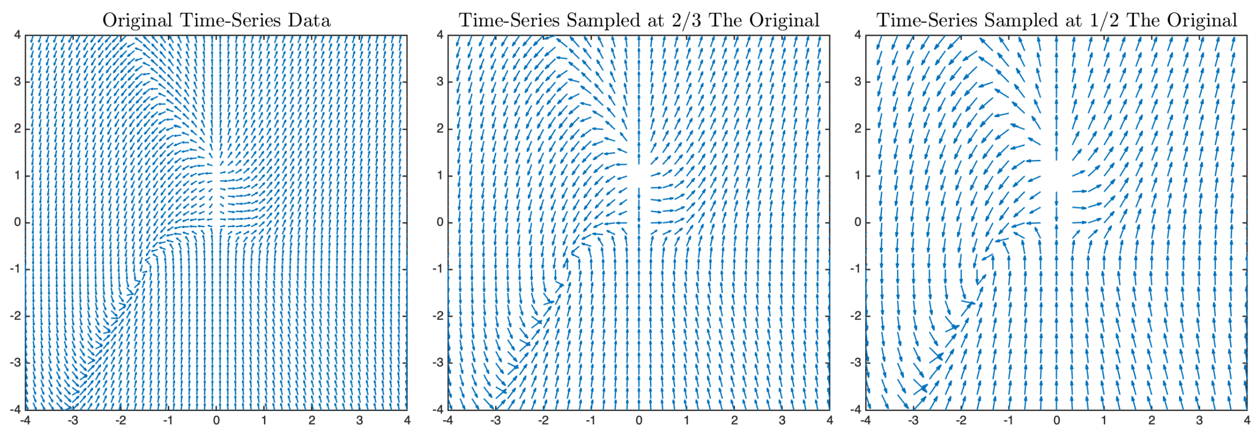

Figure 1: Graphical depiction of the phase plane analysis for a given 2-dimensional nonlinear system.

2 Deliverables

2.1 Modeling

Your task in this lab is to develop a MATLAB-based tool to produce a phase plane for a given system of differential equations (both linear and nonlinear).

2.2 Minimum requirements

Your phase plane tool must at minimum satisfy the following:

1. Allow the user to define a 2-dimensional system of both linear and nonlinear differential equations, a given "resolution" (T_s), the bounds on which they would like to investigate the system, and you will produce a phase plot similar to one of the plots in Figure 1). **All vectors in the plot should be normalized to the same length**.
2. Once you have your phase plane constructed, you will want to show a subset of trajectories on the phase plane. For whatever resolution T_s, you should divide $T_s/4$ (or more if needed) and display the solutions on the grid. Note: This grid sub-sampling will in effect be initial conditions for the system that you will need to pass to your favorite ODE solver (such as ode45). For simplicity, you might consider only using the boundary samples and adjust the time for the solution.
3. You will need to validate your solution on a minimum of:
   (a) A linear system of your choice.
   (b) A nonlinear system of your choice.
   (c) The pendulum system we investigated in class.

2.3 Bonus and Hints:

Bonus: As a bonus, create a MATLAB GUI similar to the pplane tool for graphical display of your phase plots. You might find MATLAB'S GUIDE tool useful here.

Hint: You may find the following MATLAB functions useful:

- `meshgrid`
- `linspace`
- `quiver`
- `axis`

You might notice using `quiver` that the length of your vectors vary based on the magnitude of the evaluation. Think about how you might normalize these vectors somehow so they are all the same length.

2.4 Submission

Submit in a single .pdf your code along with simulation plots illustrating the simulations (both simulation and design) verifying your control does indeed enable the quad to track a desired reference altitude. **You should use the MATLAB publisher for this task $\longrightarrow$ change the settings from html (default) to pdf.**.