Dynamic Analysis of Information Propagation in Online Social Network: SEIR Model

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***Abstract*— The main purpose of the present paper is to analyze the SEIR dynamical model of information propagation in online social networks. Existing studies, recognize the behavior of the system over time and under certain circumstances, which does not lead to a deep understanding of the system. Herein, the global dynamics of the system are analyzed by calculating the exact equilibrium points and verifying their stability. The phase diagrams and time responses are presented to demonstrate the behavior of the system for different constant coefficients. Also, by plotting basin of attraction graphs, the stability of each fixed point is further examined. These graphs illustrate how changes in the parameters can affect the system and change the stability of each equilibrium point. Obtained results reveal that the propagation of information highly depends on the system parameters, and by controlling the parameters, the behavior of the system can be controlled.**

Keywords—information propagation; social networks; equilibrium points; basin of attraction; stability analysis

I. Introduction

In recent years and with the development of social networks, information propagation is becoming easier and easier. Today, each person has a smart device; thus, they can send or receive information and interact with people from all over the world in a short time and as simple as a click. Therefore, the realization of how information propagates in a complex network and the analysis of such systems is an important focus of research.

As information propagation can be similar to the spreading of infectious diseases in some way, a similar model can be applied for both cases. The most famous models are the susceptible-infectious-recovered (SIR) epidemic model [1,2] and the susceptible-infectious-susceptible (SIS) epidemic model [3,4]. In epidemic models, there is an epidemic threshold on various complex networks that if the infection rate is above the threshold, then the infection spreads and becomes endemic (even pandemic); otherwise, the disease will die out [5-8]. With a simple translation from epidemic to rumor model vocabulary, a susceptible node corresponds to an agent who has not known the information; therefore, it is called “healthy” or “ignorant” node [9]. As an infected node can spread the disease, it can be translated to ones who forward the information to others, or “spreaders” [10]. Finally, the recovered nodes are regarded as “immune” nodes, the ones who had contacted with the information but may not be influenced by it any later. The first models of rumor spreading were introduced by Daley and Kendall which is known as the DK model [11], and Maki and Thompson [12] which illustrates a model of spreaders who have direct communication with other people. If two spreaders contact each other, the first one would become a stifler. Moreover, ignorants would convert to spreaders, after being in contact with them and if a spreader meets a stifler, the spreader would prefer to stop propagating the information. Consequently, the spreader would become a stifler. In the DK model [11], the spreaders attempt to infect the other nodes with the rumor, in pair-wise contacts between them. In the case in which a spreader meets with an ignorant, the information propagates and the ignorant would become a spreader. In the other two cases, the ones who are involved in the meeting would become stiflers. Zhao et al. [13] introduced a modified SIR rumor spreading model in the new media age with an extra condition that ignorant nodes will inevitably change their status once they become aware of a rumor. In addition, Qian et al. [14] improved the model by adopting the concept of independent spreaders and considering that nodes can obtain the rumor from different sources. Xia et al. [15] introduced an SEIR model considering the hesitating mechanism and the attractiveness and fuzziness of the content of rumors and showed that a decrease of fuzziness can effectively reduce the maximum rumor influence. Moreover, the spreading threshold is independent of the attractiveness of the rumor. Liu et al. [16] further developed the SEIR model with the hesitating mechanism using a feedback mechanism that can reduce the spread of the rumor. Also, other information propagation models can be mentioned such as the SICR model and adjusted-SICR model [17] by considering the counterattack mechanism of the rumor spreading and the influence of the self-resistance parameter. Zhao et al. [18] developed a new rumor spreading model with consideration of the mutual effect of the remembering and forgetting mechanism named SIHR. This model extends the classical SIR model by adding a direct link from ignorants to stiflers, which reduces the maximum rumor influence. Another SIR-based model is the SIRaRu model [19]. This model supplements some realistic conditions on previous rumor-spreading models and shows that the network topology exerts significant influence on rumor spreading. Zhang et al. [20] proposed a new SEIR-based model name SE2IR considering that the infected nodes hold different levels of propagation capacity. Huo and Chen introduced a modified rumor propagation model with consideration of scientific knowledge level and social reinforcement and showed that the rumor propagates faster and more widely in people without scientific knowledge, while rumor propagates slower and the final size of the rumor is smaller in people with scientific knowledge [21]. He and Liu proposed a novel competitive information dissemination macro model CISIR (Competitive Information Susceptible Infected Recovered) in online social networks. They claim that this model is reasonable and effective, and it provides a new scientific method and research approach to solve the problem of competitive propagation of different information types in social networks [22]. Yu et al. [23] introduced a new rumor propagation model considering the influence of discussants indicating that discussants have an important impact on the rumor propagation process, and divides the total population into four groups including ignorant, discussants, spreaders, and removers.

In [24], the SEIR model of information propagation in online social networks with varying total population size is examined .The difference between this model and other SEIR-based models is that it considers the changes in population size using dynamic factors such as birth and death. Furthermore, the SEIR model has an advantage over the SIR model, that is it adds an exposed compartment for individuals who have already received the information but not getting immediate notice and may become infected and contagious after an incubation period. As a result, the SEIR model with varying total population size is chosen to be analyzed. Afterwards, the global dynamics of information propagation in a complex social network are analyzed analytically as well as numerically by plotting phase diagrams and time responses. The previous works on this subject were mostly numerical and represented the time response of the system for specific parameters and under certain conditions. However, this work presents a classification of the performance of the system using the basin of attraction for different ranges of parameters and illustrates the dynamical behaviors of the system; i.e., how the change in one parameter can alternate the stability of equilibrium points; thus, the behavior of the system would change, as well. Therefore, a great understanding of the system and how it conducts for different sets of parameters can be obtained.

Our study is structured as follows. In section II, the SEIR model of information propagation is introduced and it is nondimensionalized to reduce the number of model parameters, which results in a more convenient system to analyze and understand. Also, the system is simulated numerically to have a better realization of how it works, and two different behaviors are identified. Next, Section III provides the stability analysis of equilibrium points by linearizing the system using the Jacobian matrix around each fixed point. In section IV, simulations are done on the system and the basin of attraction is plotted for different parameters, and how the change of a single parameter can affect the system is further examined. Then, the findings are validated using time responses and phase diagrams. Finally, section V presents the conclusions.

II. Model and Methods

A. SEIR Model of Information Propagation

The SEIR model is used for analyzing the characteristics of information propagation in online social networks with varying total population size [24].

To establish this model, online social network users (nodes), can be classified into four categories: healthy nodes (denoted as S), latent nodes (denoted as E), transmission nodes (denoted as I), and immune nodes (denoted as R). Also, the number of total population of users can be defined as N(t) = S(t) + E(t) + I(t) + R(t) and it may change with time.

The meaning of these four categories of subjects is explained as follows:

- the S(t) healthy nodes are the group of users, whose news columns have not yet received any news about the information, up to time t;
- the E(t) latent nodes are those whose news columns have received the information already but not getting immediate notice. This group of nodes always preserves the chance of noticing the information;
- I(t) represent the number of users, who receive, read, believe and forward the message that contains the information, onto their status bars in a unit time at time t;
- R(t) is the number of users, who had contacted with the information but may not be influenced by it any later.

The SEIR model can be represented as the following system of differential equations

$$\begin{aligned}\dot{S} &\equiv \frac{dS}{dt} = \beta.N - \delta.S - \frac{\lambda.I.S}{N}\\ \dot{E} &\equiv \frac{dE}{dt} = \frac{\lambda.I.S}{N} - (\varepsilon+\delta)E\\ \dot{I} &\equiv \frac{dI}{dt} = \varepsilon.E - (\gamma+\delta)I\\ \dot{R} &\equiv \frac{dR}{dt} = \gamma.I - \delta.R\end{aligned} \tag{1}$$

Where β, birth, is the unit time rate of new user registrations in the online social network, δ, death or deactivation, is the user deactivation rate, ε is the probability or speed for the latent nodes turning into the transmission nodes, γ is the probability or speed for the transmission nodes turning into the immune nodes and λ indicates the frequency of new messages received by an average user in the online social network.

The number of initial transmission nodes that make up the first batch of social network users propagating the information considered as I_0 ($I(t=0)=I_0$).

Because N(t) = S(t) + E(t) + I(t) + R(t) is time-dependent, using Eq. (1) we have

$$\begin{aligned}\dot{N} &= \dot{S}+\dot{E}+\dot{I}+\dot{R}\\ &= \left[\beta.N - \delta.S - \frac{\lambda.I.S}{N}\right] + \left[\frac{\lambda.I.S}{N} - (\varepsilon+\delta)E\right]\\ &\quad + [\varepsilon.E - (\gamma+\delta)I]\\ &\quad + [\gamma.I - \delta.R]\\ &= (\beta-\delta)N\end{aligned} \tag{2}$$

As expected, this result implies that the evolution of the total population N(t) can be determined by the difference between the birth and deactivation rates of users in the online social network. Equation (2) can be easily solved as follows:

$$N = N_0 \, exp[(\beta - \delta)t] \tag{3}$$

Here N_0 is the integer representing the total number of users in the online social network at some beginning moment [25].

B. Nondimensionalization

To reduce the number of parameters in the model, the system was nondimensionalized by defining the following parameters.

$$s = \frac{S}{N}, e = \frac{E}{N}, i = \frac{I}{N}, r = \frac{R}{N} \tag{4}$$

The summation of these fractions would be the total density of users, so $s + e + i + r = 1$. By substituting Eq. (4) into equation set (1), the following non-dimensional equations are obtained.

$$\begin{aligned} s' &= \beta - \delta s - \lambda is \\ e' &= \lambda is - (\varepsilon + \delta)e \\ i' &= \varepsilon e - (\gamma + \delta)i \\ r' &= \gamma i - \delta r \end{aligned} \tag{5}$$

The first three equations are independent of the fourth equation and form a consistent system by themselves and r can still be determined from $r = 1 - (s + e + i)$.

By setting the right-hand of the equation set (5) equal to zero, it is revealed that the system has two equilibrium points; the first one, e_1, is (β/δ,0,0,0), which indicates the situation with no information spreading, and the second one, which is called e_2, is as follows:

$$\left(\frac{(\varepsilon+\delta)(\delta+\gamma)}{\varepsilon\lambda}, -\frac{\varepsilon(-\beta\lambda+\delta^2+\delta\gamma)+\delta^2(\delta+\gamma)}{\varepsilon\lambda(\varepsilon+\delta)}, -\frac{\varepsilon(-\beta\lambda+\delta^2+\delta\lambda)+\delta^2(\delta+\gamma)}{\lambda(\varepsilon+\delta)(\delta+\gamma)}, -\frac{\gamma(\varepsilon(-\beta\lambda+\delta^2+\delta\gamma)+\delta^2(\delta+\gamma))}{\delta\lambda(\varepsilon+\delta)(\delta+\gamma)}\right)$$

The time response of the system is plotted numerically to have a better understanding of how it works. The density of transmission nodes is set equal to 0.01 ($i_0 = 0.01$), meaning that when the information starts to propagate, only a small number are informed about it. On the other hand, other users are not yet aware of the information, so they are considered as healthy nodes, and it was assumed that healthy nodes comprise 99% of the population of the social network. The values of system parameters are considered as λ = 1, β = 0.2, γ = 0.7, δ = 0.2, and ε = 0.7. As a result, the system fixed points would be (1, 0, 0, 0) and (1.1571, -0.0349, -0.0272, -0.0951). The system time response and phase portrait are plotted in Fig. 1a and 1b, respectively. It can be seen from Fig. 1a, the density of online social network users whose news columns have not yet received any news about the information decreases at first, and then it increases. The densities of other nodes, on the other hand, are acting exactly the opposite; but with smaller slope while increasing. At first 10 units of time, the more users are informed of the news, the more s(t) decreases. In t = 10 to t = 20, new users are joining who do not know about the information, which causes s(t) to increase.

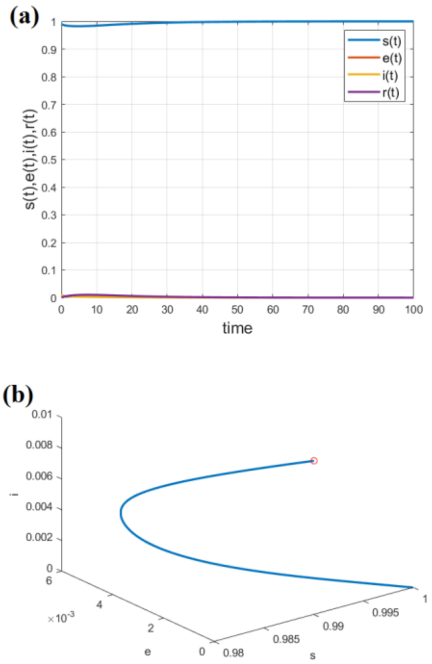

Fig. 1. (a) time response; (b) phase diagram

Now, let us select different values for the system parameters to see a completely different behavior of the system. The parameters are considered as λ = 1, β = 0.2, γ = 0.1, δ = 0.2, and ε = 0.8. Consequently, the fixed points are (1, 0, 0, 0) and (0.3750, 0.1250, 0.3333, 0.1667). The system time response and phase portrait are plotted in Fig. 2a and 2b, respectively. As shown in Fig. 2a, the density of healthy nodes is decreasing until around t = 23. At this moment, it reaches its minimum, and then with the new users showing up, it undergoes a slight increase and then settles at its equilibrium. Although the joining of new users slightly increases the density of the healthy nodes, it cannot help the density of healthy nodes to reach its initial value. Therefore, it can be said that the information has propagated, and many users have received it. On the other hand, as the information propagates, the density of transmission nodes and the density of the latent nodes increase gradually. At around t = 20, the most rapid propagation of information occurs with the transmission node density reaching its peak. However, with latent nodes turning into immune nodes, the immune node density surpasses the density of latent nodes. Therefore, the density of transmission nodes faces a slight decrease and finally reaches its equilibrium with a larger value than its initial value, which means there was a significant increase in the density of transmission nodes. After t = 40, it can be seen that the density of each type of node faces numerically insignificant changes, so it can be concluded that the information will no longer propagate. As seen in Fig. 1 and Fig. 2, the system may show different dynamical behaviors depending on its parameters. It is worthy to analyze this behavior analytically and categorize the system's dynamical behavior based on its parameters which will be performed in the next section by stability analysis of the system's fixed points.

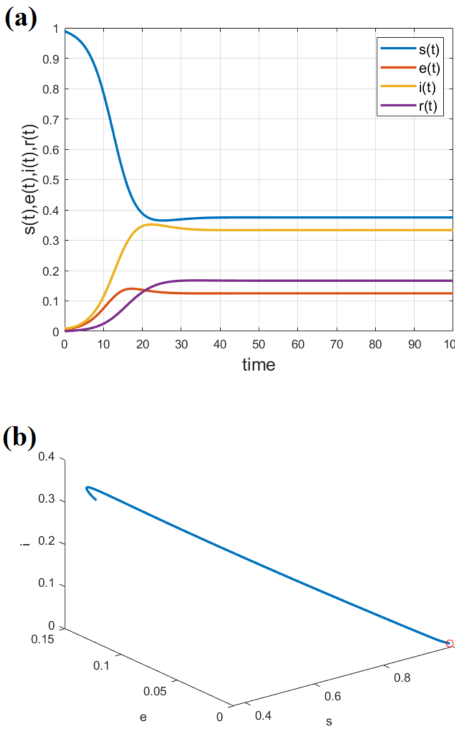

Fig. 2. (a) time response; (b) phase diagram

III. Stability Analysis of Equilibrium Points

For stability analysis, the system was linearized using the Jacobian matrix [26]:

$$J_{(x,y,z)} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix}$$

As previously mentioned, the system can be treated as a system of three differential equations and the fourth equation can be ignored as its solution can be obtained from $r = 1 - (s + e + i)$. Consequently, The Jacobian matrix of the first three equations of system (5) is presented as:

$$J_{(s_0,e_0,i_0)} = \begin{bmatrix} -\delta - \lambda.i_0 & 0 & -\lambda.s_0 \\ \lambda.i_0 & -(\varepsilon+\delta) & \lambda.s_0 \\ 0 & \varepsilon & -(\gamma+\delta) \end{bmatrix} \tag{6}$$

By calculating the eigenvalues of the Jacobian matrix around each equilibrium point, their stability can be examined.

For equilibrium point e_1, the Jacobian matrix becomes

$$J_{(1,0,0)} = \begin{bmatrix} -\delta & 0 & -\lambda \\ 0 & -(\varepsilon+\delta) & \lambda \\ 0 & \varepsilon & -(\gamma+\delta) \end{bmatrix} \tag{7}$$

and its eigenvalues are

$$\lambda_1 = -\delta$$
$$\lambda_2 = \frac{1}{2}\left(-2.\delta - \sqrt{\gamma^2 - 2.\gamma.\varepsilon + 4.\lambda.\varepsilon + \varepsilon^2} - \gamma - \varepsilon\right)$$
$$\lambda_3 = \frac{1}{2}\left(-2.\delta + \sqrt{\gamma^2 - 2.\gamma.\varepsilon + 4.\lambda.\varepsilon + \varepsilon^2} - \gamma - \varepsilon\right)$$

Without losing generality, the activation rate is considered equal to the deactivation rate in the social network ($\beta = \delta = 0.2$). This assumption means that the total population, N(t), does not change over time. By setting $\delta = 0.2$, λ_1 will be negative. but both λ_2 and λ_3 can be either positive or negative. As long as all of the above eigenvalues are negative, e_1 is stable, and e_2 is unstable. By finding the eigenvalues of the Jacobian matrix around e_2, it can be realized that whenever all of its eigenvalues are negative (e_2 is stable), e_1 is unstable. In other words, with different parameters, there is one and only one stable equilibrium point, which is either e_1 or e_2.

When e_1 is stable, it means that in this situation, no information spreading occurs, but when e_2 is stable, there is a condition in which information propagates.

IV. Simulations and Results

By Setting $\lambda = 1.5$, 1, and 0.5, the model is analyzed based on the variation of γ and ε as the bifurcation parameters. As γ is the probability for the transmission nodes turning into the immune nodes, and ε is the probability for the latent nodes turning into the transmission nodes, these two parameters can change between 0 and 1. In what follows, two different behaviors are shown. In one of them, e_1 is unstable, and e_2 is stable. This case is illustrated with blue color. In the second case, e_1 is stable, and e_2 is unstable. The red color represents this behavior. The former indicates a situation where the information propagates, and a considerable number of users receive the information, but the latter shows a condition in which no information spreading occurs.

As it can be seen from Fig. 3a, with such parameters, in most cases point (1,0,0) is the stable fixed point, which means no information spreads. With low probability for the transmission nodes turning into the immune nodes (γ) and high probability for the latent node turning into the transmission nodes (ε), still, information can propagate, but with an increase in γ, the system will face a situation in which information propagation becomes impossible. This result shows that the system is highly dependent on γ.

In Fig. 3b and c, λ was increased, and it is obvious that the number of cases in which the information can spread increased, as well. Here, the important point to note is that this system is highly dependent on parameter λ as well as γ. The reason is that λ indicates the frequency of new messages received by an average user, so it has an important role in information propagation.

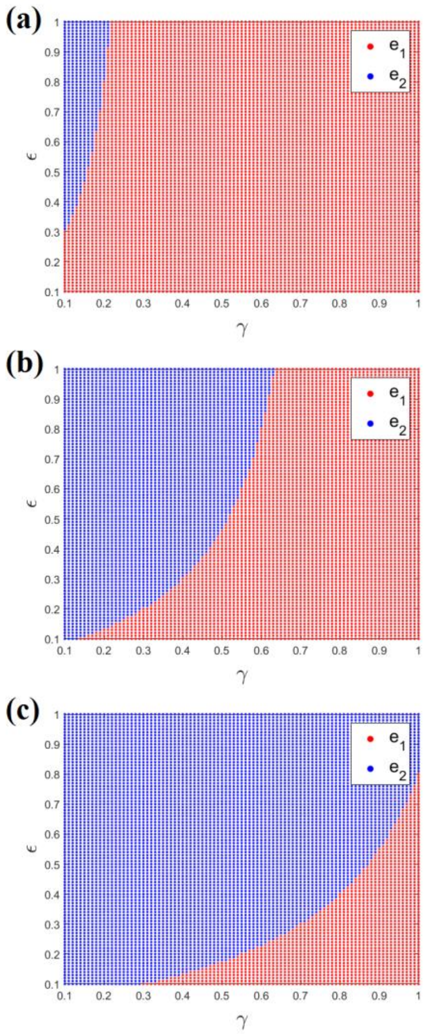

Fig. 3. basin of attraction of two fixed points.
$\beta = \delta = 0.2$ (a) $\lambda = 0.5$; (b) $\lambda = 1$; (c) $\lambda = 1.5$

In order to have a better understanding of the impact of different parameters, the simulation was performed for a different set of parameters. This time, system parameters were set as $\gamma = 0.25$, 0.5, and 0.75, while λ and ε were considered as the bifurcation parameters. Once again, the impact of γ and λ on the behavior of the system can be seen in Fig. 4. On the other hand, in both figures, it is shown that ε has a smaller effect on the system's dynamical behavior in comparison with γ and λ.

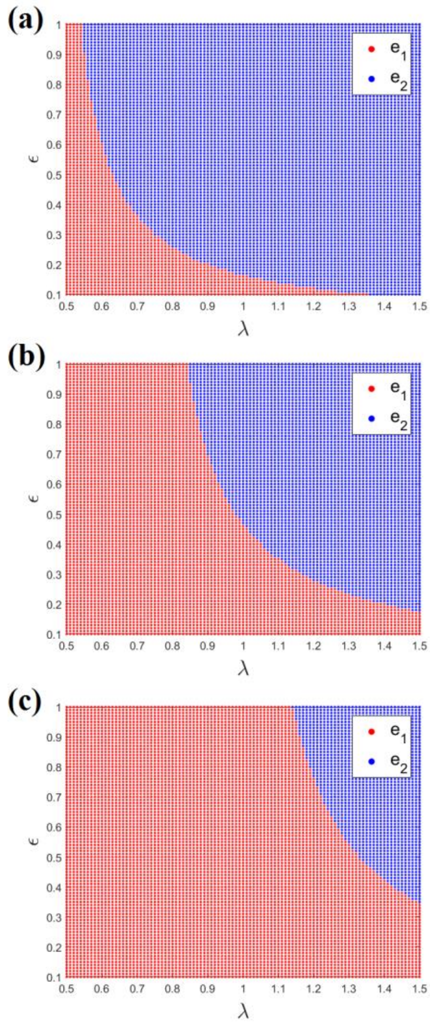

Fig. 4. basin of attraction of two fixed points.
$\beta = \delta = 0.2$ (a) $\gamma = 0.25$; (b) $\gamma = 0.5$; (c) $\gamma = 0.75$

For further analysis, the time response and phase diagram (Fig. 5) of the system were plotted. In order to plot Fig. 5, first, the initial value for transmission nodes density was selected as I = 0.01. Therefore, the density of the healthy nodes is close to 1 (S = 0.99). Then, a point from the red region of Fig. 4 was randomly chosen; e.g. $\lambda = 0.7$, $\varepsilon = 0.3$, $\beta = \delta = 0.2$, and $\gamma = 0.5$ were picked and with the mentioned values, the time response chart, and phase diagram were plotted. As expected, the system was attracted to e_1. Although at first there is a slight decrease in the density of healthy nodes, as transmission nodes change into immune nodes, the density of immune nodes increases and it prevents the propagation of information. After this time, with the registration of new users, the density of healthy nodes climbs up, and after some time it reaches 1, which is the equilibrium point for the density of healthy nodes. The phase diagram (Fig. 5b) further proves our findings.

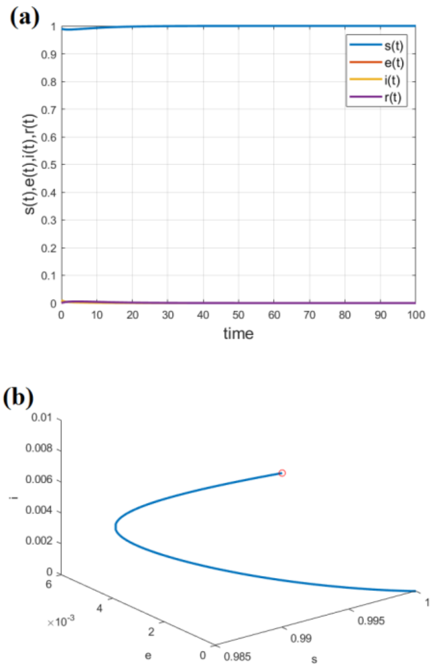

Fig. 5. (a) time response; (b) phase diagram

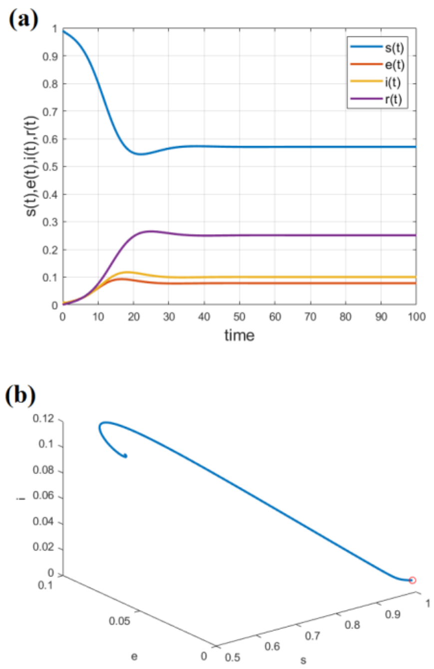

Fig. 6. (a) time response; (b) phase diagram

Another time response chart and phase diagram, with the same initial conditions, but different parameters (Fig. 6), were also plotted. This time, a point from the blue region of Fig. 4 was chosen; e.g. $\lambda = 1.5$ and $\varepsilon = 0.9$ were picked with the same values for β, δ, and γ as before. Once again, the time response chart and phase diagram were plotted. As expected, the system was attracted to e_2 and the information propagates to some extent. Until t = 20, the density of healthy nodes sharply decreases, which indicates the first round of information propagation. Through this period, the density of immune nodes increases at a lower rate than the density of healthy nodes. Also, a gradual increase in the density of latent nodes and transmission nodes can be seen. Around time t = 20, the density of healthy nodes reaches the minimum. About the same time, the density of immune nodes reaches its maximum and somehow prevents information propagation. By moving forward in time, more users register in the social network so the density of healthy nodes faces a slight increase and then reaches its equilibrium. Also, the density of other nodes undergoes a slight decrease and reaches its equilibrium, as well. Therefore, the information no longer propagates. The phase diagram (Fig. 6b), further proves our findings.

V. CONCLUSION

In this paper, the SEIR model was examined by plotting the phase planes and time responses. Four categories of users are defined as healthy nodes (S), latent nodes (E), transmission nodes (I), and immune nodes (R). It was shown that the system has two equilibrium points, both changing with different values for the parameters. It was seen that the joining of new users plays a really important role in the behavior of the system, and every small change in constant-coefficient can have very significant effects on users' performance. With an increase in the conversion rate from latent nodes to transmission nodes and frequency of newly received messages, and by reducing the probability of the transmission nodes converting into immune nodes, the information would spread more. But, in long term, and with the joining of new users who do not know anything about the information, the fluctuation ends, and the information stops spreading, which means the system has reached its equilibrium. The dynamic analysis of information propagation introduced here is capable of presenting different behavior even with changing only one constant coefficient.

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