RBF Quadrature for Neural Fields

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Summary

Goal: To create and test a neural field solver using radial basis function quadrature. The method should be

- High-order accurate
- Stable
- Geometrically flexible
- Fast (low complexity)

In the future, we will extend this to realistic curved 2D spatial domains.

Neural Field Models

- Tissue level models
- Integro-differential equation(s)
- Integral kernel represents neural network connectivity
- Non-linear firing rate function captures non-linear neural dynamics

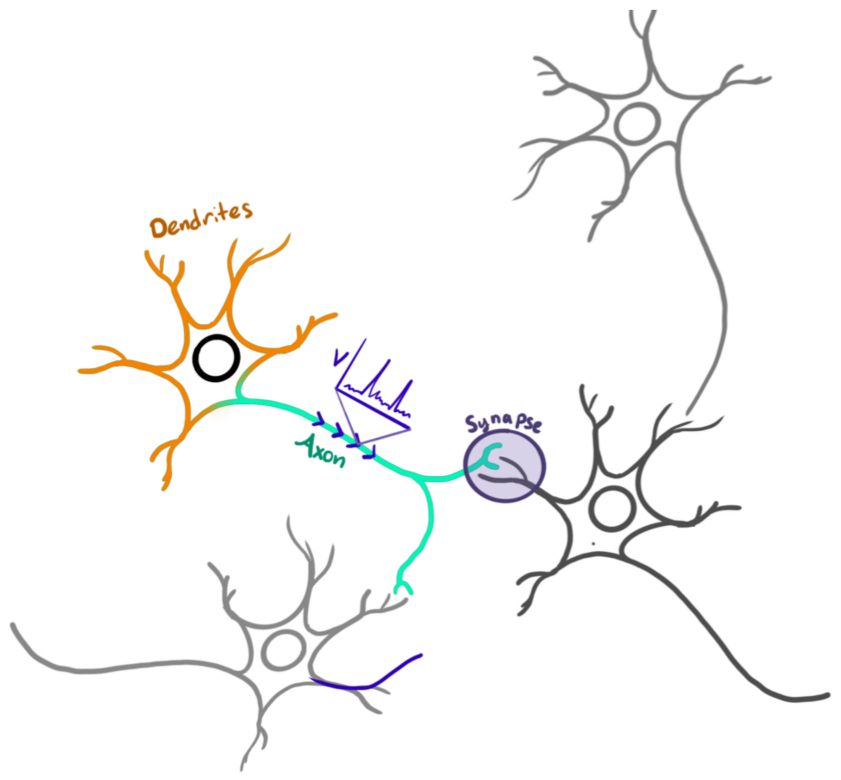


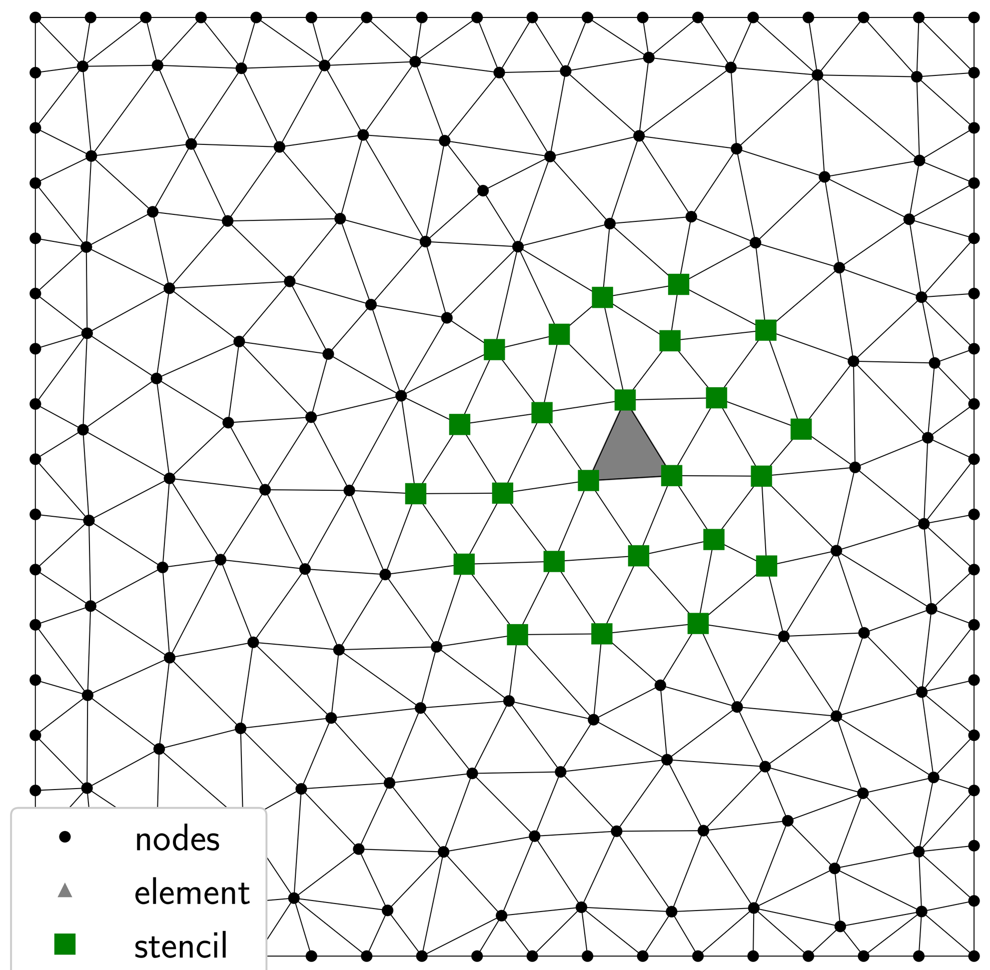
Image by Heather Cihak.

$$\partial_t u(t,\mathbf{x}) = -u + \iint_\Omega w(\mathbf{x},\mathbf{y}) f[u(t,\mathbf{y})]\ d\mathbf{y}$$

- $u(t,\mathbf{x})$ — Neural activity
- $w(\mathbf{x},\mathbf{y})$ — Connectivity kernel
- $f(\cdot)$ — Non-linear firing rate function
- $\Omega = [0,1]^2$ for now.

Radial Basis Function Quadrature Formuale

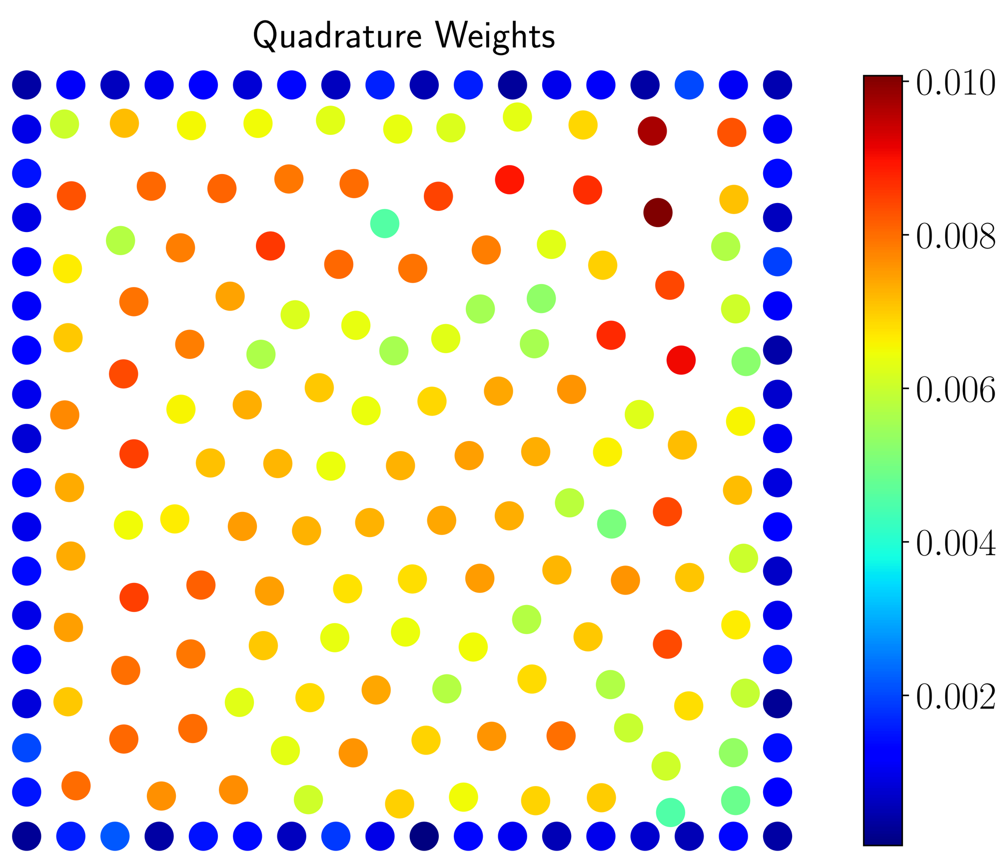
- Place N nodes in Ω
- Partition Ω into elements
- For each element
  - select the k nearest nodes (the stencil),
  - interpolate Lagrange functions,
  - integrate over the element,
  - sum over interpolants
- sum over elements



The local interpolants have the form

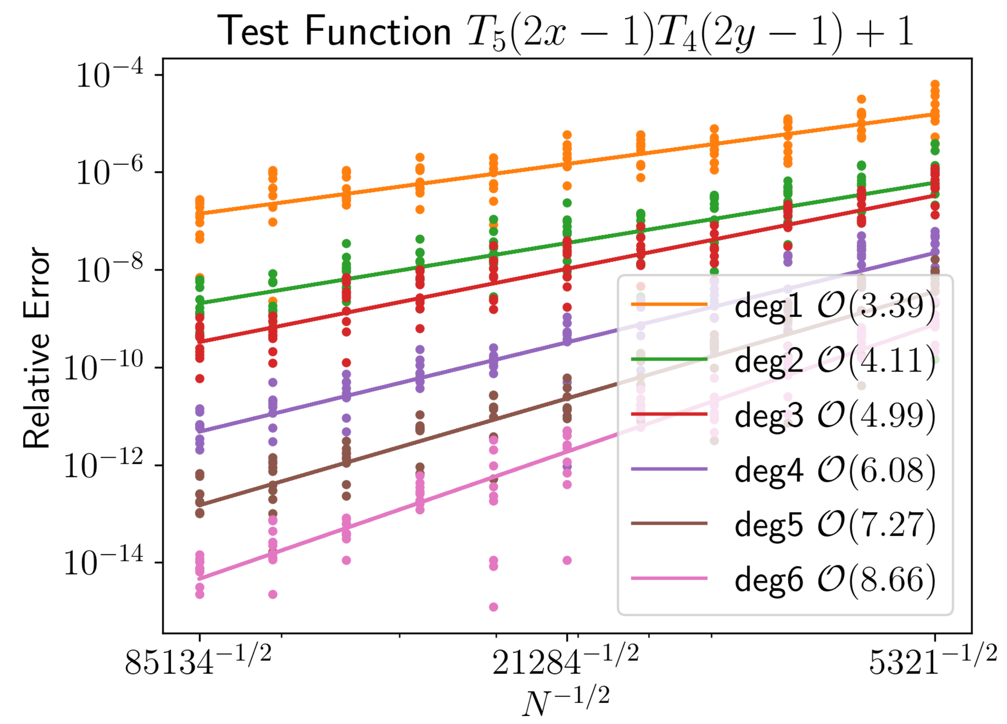
$$s(\mathbf{x}) = \sum_{i=1}^{k} c_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) + \sum_{j=1}^{m} \gamma_i \pi_j(\mathbf{x})$$

- ϕ - radial basis function (eg. $\phi(r) = r^3$)
- $\{\pi_j\}_{j=1}^m$ - polynomial basis
- k interpolation conditions
- m moment conditions



Quadrature Convergence

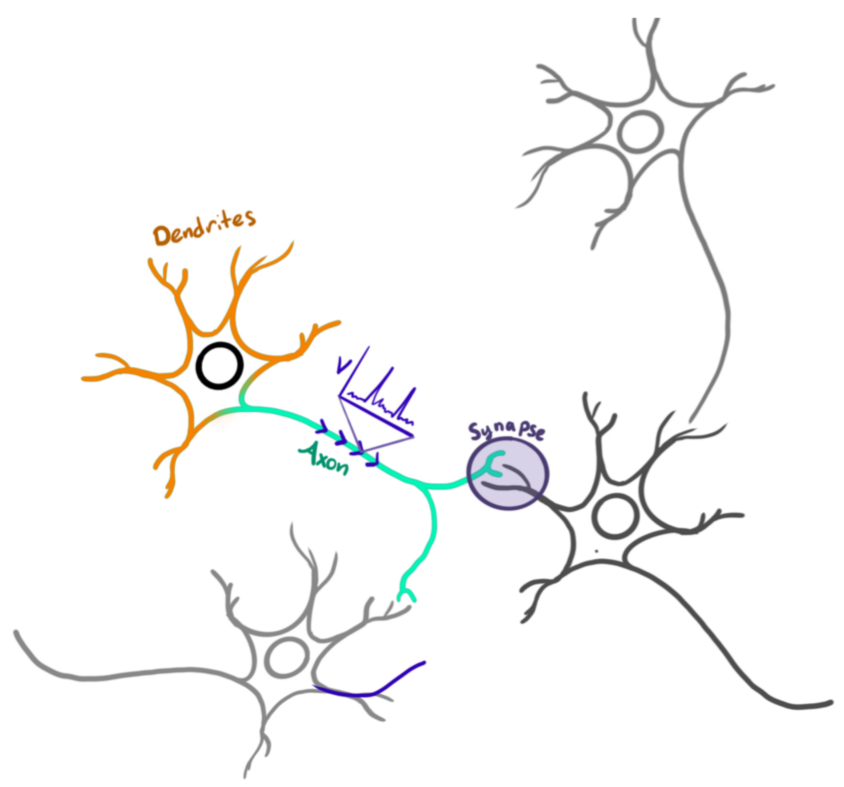
- No proven convergence rate.
- We expect at least $\mathcal{O}(d+1)$, where d is the degree of polynomial basis. (Think trapezoidal rule.)
- We test this on a product of Chebyshev polynomials.



- Rates better than expected.

Projection Method

- **Scientific Question**



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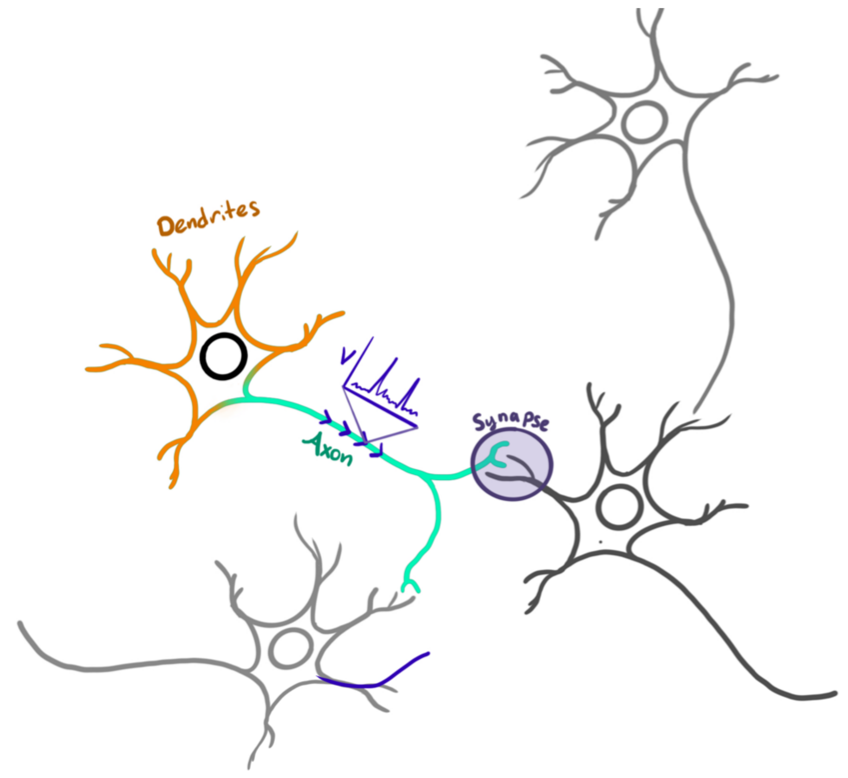
Neural Field IVP

- **Scientific Question**



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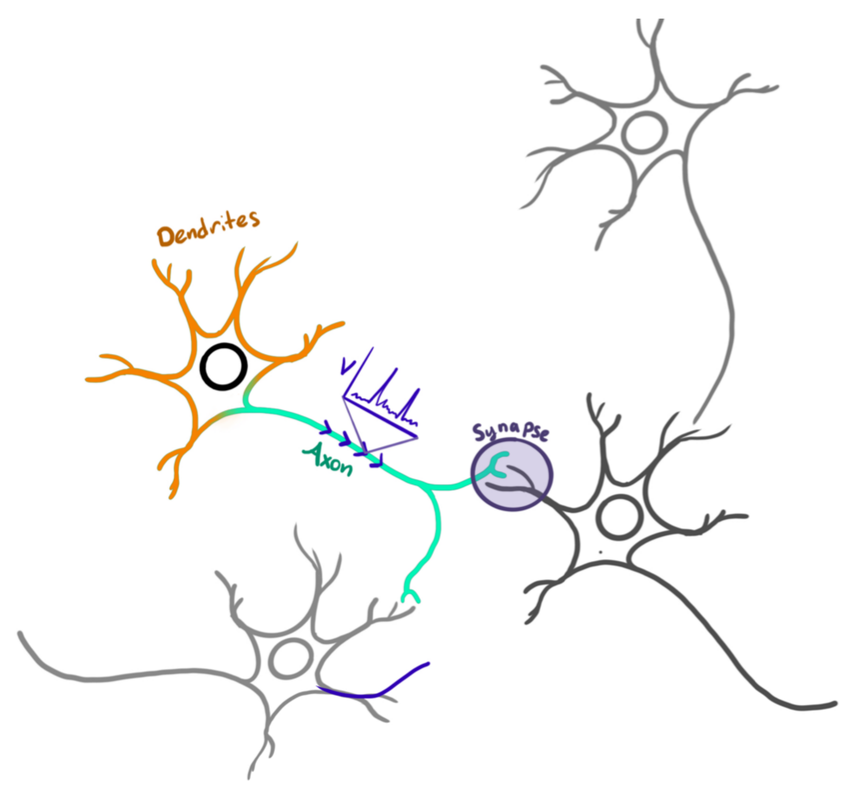
References and Funding

- Tsodyks, et al. (1998) Neural Computation
- Kilpatrick & Bressloff (2010) Physica D
- Kilpatrick & Ermentrout (2012) Phys. Rev. E This work was supported by NSF DMS-2207700.



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