1 Proof

A complete CFG of a functional language, $CCFG = (N, E)$ with N being a set of nodes and E a set of ordered pairs of nodes representing edges, has the following properties:

1. Acyclic, since there is no loop structure
2. Directed
3. Connected
4. Binary, $\forall n \in N, 0 \leq deg^+(n) \leq 2$. Since the decisions are boolean, there are maximum two outgoing edges possible for each node.
5. Rooted, $|\{n \in N | deg^-(n) = 0\}| = 1$. There is one single root for the graph, the *source node*, and it has only one outgoing edge.
6. $|\{n \in N | deg^+(n) = 0\}| = 1$; there is one *sink node*.
7. Each edge leaving a node $n \in N$ such $deg^+(n) = 2$, is connected to a node that has exactly one incoming edge. In other words, each branch starts with a non-joint node which is a node with one single incoming edge.

To prove "if every node $n \in N$ is visited then all the edges $e \in E$ are visited", we first prove that every outgoing edge is visited. Then, by cases, we prove that all the ingoing edges are visited.

For every node $n \in N$ that is not the *source node* nor the *sink node*, with one outgoing edge e:

1. If every node is visited then n is visited
2. Since the CFG is connected and n is not the *sink node*, If the execution flow reaches n then it should exit from the only edge e
3. If n is visited then e is visited

For every node $n, n1, n2 \in N$ that is not the *source node* or the *sink node*, that n has two outgoing edges $e1 = (n, n1)$ and $e2 = (n, n2)$:

1. If every node is visited then n,$n1$ and $n2$ are all visited
2. According to the property (7), since $deg^+(n) = 2$, We have $deg^-(n1) = 1$ and $deg^-(n2) = 1$. Hence, $e1$ is the only way to reach $n1$ and $e2$ is the only way to each $n2$
3. If $n1$ is visited then $e1$ is visited.
4. If $n2$ is visited then $e2$ is visited.
5. If n, $n1$ and $n2$ are all visited then $e1$ and $e2$ are visited

Since the *source node* has only one outgoing edge, if it is visited then its outgoing edges are visited too. The *sink node* has no outgoing edge.

Thus, if every node is visited then all the respective outgoing edges are visited. Since the set of all the outgoing edges is equal to E, if every node $n \in N$ is visited then all the edges $e \in E$ are visited.