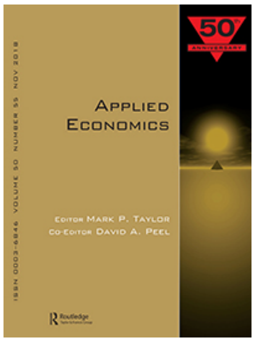

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Some stylized facts of the cryptocurrency market

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ABSTRACT
We examine the stylized facts of eight forms of cryptocurrencies representing almost 70% of cryptocurrency market capitalization. In particular, the empirical results show that (1) there exists heavy tails for all the returns of cryptocurrencies; (2) the autocorrelations for returns decay quickly, while the autocorrelations for absolute returns decay slowly; (3) returns of cryptocurrencies display strong volatility clustering and leverage effects; (4) Hurst exponent for volatility is more volatile than that of the returns, while they all suggest the long-range dependence phenomena; and (5) there exists power-law correlation between price and volume.

KEYWORDS
Stylized facts; cryptocurrency market; volatility clustering; heavy tails; autocorrelations; long-range dependence

JEL CLASSIFICATION
G12; G14

I. Introduction

Though in its infancy, the market capitalization of cryptocurrency has dramatically increased and it attracts attentions from both practitioners and academics. For example, Bitcoin already has a market capitalization of 206.76 billion U.S. dollars on 20 January 2018 and the co-founder of Ripple once became one of the wealthiest persons in the world.[1] In particular, according to the statistics in https://coinmarketcap.com, there are more than 1400 forms of cryptocurrencies and the Bitcoin only captures around 35% of the total market capitalization. Stylized facts refer to some important statistical properties of random variations of assets prices, including heavy tails, autocorrelations, volatility clustering, leverage effect, etc. (Cont 2001). Meanwhile, understanding the stylized facts of asset returns is crucial for evaluating econometric models and constructing financial theories (Reiter 1995; Cont 2001). However, a comprehensive investigation of the stylized facts of the cryptocurrency market is unexplored. In this article, we fill this gap by investigating the heavy tails, autocorrelations, volatility clustering, leverage effect, long-range dependence and power-law correlation for eight forms of cryptocurrencies, i.e. Bitcoin (BTC), Dash (DASH), Ethereum (ETH), Litecoin (LTC), NEM (XEM), Stellar (XLM), Monero (XMR) and Ripple (XRP).

Existing literature on the cryptocurrency mainly focuses on Bitcoin and can be divided into three aspects. The first is to analyse the inefficiency of Bitcoin (Almudhaf 2018; Alvarez-Ramirez et al., 2018; Nadarajah and Chu 2017; Tiwari et al. 2018). Urquhart (2016) employs the Ljung–Box test, R/S Hurst Exponent and other methods to analyse the inefficiency of Bitcoin and conclude that Bitcoin is not weakly efficient in the entire sample period but may be in the process of moving to efficient. Bariviera (2017) adopts the rolling window detrended fluctuation analysis (DFA) method, finding that the volatility of Bitcoin has long-range dependence throughout the sample period, suggesting the inefficiency. The second refers to the investigations on the pricing dynamics of Bitcoin and its correlations with other financial assets (Baek and Elbeck 2015; Baur et al., 2017a; Blau 2018; Bouri et al. 2017a, 2017b; Chen and He 2013; Cheung, Roca, and Su 2015; Dyhrberg 2016; Feng, Wang, and Zhang 2018; Śmiech and Papież 2017). Dyhrberg (2016) shows than Bitcoin can be used as a hedge against American dollar in the short term and Bouri et al. (2017c) find that

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[1]Data source: (1) https://coinmarketcap.com; (2) https://www.cnbc.com/2018/01/04/ripple-co-founder-is-now-richer-than-the-google-founders-on-paper.html.

Bitcoin can only serve as a strong safe haven against Asian stocks. Ciaian, Rajcaniova and Kancs (2016) conclude that Bitcoin prices are driven by market forces and attractiveness to investors, not macroeconomic factors. Baur et al. (2017b) find no correlations between Bitcoin and other traditional assets, either during normal times or during financial crisis, and conclude that Bitcoin could be used to speculate. Corbet, Lucey and Yarovya (Forthcoming) find that there exist pricing bubbles for Bitcoin from January 2009 to November 2017. The third focuses on the long-range dependence of Bitcoin returns (Balcilar et al. 2017; Lahmiri and Bekiros 2018; Lahmiri, Bekiros, and Salvi 2018; Urquhart 2017). For example, Bariviera et al. (2017) study the long-range dependence of the Bitcoin returns at different timescales by means of rolling window DFA, and conclude that different time scales have similar long-range dependence. Jiang, Nie and Ruan (Forthcoming) also use the rolling window method to estimate the generalized Hurst exponent, proving the existence of long-range dependence in the Bitcoin market. With a few exceptions, there are also some studies on banning effect, Google searches, models fitting and price controls of Bitcoin (Gandal et al., Forthcoming; Hendrickson and Luther 2017; Katsiampa 2017; Yelowitz and Wilson 2015).

Our article contributes to the existing literature by giving the first empirical analysis of the heavy tails, autocorrelations, volatility clustering, leverage effect, long-range dependence and power-law correlation for eight forms of cryptocurrencies, which account for 67.29% of cryptocurrency market capitalization. In particular, we find heavy tails for all the cryptocurrencies' returns and the absence of autocorrelations and the slow decay of autocorrelations are confirmed. Besides, with the GARCH model, the GJR model, the DFA and the detrended moving average cross-correlation analysis (DMCA), the empirical results further reveal volatility clustering, leverage effect and long-range dependence for the returns of cryptocurrencies.

The remainder of this article is organized as follows. Section II describes the methodology. Section III presents the data and empirical results and Section IV concludes.

II. Methodology

In this section, we illustrate the methodologies used to describe the statistical properties of cryptocurrencies' returns, i.e. heavy tails, autocorrelations, volatility clustering and long-range dependence.

Skewness, kurtosis and Jarque–Bera test

We begin by testing the normality of the returns of cryptocurrencies with the methods of skewness, kurtosis and Jarque–Bera test. The calculations of skewness and kurtosis are defined respectively as

$$s = \frac{E(x-\mu)^3}{\sigma^3} \tag{1}$$

$$k = \frac{E(x-\mu)^4}{\sigma^4} \tag{2}$$

where μ is the mean of time series x, σ is the SD of x, and $E(t)$ represents the expected value of the quantity t. Skewness and kurtosis compute a sample version of this population value.

The null hypothesis of Jarque–Bera test is that the sample data come from a normal distribution with an unknown mean and variance. The Jarque–Bera test estimates the goodness of fit of the sample data for skewness and kurtosis. The test is specifically designed for alternatives in the Pearson system of distributions. The test statistic is calculated as

$$\mathrm{JB} = \frac{n}{6}\left(s^2 + \frac{(k-3)^2}{4}\right) \tag{3}$$

where n is the sample size, s is the sample skewness, and k is the sample kurtosis. For large sample sizes, the test statistic has a chi-square distribution with two degrees of freedom.

Autocorrelations

Autocorrelation are the correlations of signals with delayed copies of themselves. We measure the correlation between y_t and y_{t+k}, where $k = 0, \ldots, K$ and y_t is a stochastic process. The model for the autocorrelation for lag k is

$$r_k = \frac{1}{c_0} \cdot \frac{1}{T} \sum_{t=1}^{T-k} (y_t - \bar{y})(y_{t+k} - \bar{y}) \tag{4}$$

where c_0 is the sample variance of the time series.

GARCH model

We employ the GARCH model to test volatility clustering. A GARCH model is a statistical model that addresses conditional heteroscedasticity. Specifically, this model posits that the current conditional variance is the sum of the past conditional variances (the GARCH component or polynomial) and the past squared innovations (the ARCH component or polynomial). In particular, considering the time series

$$y_t = \mu + \varepsilon_t \tag{5}$$

where $\varepsilon_t = \sigma_t z_t$. The GARCH$(P, Q)$ conditional variance process, σ_t^2, has the form

$$\sigma_t^2 = \kappa + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{Q} \alpha_k \varepsilon_{t-k}^2 \tag{6}$$

In lag operator notation, the model is defined as

$$\left(1 - \sum_{j=1}^{P} \beta_j L^j\right) \sigma_t^2 = \kappa + \left(1 - \sum_{k=1}^{Q} \alpha_k L^k\right) \varepsilon_t^2 \tag{7}$$

For stationarity and positivity, GARCH models use these constraints: (1) $\kappa > 0$, (2) $\beta_j > 0$, $\alpha_k > 0$, (3) $\sum_{j=1}^{P} \beta_j + \sum_{k=1}^{Q} \alpha_k < 1$.

In particular, the ARCH(Q) model is identical to GARCH$(0, Q)$ model and we need to first confirm the ARCH effect before implementing the GARCH model.

GJR model

The Glosten, Jagannathan and Runkle (1993) model is a generalized GARCH model and suitable for modelling asymmetric volatility clustering. Specifically, the model assumes that the current conditional variance is the sum of past conditional variances (the GARCH component or polynomial), past squared innovations (the ARCH component or polynomial) and past squared, negative innovations (the leverage component or polynomial). Based on GARCH model, the conditional variance process σ_t^2 satisfies the following equation:

$$\sigma_t^2 = \kappa + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{Q} \alpha_k \varepsilon_{t-k}^2 + \sum_{k=1}^{Q} \xi_k I[\varepsilon_{t-k} < 0] \varepsilon_{t-k}^2 \tag{8}$$

where $I(\cdot)$ is an indicator function and ξ_k is the leverage component coefficient. If all leverage coefficients are zero then the GJR model is identical to the GARCH model. The model is stationary if the coefficients satisfy $\xi < 2(1 - \alpha - \beta)$ (Rodríguez and Ruiz 2012).

Detrended fluctuation analysis

To investigate the long-range dependence of cryptocurrencies' returns, we employ the DFA, which is advocated by Peng et al. (1994). This method can measure long-range power-law correlation exponents related to memory effects in noisy signals and nonstationary time series using one quantitative parameter. The DFA procedure described by Peng et al. (1995) contains three steps. Let x_t denote a time stochastic series of length N.

Step 1: We determine the following profile

$$y(k) = \sum_{t=1}^{k} [x_t - \bar{x}],\ k = 1, \ldots, N \tag{9}$$

where $\bar{x}$ is the average value of time series x_t.

Step 2: The integrated time series $y(k)$ can be used to analyse long-range correlations in x_t readily by getting rid of trends. We divide the profile $y(k)$ into $N_s \equiv int(N/s)$ non-overlapping segments (indexed by $m = 1, \ldots, N$) of equal length s.

Step 3: The local trend of the series y_{fit} is determined by least square polynomial fitting, and then subtracted from the segment profiles to yield the detrended fluctuations. Then the fluctuation function can be written as

$$F(m) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} [y(i) - y_{fit}(i, m)]^2} \tag{10}$$

This procedure is repeated for m times. The fluctuation function $F(m)$ behaves as a power-law of m.

$$F(m) \sim m^H \tag{11}$$

where H is the well-known Hurst exponent. Consequently, the exponent is computed by regressing $\ln(F(m))$ on $\ln(m)$. Thus the long-range anti-correlated is evident of $0 < H < 0.5$, uncorrelated is evident of $H = 0.5$ and (positively) long-range correlated is evident of $0.5 < H < 1$.

Detrended moving average cross-correlation analysis

To measure power-law cross-correlation between price and volume, which is another typical stylized fact (He et al. 2014; He and Wen 2015), we use DMCA proposed by He and Chen (2011). Let us assume there are two time series $\{x_i\}$ and $\{y_i\}$, $i = 1, 2, \ldots, N$, where N is the length of each time series. Then, we calculate the integrated series, $X_i = \sum_{j=1}^{i} x_j$, $Y_i = \sum_{j=1}^{i} y_j$, $i = 1, 2, \ldots, N$. For a window of size, the moving average is given by

$$\bar{X}_{n,i} = \frac{1}{n} \sum_{j=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} X_{i-j},\ \bar{Y}_{n,i} = \frac{1}{n} \sum_{j=[-(n-1)\theta]}^{[(n-1)(1-\theta)]} Y_{i-j} \tag{12}$$

where θ is a position parameter, i.e. $\theta = 0$ (backward moving average), $\theta = 0.5$ (centred moving average) and $\theta = 1$ (forward moving average). Then, the detrended covariance can be obtained:

$$F^2(n) = \frac{1}{N-n+1} \sum_{j=[n-\theta(n-1)]}^{[N-\theta(n-1)]} \left(X_i - \bar{X}_{n,i}\right)\left(Y_i - Y_{n,i}\right) \tag{13}$$

If two series are long-range cross-correlated, we obtain a power-law relationship as follows:

$$F(n) \propto n^H \tag{14}$$

The Exponent H can describe the power-law cross-correlation relationship between the two related time series.

III. Data and empirical results

Data

The cryptocurrency data are crawled from the website https://coinmarketcap.com, which including the opening, closing, highest and lowest prices for more than 1500 forms of cryptocurrencies from 28 April 2013 (the earliest available date) to 30 April 2018 (the access date). In particular, we focus on the Top 20 cryptocurrencies (according to their market capitalization on 15:00 pm UTC, 1 May 2018) and require a 2-year sample for the stylized facts tests, which leave us with eight forms of cryptocurrencies. The selected eight cryptocurrencies account for 67.29% of the total capitalization; therefore, our sample is an appropriate representation of the cryptocurrency market. Figure 1 illustrates market share of each cryptocurrency and accumulative shares. The cryptocurrencies' returns are calculated as

$$r_{i,t} = \ln\left(\frac{cp_{i,t}}{cp_{i,t-1}}\right) \tag{15}$$

where $cp_{i,t}$ indicates the closing price on trading day t for cryptocurrency i.

According to Garman and Klass (1980), we also employ the following model to calculate the range-based volatility for each cryptocurrency.

$$v_{i,t} = \frac{1}{2} hl_{i,t}^2 - (2\ln 2 - 1) oc_{i,t}^2 \tag{16}$$

where $hl_{i,t}$ is the difference in natural logarithm between the highest and lowest prices on day t for cryptocurrency i, $oc_{i,t}$ is the difference in natural logarithm between the opening and closing prices on day t for cryptocurrency i. Table 1 reports the basic information for cryptocurrencies, including starting date, ending date, market share and market capitalization (denominated in USD). Table 2 reports the statistical properties of returns of cryptocurrencies and we find that all the cryptocurrencies have positive returns and kurtosis values are greater than 3 during the sample period. The SD of the return of Bitcoin is smaller than other cryptocurrencies. For clear illustrations, we also plot the price dynamics in natural logarithmic form in Figure 2 and the returns of the selected cryptocurrencies in Figure 3, and they both show significant fluctuations. As is evident from Figure 2, the price of major cryptocurrencies has risen very rapidly since 2017. Since the prices of XEM, XLM and XRP are less than 1, their y-axis is negative.

Empirical results

Heavy tails

Table 2 reports the skewness, kurtosis and Jarque–Bera statistics for the returns of cryptocurrencies. Except for the Bitcoin, all the cryptocurrencies have positive skewness and the kurtosis values

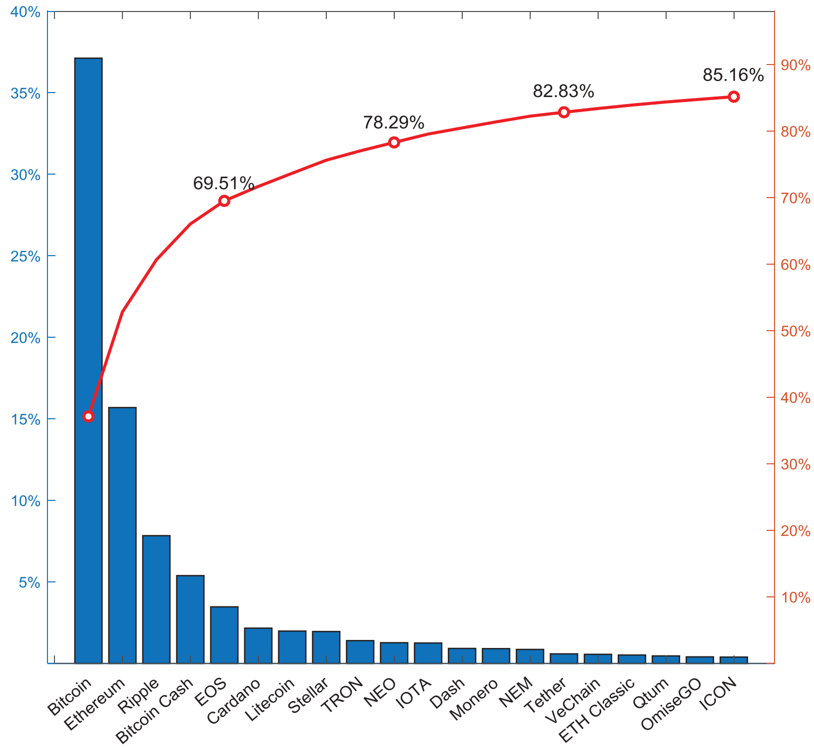

Figure 1. Market share of each cryptocurrency and accumulative value.

Table 1. Brief description of cryptocurrencies.

Cryptocurrency	Symbol	Start date	End date	Capitalization	Percentage	Accumulated
Bitcoin	BTC	28 April 2013	30 April 2018	1.52E+11	37.12	37.12%
Ethereum	ETH	07 August 2015	30 April 2018	6.43E+10	15.69	52.82%
Ripple	XRP	04 August 2013	30 April 2018	3.21E+10	7.84	60.65%
Litecoin	LTC	28 April 2013	30 April 2018	8.14E+09	1.99	62.64%
Stellar	XLM	05 August 2014	30 April 2018	8.03E+09	1.96	64.60%
Dash	DASH	14 February 2014	30 April 2018	3.78E+09	0.92	65.52%
Monero	XMR	21 May 2014	30 April 2018	3.73E+09	0.91	66.43%
NEM	XEM	01 April 2015	30 April 2018	3.53E+09	0.86	67.29%

Table 2. Statistical properties of returns of cryptocurrencies.

Symbol	Obs.	Mean	Median	SD	Max	Min	Skewness	Kurtosis	Jarque–Bera
BTC	1828	0.0023	0.0021	0.0450	0.3575	−0.2662	−0.1885	10.7615	4599.11*
DASH	1536	0.0046	−0.0019	0.0850	1.2706	−0.4676	3.0057	43.2100	105,790.73*
ETH	996	0.0068	−0.0003	0.0721	0.4123	−0.3155	0.5151	7.3473	828.33*
LTC	1828	0.0019	0.0000	0.0693	0.8290	−0.5139	1.7768	27.9612	48,418.24*
XEM	1125	0.0066	0.0000	0.0937	0.9956	−0.3615	1.8427	18.0965	11,319.71*
XLM	1364	0.0038	−0.0028	0.0850	0.7231	−0.3664	1.9441	17.1158	12,183.55*
XMR	1439	0.0035	0.0000	0.0781	0.5846	−0.3782	0.6542	8.6084	1988.59*
XRP	1730	0.0029	−0.0027	0.0802	1.0274	−0.6163	2.0000	29.6288	52,267.28*

* indicates statistical significance at 1% level.

are larger than 3. The Jarque–Bera statistics reject the null hypothesis of normal distribution at 1% significant level, indicating that all the returns are heavy-tail distributed. Besides, the natural logarithmic distribution fitting and Q–Q plot are illustrated in Figures 4 and 5, respectively, illustrating that the returns of cryptocurrencies have obvious heavy tails. We further present the Hill tail index estimator (Hill 1975) in Figure 6, which is a more stable indicator and shows heavy tails in eight cryptocurrencies. For more clarity, we only report the results of the largest observations in Figure 6. Taken together, we find strong evidence of heavy tails for the returns of cryptocurrencies.

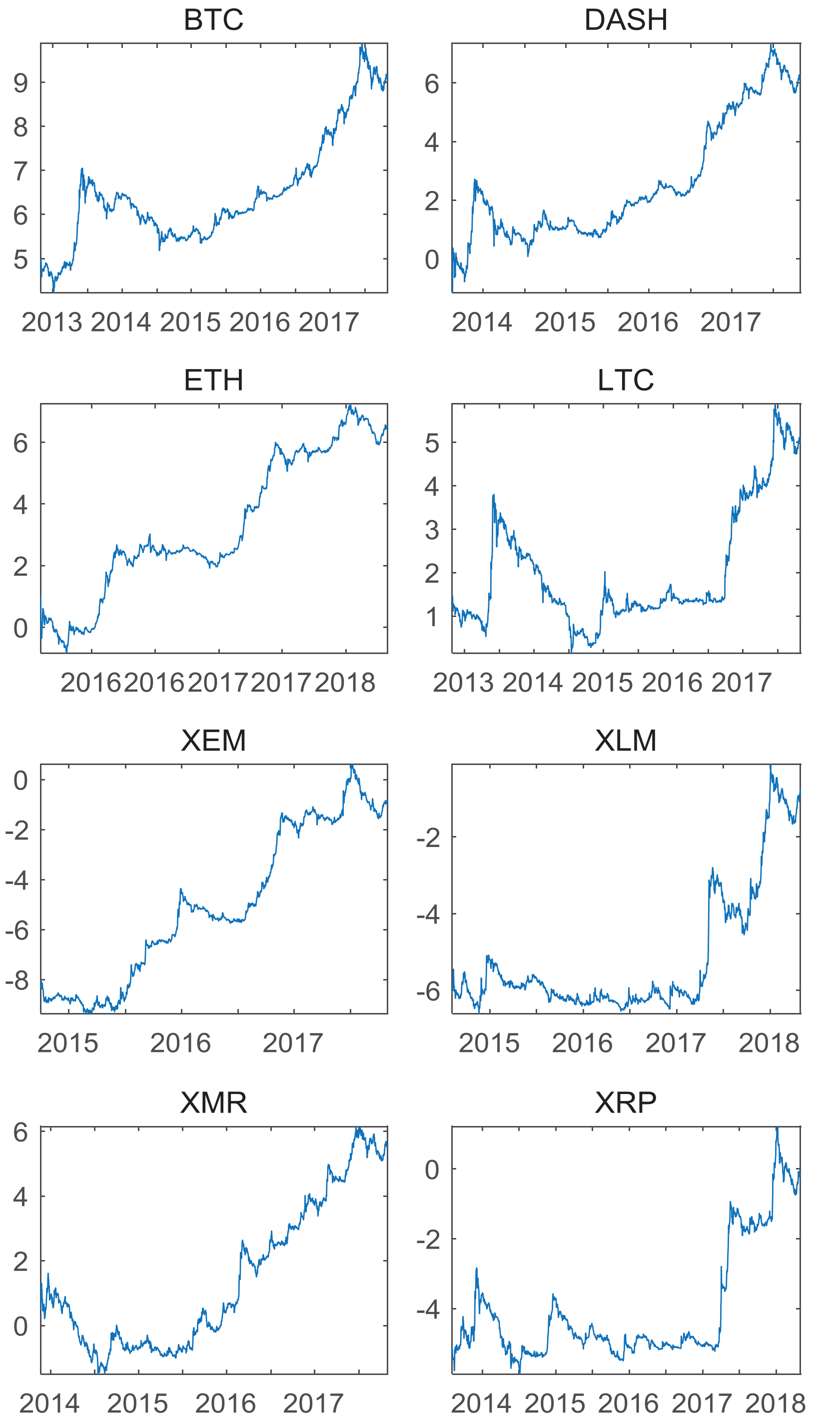

Figure 2. Natural logarithmic prices of the cryptocurrencies.

Autocorrelations

Figure 7 illustrates the autocorrelations of returns (red lines with solid circles) and Figure 8 shows that of returns in absolute form (blue lines with pentagons) of cryptocurrencies. The blue straight line near 0 in y-axis is approximate confidence bounds. As is clearly illustrated, the autocorrelations for returns decay quickly, while the autocorrelations for absolute returns decay slowly.

Volatility clustering

According to Bollerslev (1986), the GARCH(1, 1) model is a parsimonious characterization of

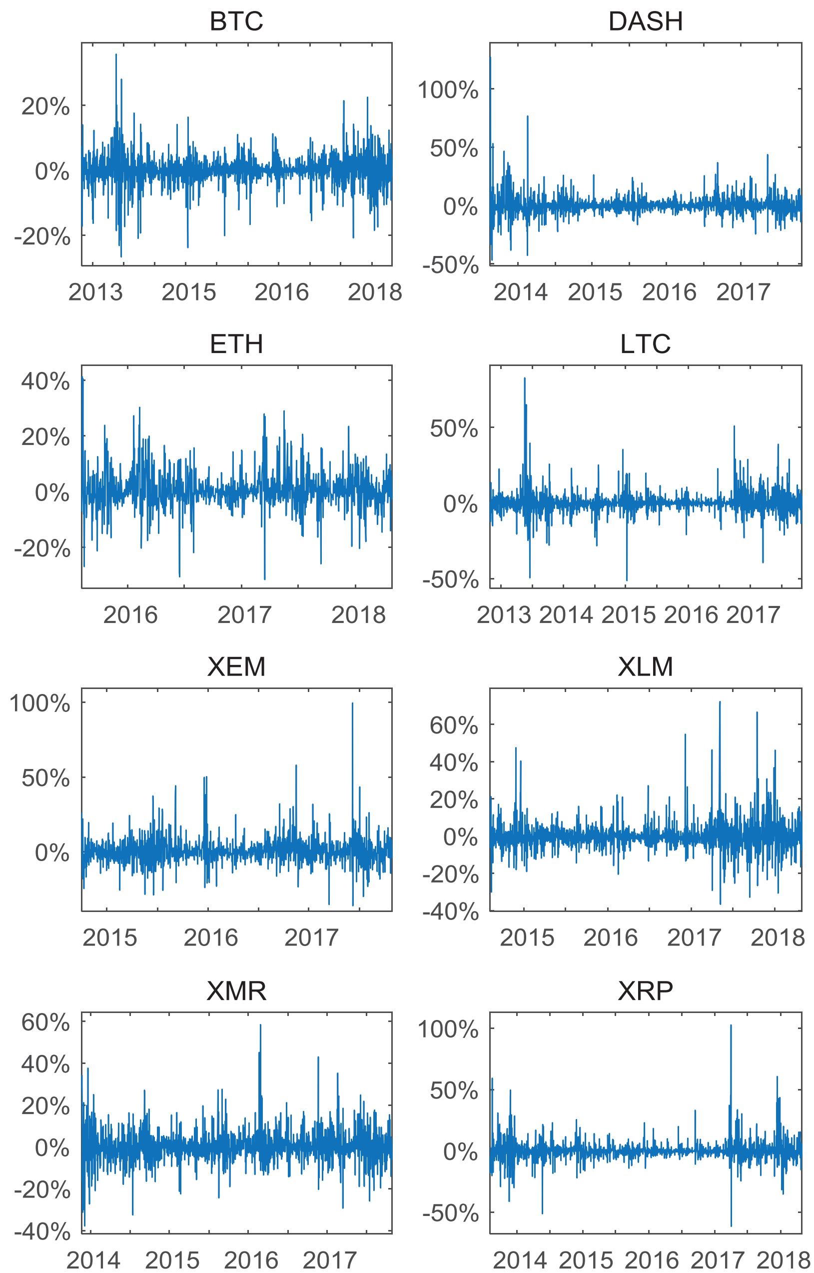

Figure 3. Returns of the selected cryptocurrencies.

conditional variance, which suits many financial time series quite well. Therefore, we implement the GARCH(1, 1) model to detect the volatility clustering. In particular, if α and β are significantly positive, then the volatility of the previous period is strongly correlated with the fluctuations of the later period, i.e. the volatility clustering. And if $\alpha + \beta$ is close to 1, it shows that there is a strong persistence of volatility shocks. Figure 3 clearly illustrates that large price changes tend to be followed by large price changes. Table 3 reports the ARCH test and estimation of GARCH(1, 1) model. We mainly find that all the cryptocurrencies pass the ARCH test, and both α and β are significantly positive and $\alpha + \beta$ is very close to 1. Therefore, we can conclude that returns of cryptocurrencies display strong volatility clustering effects.

Leverage effect

There is an empirically observed fact that the negative shocks at time $t - 1$ have stronger impact on the variance of time t than the positive shocks, which the plain GARCH model fails to capture

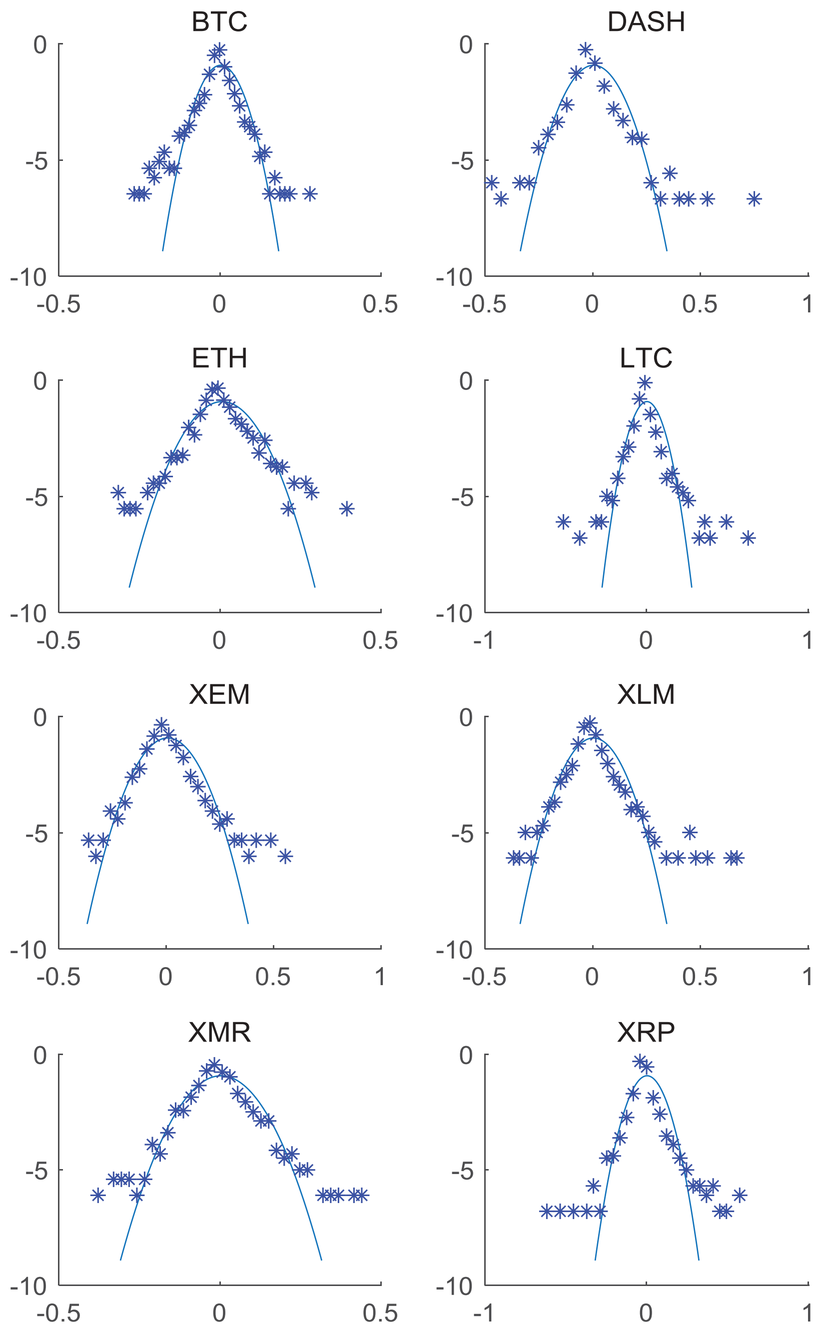

Figure 4. Natural log distribution of each cryptocurrency and log normal distribution fit.

while GJR model can. As the negative impact will lead to increased leverage and risk, this asymmetry is called the leverage effect. Table 4 illustrates the estimation of GJR$(1,1)$ model. Since the value of $1-\alpha-\beta-0.5\xi$ is greater than zero, the model is stationary for eight cryptocurrencies. The leverage (ξ) of ETH and XEM is positive, whereas that of the other six cryptocurrencies is negative. As is clearly shown, the leverage of XRP is the largest, while BTC has the least value. As XRP rose by more than 15 times in less than a month at the end of 2017, we speculate that the difference in leverage effects may be related to fluctuations in the rise and fall of themselves.

Long-range dependence

According to Bariviera (2017), we use six points to estimate the Hurst exponent. The points for regression estimation are $m=\{4,\ 8,\ 16,\ 32,\ 64,\ 128\}$. The rolling window method works as follows: we first calculate the Hurst exponent for the first 500 returns and then calculate Hurst exponent from the

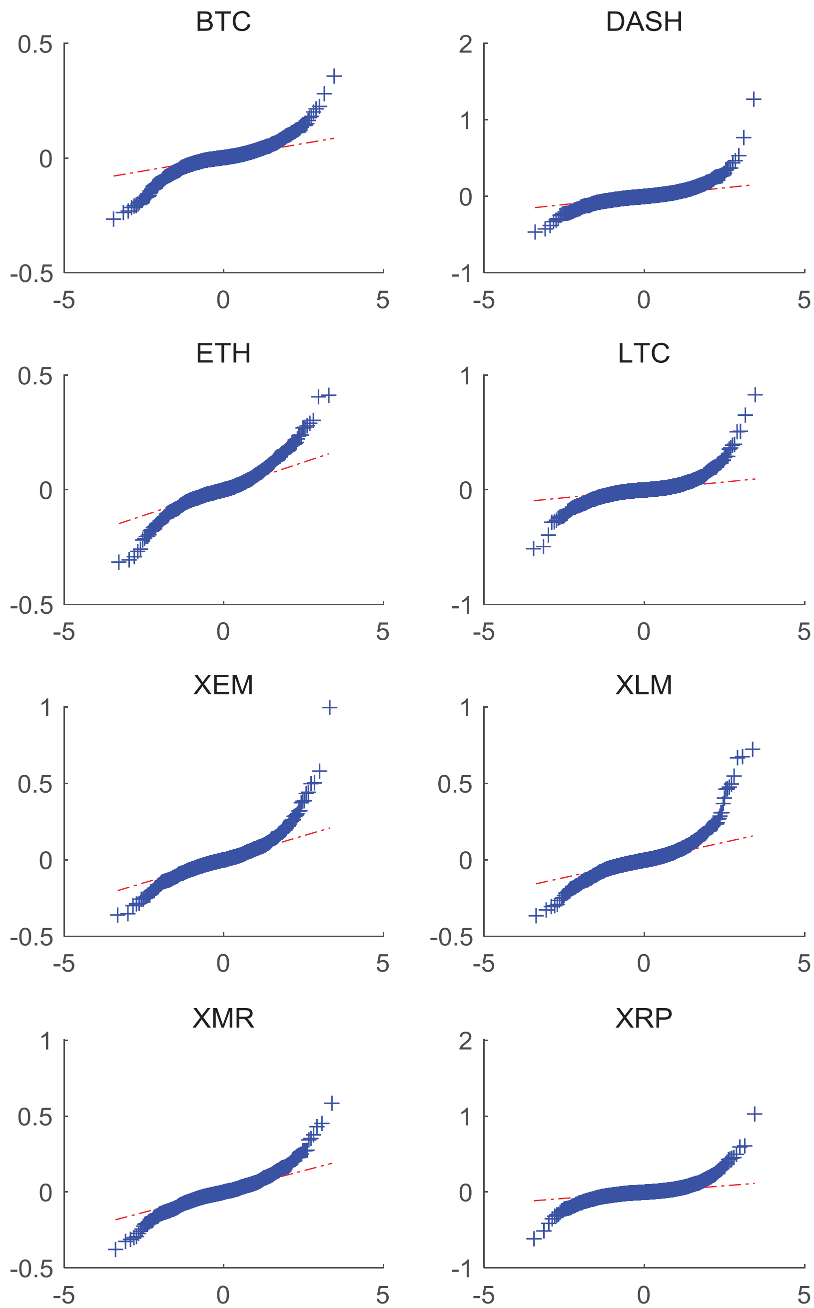

Figure 5. Q–Q plot of the returns of cryptocurrencies.

second to the 501th returns, and continue this process until the end of the data. Figure 9 illustrates the time-varying Hurst exponent of daily returns and range-based volatility computed by rolling window DFA method.

We mainly find that the long-range dependence of the returns and volatility of different currencies varies widely. First, the Hurst exponent of Bitcoin shows a persistent behaviour with its value greater than 0.5 before 2015 and wander around 0.5 after 2015, indicating that Bitcoin is in the process of moving towards to efficient market. However, there are no clear trends for other cryptocurrencies throughout the sample period. Second, we analyse the long-range dependence of volatility. Hurst exponents of volatility of BTC, ETH and XEM show exhibits long-range dependence in all sample periods. And Hurst exponents of DASH, LTC, XLM, XMR and XRP are volatile, indicating that long-range dependence only appears in a certain range of segments. Finally, we observe that the long-range dependences of returns and volatility

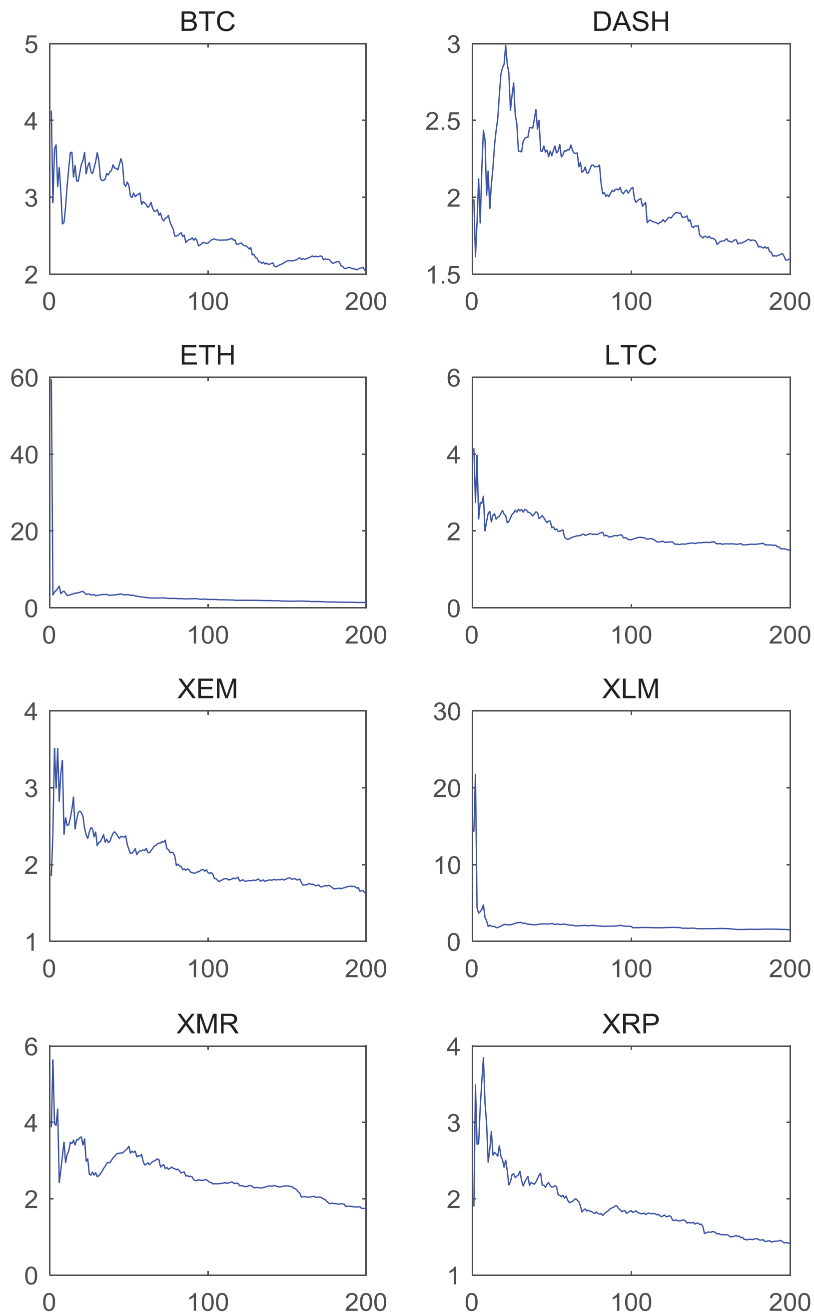

Figure 6. Tail index of the cryptocurrencies.

are very different, but the explanation behind this is beyond the scope of this article. Table 5 reports the descriptive statistics of Hurst exponents of returns and volatility, which indicates that all the average of Hurst exponents of range-based volatility are larger than those of returns. We also employ R/S Hurst method to confirm the robustness for the full sample period (He 2010; He and Qian 2012; He and Wen 2013), and the last column of Table 5 reports the R/S Hurst results, which is consistent with the rolling window DFA and shows that there exists long-range dependence for returns and range-based volatility.

Power-law correlation

Power-law correlation between price and volume is another typical stylized fact in financial assets. We measure the cross-correlation between returns in model (15) and change of volume, which is calculated by $v_{i,t} = \ln\left(\frac{volume_{i,t}}{voluem_{i,t-1}}\right)$, where $volume_{i,t}$ indicates the trading volume on trading day t for

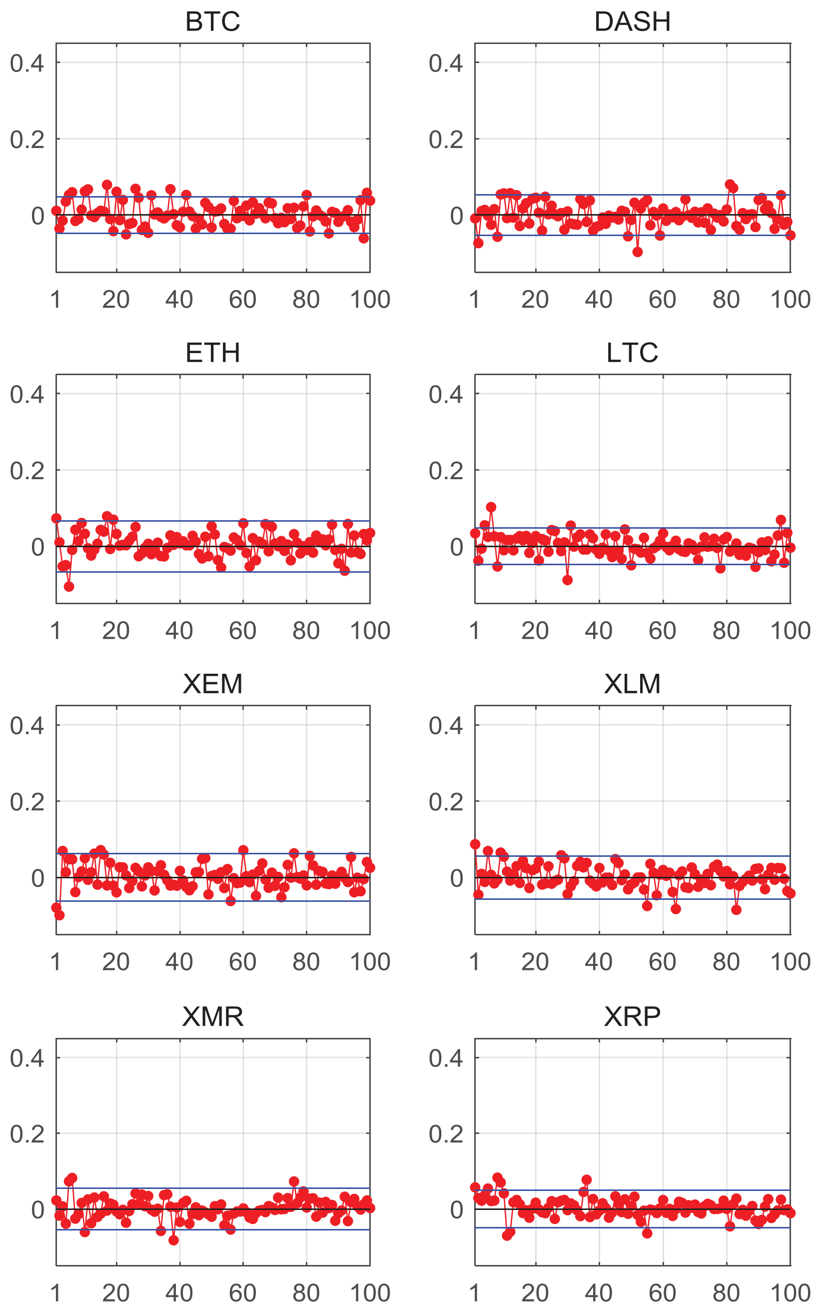

Figure 7. Autocorrelations of returns of the cryptocurrencies.

cryptocurrency i. Following He and Chen (2011), we also report the full sample period cross-correlation exponents of DMCA when $\theta = 0, 0.5$ and 1 in Table 6. All the exponents are greater than 0.5, indicating that the price and volume of cryptocurrencies are power-law cross-correlated.

IV. Conclusions

This article is in line with the stylized statistical properties of agent-based and empirical researches (Franke and Westerhoff 2012, 2016; Schmitt and Westerhoff 2017a, 2017b; Westerhoff and Franke 2012) and extends the exploration for new assets. In this article, we give the first empirical analysis on the stylized facts of eight forms of cryptocurrencies. We mainly find that there are heavy tails, absence of autocorrelations, volatility clustering, leverage effect and long-range dependence for the returns of cryptocurrencies. In addition, power-law correlation between price and volume is also revealed. These results have practical implications for investors. In

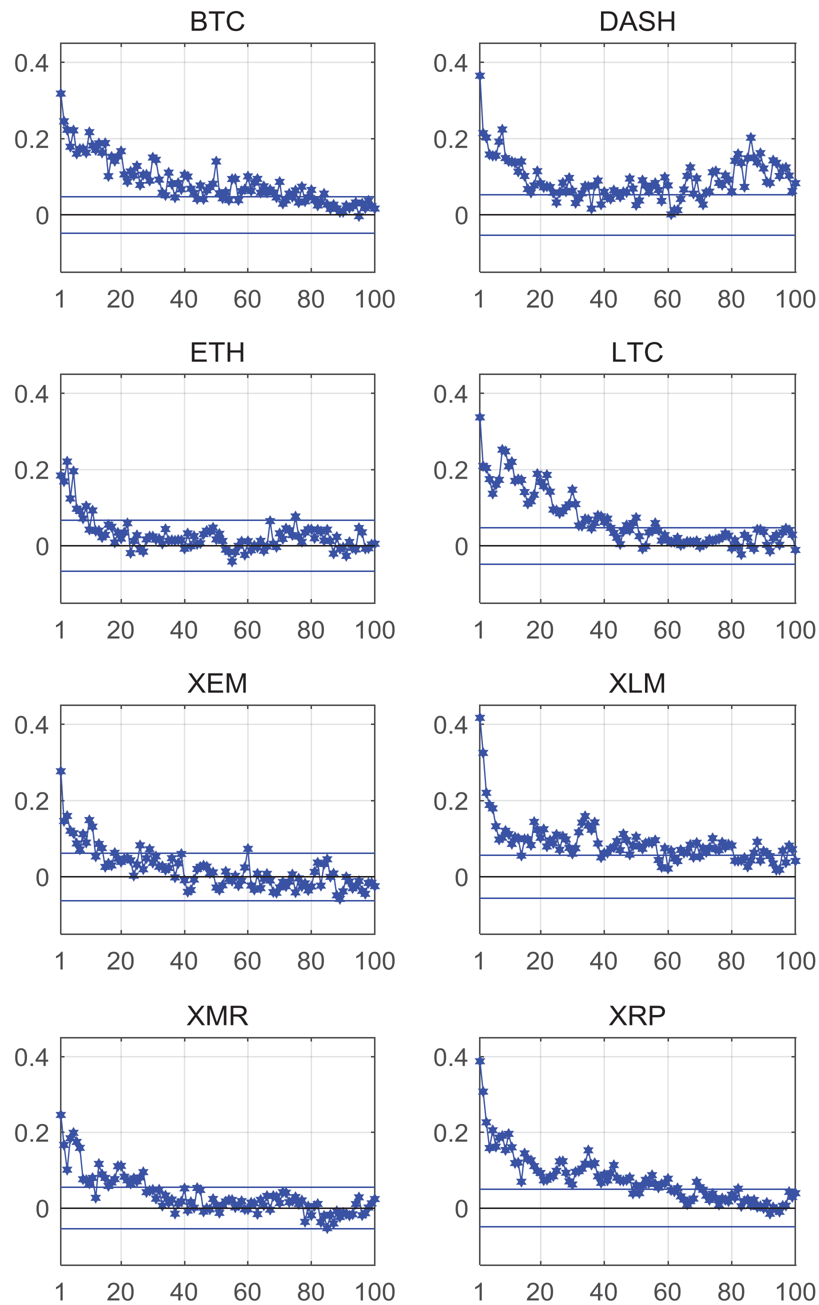

Figure 8. Autocorrelations of absolute returns of the cryptocurrencies.

Table 3. ARCH test and estimation of GARCH(1, 1) model.

Symbol	ARCH test	ARCH(α)	GARCH(β)	$\alpha+\beta$
BTC	176.2965 (0.0000)*	0.1401	0.8599	1.0000
DASH	32.1974 (0.0000)*	0.3392	0.6608	1.0000
ETH	34.5057 (0.0000)*	0.2755	0.7081	0.9837
LTC	70.055 (0.0000)*	0.0957	0.8851	0.9808
XEM	25.3427 (0.0000)*	0.5492	0.4508	1.0000
XLM	206.5109 (0.0000)*	0.2107	0.7670	0.9777
XMR	40.2332 (0.0000)*	0.1277	0.8170	0.9447
XRP	130.5763 (0.0000)*	0.4045	0.5955	1.0000

* indicates statistical significance at 1% level.

Table 4. The estimation of GRJ(1, 1) model.

Symbol	Constant	ARCH (α)	GARCH (β)	Leverage (ξ)	$1-\alpha-\beta-0.5\xi$
BTC	0.0000	0.1392	0.8609	−0.0001	0.0000
DASH	0.0004	0.3754	0.6571	−0.0656	0.0003
ETH	0.0003	0.2696	0.7070	0.0175	0.0147
LTC	0.0001	0.1233	0.8893	−0.0693	0.0221
XEM	0.0013	0.4951	0.4486	0.1126	0.0000
XLM	0.0004	0.2632	0.7674	−0.1570	0.0479
XMR	0.0003	0.1606	0.8431	−0.1057	0.0491
XRP	0.0005	0.6253	0.5355	−0.3216	0.0000

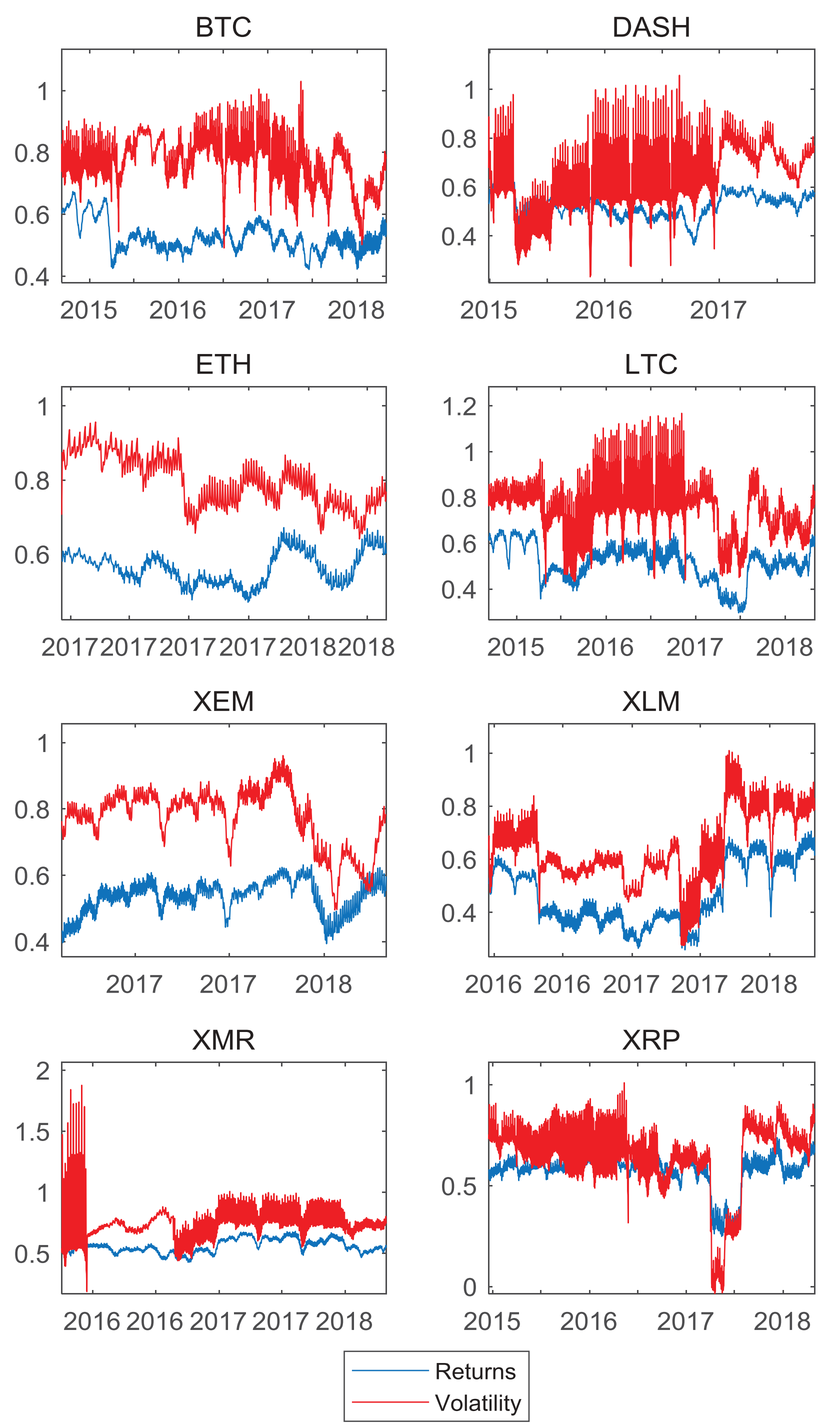

Figure 9. Hurst exponents of returns and range-based volatility of cryptocurrencies.

Table 5. Statistical properties of Hurst estimates.

	Obs.	Mean	Median	SD	Max	Min	R/S Hurst
Panel A: Hurst exponents of returns							
BTC	1329	0.5237	0.5142	0.0497	0.6737	0.4216	0.6092
DASH	1037	0.5232	0.5198	0.0501	0.6644	0.3626	0.6188
ETH	498	0.5634	0.5636	0.0437	0.6716	0.4702	0.6050
LTC	1329	0.5124	0.5191	0.0799	0.6622	0.2959	0.6261
XEM	626	0.5352	0.5454	0.0505	0.6321	0.3938	0.6142
XLM	865	0.4664	0.4248	0.1177	0.7062	0.2592	0.5645
XMR	940	0.5603	0.5522	0.0557	0.6799	0.4273	0.5907
XRP	1231	0.5801	0.5969	0.0926	0.7381	0.2486	0.6337
Panel B: Hurst exponents of range-based volatility							
BTC	1330	0.7656	0.7676	0.0830	1.0302	0.4911	0.7887
DASH	1038	0.6329	0.5948	0.1494	1.0576	0.2322	0.5718
ETH	499	0.7953	0.7908	0.0702	0.9567	0.6410	0.7569
LTC	1330	0.7394	0.7470	0.1303	1.1671	0.4235	0.8244
XEM	627	0.7744	0.7984	0.0904	0.9610	0.4957	0.7789
XLM	866	0.6419	0.6169	0.1342	1.0096	0.2757	0.6431
XMR	941	0.7360	0.7293	0.1538	1.8764	0.1868	0.6888
XRP	1232	0.6306	0.6549	0.1807	1.0101	−0.0402	0.6765

Table 6. The cross-correlation exponents of DMCA.

Symbol	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
BTC	0.8027	0.6299	0.8101
DASH	0.8344	0.6649	0.8552
ETH	0.8023	0.6395	0.8302
LTC	0.7926	0.6588	0.8060
XEM	0.7770	0.6334	0.8307
XLM	0.8034	0.6829	0.8419
XMR	0.8251	0.6660	0.8350
XRP	0.7764	0.7265	0.7989

particular, investors who are interested in investing in cryptocurrency market should take account of the stylized facts when constructing the portfolios. However, the explanation on the distinctions between returns and volatility on long-range dependence is unsolved. We leave this for future research.

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