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Portfolio diversification with virtual currency: Evidence from bitcoin[☆]

Khaled Guesmi[a,*], Samir Saadi[a,b], Ilyes Abid[d], Zied Ftiti[e]

[a] IPAG Business School, France
[b] University of Ottawa, Canada
[c] ISC Paris Business School, France
[d] EDC Paris Business School, France

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ABSTRACT

The paper investigates the proprieties of Bitcoin in the financial markets. Specifically, we explore the conditional cross effects and volatility spillover between Bitcoin and financial indicators using different multivariate GARCH specifications. The nature of interaction between Bitcoin and financial variables and their transmission mechanisms are taken into account when analyzing the diversification and hedging effectiveness across gold asset and stock market. Our findings suggest that all models confirm the significant returns and volatility spillovers. More importantly, we find that VARMA (1,1)-DCC-GJR-GARCH is the best-fit model for modeling the joint dynamics of a variety of financial assets. We also show that a short position in the Bitcoin market allows hedging the risk investment for all different financial assets. Finally, hedging strategies involving gold, oil, equities and Bitcoin reduce considerably the portfolio's risk, as compared to the risk of the portfolio made up of gold, oil and equities only.

1. Introduction

Cryptocurrencies are commonly perceived as a disruptive technology that raises both hopes and fears in the minds of different categories of stakeholders within the economy.[1] In fact, Cryptocurrencies offer several potential benefits as innovative and efficient payment system but at the same time, they are the source of potential risks that could harm investors, consumers, businesses, financial systems and even the national security. The mixed views on cryptocurrencies and their future continue to be the driving force behind the excessive volatility of their market values. This has in turns attracted a growing interest from researchers in demystifying the complex world of cryptocurrencies which remains ambiguous and puzzling for the majority of market participants. Aiming to extend earlier research efforts, the present paper examines the conditional cross effects and volatility spillover between the most prominent cryptocurrency (i.e., Bitcoin) and a set of financial indicators.

With a market capitalization close to seventeen billion dollars, Bitcoin is by far the most widely known cryptocurrency followed by its traditional competitors the Ethereum, Ripple, Litecoin and more recently Bitcoin Cash.[2] Similar to other virtual currencies, Bitcoin is a decentralized, peer-to-peer network that allows for the proof and transfer of its ownership without the need for an intermediary. By removing the need for intermediaries Bitcoin provides a more efficient infrastructure for the transfer of money, allowing for allow cheaper and faster payments. This lack of a regulatory framework became the central issue of the bulk of cryptocurrency literature focussing primarily on Bitcoin price mechanism and its ability to develop into an alternative monetary system (e.g., Becker et al., 2013; Brandvold, Molnár, Vagstad, & Valstad, 2015; Ciaian, Rajcaniova, & Kancs, 2016; Dwyer, 2015; Rogojanu & Badea, 2014; Segendorf, 2014).

The sudden surge of volatility in the exchange rate of Bitcoin by the end of 2013 triggered another strand of studies examining the speculative feature of Bitcoin (Baek & Elbeck, 2015; Dyhrberg, 2016b; Kristoufek, 2015; Yermack, 2015). For example, Kristoufek (2015) shows that Bitcoin price cannot be explained by economic theories, it is instead driven by speculation. In the same vein, Yermack (2015) concludes that Bitcoin resembles more like a speculative investment than like a true currency. Yermack's remarks were corroborated by influential economists such as Robert Shiller, Nouriel Roubini and Stephen

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* Corresponding author.
E-mail address: khaled.guesmi@ipag.fr (K. Guesmi).

[1] Throughout the paper, we use the terms ‘cryptocurrencies’, ‘virtual currencies’ and ‘digital currencies’ interchangeably.

[2] As of January 01, 2018, there are 1519 types of cryptocurrencies, twenty-four of which have market capitalization that exceeds one billion dollars (Coinmarketcap, 2018).

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Roach who labelled Bitcoin as a typical example of a bubble waiting to burst (Monaghan, 2018). In an earlier statement, Alan Greenspan, the former Federal Reserve chairman stated that "You have to really stretch your imagination to infer what the intrinsic value of bitcoin is. I haven't been able to do it. Maybe somebody else can." (Kearns, 2013).

The mystery around Bitcoin and its underpinning technology (i.e. blockchain) sparkled another debate on whether Bitcoin is a currency or a commodity (Buchholz, Delaney, Warren, & Parker, 2012; Dyhrberg, 2016a,b; Adjohani, Chourou, & Saadi, 2017; Cheah & Fry, 2015; Katsiampa, 2017; Pieters & Vivanco, 2017). Dyhrberg (2016a), for example, shows that Bitcoin exhibits many similarities to both the U.S. dollar and gold. Dyhrberg (2016b) adds that Bitcoin offers comparable opportunities of hedging and safe haven as gold, and can be categorized somewhere in between gold and the US dollars. Baur, Dimpfl, and Kuck (2018), however, criticize the findings of Dyhrberg (2016a,b) and show that Bitcoin has distinctively different time series characteristics in comparison to other assets including gold and the US dollar.

Following periods of financial turmoil experienced over the last decade, investors continue to look for alternative investment instruments that can offer diversification and/or hedging advantages. As it was the case with commodities in early 2000s, and because of its high average return and low correlation with major financial assets, Bitcoin could be a useful tool for portfolio management. However little is known from the literature on whether bitcoin can offer diversification advantages for a global market portfolio (Bouri, Peter Molnár, Azzi, Roubaud, & Hagfors, 2017). Dyhrberg (2016a) examined the nature of the relation between Bitcoin and FTSE stock index, USD/EUR and USD/GBP exchange rates. Dyhrberg (2016b) explores the hedging capabilities of Bitcoin using a T-GARCH model, and finds that Bitcoin could be a hedging tool against the stock markets during period of turmoil. In line with Dyhrberg (2016b) show that Bitcoin could serve as a hedging tool against the U.S. dollar in the short run. Using a wavelet and quantile regression, Bouri et al. (2017) investigate the hedging behavior of the Bitcoin through analyzing a positive relationship between Bitcoin and global uncertainty. Bouoiyour and Selmi (2017) argue that Bitcoin exhibits the properties of weak safe haven in the short and long terms.

In this paper, we extend the emerging literature on cryptocurrencies by analyzing the dynamic between Bitcoin and a selection of financial assets and see whether Bitcoin offers diversification and risk management benefits to investors. To do so we use a larger sample and recent time series data. In fact, recent evidence suggests that a fledgling relationship between Bitcoin and gold has emerged only by the end of 2017 (Landsman, 2018). Moreover, we propose a new empirical design based on various specifications to examine the conditional volatility dynamic of Bitcoin and other financial assets, and measure the conditional cross effects and volatility spillover between them. All the proposed models are rooted from VARMA (1,1)-DCC-GARCH by adding some potential characteristics likely to best describe the interaction between Bitcoin and financial assets.[3] The nature of interaction between Bitcoin and financial variables and their transmission mechanisms are taken into account when analyzing the diversification and hedging effectiveness across the financial assets.

Our empirical results suggest VARMA (1,1)-DCC-GJR-GARCH as the best model specification to describe the joint dynamics of Bitcoin and different financial assets. Second, we show that a short position in the Bitcoin market allow hedging the risk investment against all different financial assets. We also find that hedging strategies involving gold, oil, emerging stock markets and Bitcoin reduce considerably a portfolio's risk (variance), as compared to the risk of a portfolio composed of gold, oil and stocks from emerging stock only. Taken together, our results show that Bitcoin may offer diversification and hedging benefits for investors. The reminder of this paper is organized as follows. Section 2 discusses our empirical design. Section 3 presents the data and empirical findings. Section 4 concludes the paper.

2. Empirical design

Since the seminal paper of Engle (1982), ARCH models become the most popular specifications in modeling time series volatility. A related stream of literature has emerged to take into account some stylized facts of financial time-series, such as asymmetric and long memory effects (e.g., Engle & Ng, 1993; Glosten, Jagannathan, & Runkle, 1993; Nelson, 1991; Sentana, 1995). Building on Engle (1982), multivariate GARCH models have been developed offering a superior capability in modeling volatility dependence between time series. Examples of multivariate GARCH models includes, among others, CCC-GARCH model (Bollerslev, 1990), BEKK-GARCH (Engle & Kroner, 1995), DCC-GARCH (Engle, 2002).

GARCH models have become popular tools in modeling volatility of time series data (Agnolucci, 2009; among others). In our study, we implement various specifications of GARCH model to investigate the volatility spillover effect between Bitcoin and exchange rates, stock market, and commodity series, in pairs. In particular, we employ the following four base specifications to model the volatility dependence: *VARMA (1,1)-DCC-GARCH*, *VARMA (1,1)-DCC-EGARCH*, *VARMA (1,1)-DCC-GARCH*, *VARMA (1,1)-cDCC-FIAPARCH*, and the *VARMA (1,1)-DCC-GJR-GARCH* models. We have also used several extensions of the above models by including additional characteristics that might exist in the spillover effect between financial series.[4] The most appropriate model will be identified and used to analysis Bitcoin's portfolio diversification and hedging effectiveness.

2.1. The VARMA (1,1)-DCC-GARCH model

VARMA-DCC-GARCH model was proposed by Bollerslev (1990) allowing for the correlation to be time varying. Below we present the basic specification of an AR (1)-DCC-GARCH (1,1) model. Let r_t be a time series. Its conditional mean is specified as follows:

$$w_t^{i/Bitcoin} = \begin{cases} r_t = c + \Omega r_{t-1} + \mathscr{E}_t \\ \varepsilon_t = H_t^{1/2}\eta_t \end{cases} \tag{1}$$

where c is the vector of constant terms. r_t is the vector of returns on the financial assets and Bitcoin prices, respectively. Ω refers to a (n × n) matrix of coefficients. ε_t is the vector of the error terms of the conditional mean equations for different financial variables and bitcoin returns respectively. η_t refers to a sequence of independently and identically distributed (*i.i.d.*) random errors, and H_t is the conditional variance-covariance matrix of different financial variables and bitcoin returns, and it is defined as:

$$H_t = D_t P_t D'_t$$

H_t denotes the variance-covariance matrix of returns at time *t*. P_t defines the (n × n) symmetric matrix of dynamic conditional correlations. D_t denotes the diagonal matrix of conditional standard deviations for each return series. The matrix D_t is determined from the following Eq. (2) through estimating an univariate GARCH process.

[3] Specifically, we propose the following 16 model specifications: VARMA(1,1)-BEKK-AGARCH, VARMA(1,1)-DCC-GARCH, VARMA(1,1)-DCC-EGARCH, VARMA(1,1)-DCC-GJR-GARCH, VARMA(1,1)-DCC-FIAPARCH, VARMA(1,1)-cDCC-GARCH, VARMA(1,1)-cDCC-EGARCH, VARMA(1,1)-cDCC-FIGARCH, VARMA(1,1)-cDCC-GJR-GARCH, VARMA(1,1)-ADCC-GARCH, VARMA(1,1)-ADCC-EGARCH, VARMA(1,1)-ADCC-FIGARCH, VARMA(1,1)-cADCC-GARCH, VARMA(1,1)-cADCC-EGARCH, VARMA(1,1)-cADCC-GJR-GARCH, VARMA(1,1)-cADCC-FIGARCH.

[4] These models are VARMA(1,1)-ADCC-GARCH, VARMA(1,1)-ADCC-EGARCH, VARMA(1,1)-ADCC-FIGARCH, VARMA(1,1)-cADCC-GARCH, VARMA(1,1)-cADCC-EGARCH, VARMA(1,1)-cADCC-GJR-GARCH, VARMA(1,1)-cADCC-FIGARCH, where DCC, c-DCC, ADCC denotes dynamic conditional correlation, copula dynamic conditional correlation, asymmetric dynamic conditional correlation.

$$h_{ii,t} = w_i + \alpha_i \mathscr{E}^2_{ii,t-1} + \beta_i h_{ii,t-1} \quad (2)$$

Q_t is a (n × n) variance-covariance matrix of standardized residuals $u_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ which is defined as follows

$$Q_t = (1 - \theta_1 - \theta_2)\overline{Q} + \theta_1 u_{t-1}u'_{t-1} + \theta_2 Q_{t-1} \quad (3)$$

where $\overline{Q} = E(u_t u_t')$ to a (n × n) symmetric positively-defined matrix of the unconditional variance-covariance of standardized residuals. θ_1 and θ_2 are the unknown parameters to be estimated. The sum of θ_1 and θ_2 should be less than one in order to insure positivity of the matrix Q_t. The DCC process relies on the decomposition of the conditional covariances as the product of conditional standard deviations and conditional correlations between two markets i and j such that

$$h_{ij,t} = c + \sqrt{h_{ii,t}h_{jj,t}} \quad (4)$$

Therefore, for a pair of markets i and j, their conditional correlation at time t can be written as

$$\rho_{ij,t} = \frac{(1-\theta_1-\theta_2)\overline{q}_{ij} + \theta_1 u_{i,t-1}u_{j,t-1} + \theta_2 q_{ij,t-1}}{[(1-\theta_1-\theta_2)\overline{q}_{ii} + \theta_1 u^2_{i,t-1} + \theta_2 q_{ii,t-1}]^{\frac{1}{2}}[(1-\theta_1-\theta_2)\overline{q}_{jj} + +\theta_1 u^2_{j,t-1} + \theta_2 q_{jj,t-1}]^{\frac{1}{2}}} \quad (5)$$

where q_{ij} is the element on the ith line and jth column of the matrix Q_t.

2.2. The VARMA (1,1)-DCC-EARCH model

The difference with respect to the previous model (i.e. VARMA-DCC-GARCH) is that the *VARMA (1,1)*-DCC-EARCH accounts for asymmetric volatility spillovers. This asymmetry is captured by the variance covariance matrix and can be specified as follows:

$$H_t = D_t \rho_t D'_t \quad (6)$$

Under the conditional covariance matrix, D_t is a (n × n) diagonal matrix with the time-varying standard deviations and D_i is a time-varying symmetric correlation matrix. The dynamic correlations are captured through the following asymmetric general diagonal DCC equation:

$$Q_t = \overline{\theta} + A'\overline{\theta}A - B'\overline{\theta}B - C'\overline{N}C - A'Z_{t-1}Z'_{t-1}A + B'\theta_{t-1}B + C'\eta_t\eta'_{t-1}C \quad (7)$$

where $\overline{\theta}$ and $\overline{N}$ are the conditional correlation matrices of Z_t and η_t. The model described in Eq. (6) above is a generalization of the DCC model of Engle (2002) to capture asymmetric correlations. The matrix of A, B and C are restricted to being diagonal for practicality in the estimation of model. θ_t is positive definite with probability one if $(\overline{\theta} + A'\overline{\theta}A - B'\overline{\theta}B - C'\overline{N}C)$ is positive definite. Through the final term of (5), the time-varying correlations will respond asymmetrically to positive or negative shocks in each market. Next, we scale θ_t to get the correlation matrix ρ_t:

$$\rho_t = \theta_t^{*-1}\theta_t\theta_t^{*-1} \quad (8)$$

It is noteworthy that while a multivariate EGARCH model would allow us to examine both the asymmetry effects as well as the volatility spillover between financial variables, it is not however useful to apply such specification to the conditional correlations, because it would unduly restrict the conditional correlations to be always positive and because it has too many parameters. The DCC specification in Eq. (7) does not suffer from these problems while allowing for the possibility of asymmetric effects.

2.3. The VARMA (1,1)-DCC-GJR-GARCH model

This model is a specification proposed by Glosten et al. (1993) to account for nonlinearity when modeling conditional variance of time series. It consists of assuming that negative shocks have stronger impact than the positive ones, also known as leverage effect. This type of asymmetry is modeled through the following variance covariance matrix:

$$H_t = D_t R_t D_t'$$
$$D_t = diag(\sqrt{h_{ii,t}}, \sqrt{h_{jj,t}})$$
$$R_t = (1-\theta_1-\theta_2)R + \theta_1\varphi_1 + \theta_1 R_2 \text{ with } \theta_1 + \theta_2 < 0 \quad (9)$$

where R_t is the symmetric matrix of dynamic conditional correlations. D_t is a diagonal matrix of conditional standard deviations for each of the return series, obtained from estimating the following univariate GJR-GARCH[5] process of Glosten et al. (1993):

$$h_{ii,t} = w_i + \alpha_i \mathscr{E}^2_{ii,t-1} + \beta_i h_{ii,t-1} + \gamma_i I_{i,t}\mathscr{E}^2_{ii,t-1} \quad (10)$$

where persistence is measured by the coefficients β_i and the indicator variables $I_{i,\ t}$ captures asymmetry in the estimate of coefficients γ_i. A negative value of γ_i implies that negative residuals increase the variance more than positive residuals. The unknown parameters to be estimated are θ_1 and θ_2. Therefore, for a pair of markets i and j, their conditional correlation at time t can be written as follows:

$$\rho_{ij,t} = (1-\theta_1-\theta_2)\rho_{ij} + \theta_2\rho_{ij,t-1} + \theta_1\frac{\sum_{m=1}^{M}u_{i,t-m}u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M}u^2_{i,t-m})(\sum_{m=1}^{M}u^2_{j,t-m})}} \quad (11)$$

$$u_t = \frac{\varepsilon_t}{\sqrt{h_t}}$$

The estimation of the vector of unknown parameters θ is carried out by the quasi-maximum likelihood estimation method which is robust to departures from normality under some regular conditions (Bollerslev & Wooldridge, 1992).

2.4. The VARMA (1,1)-cDCC-FIAPARCH model

The last competing model is the AR (1)-c-DCC-FIAPARCH (1,1) proposed by Aielli (2008). Aielli developed a corrected Dynamic Conditional Correlation (c-DCC) model to correct for the lack of consistency and the potential bias in the estimated parameters of Engle's (2002) DCC-GARCH model. The c-DCC-GARCH model is indeed similar to the standard DCC-GARCH model, except that the correlation process Q_t is reformulated as:

$$\boldsymbol{Q_t = (1-\theta_1-\theta_2)\overline{Q}_t + \theta_1\eta^*_{t-1}\eta^{*'}_{t-1} + \theta_2 Q_{t-1}} \text{ with } \boldsymbol{\eta^*_t = diag\{Q_t\}^{1/2}} \quad (12)$$

We use the univariate FIAPARCH of Tse (1998) to model the conditional volatility of each of the system variables and then compute their time-varying standard deviations. An extension of the APARCH model of Ding et al. (1993), FIAPARCH model allows for the asymmetric responses of volatility to positive and negative shocks as well as the long memory property of conditional volatility. The FIAPARCH model integrates a fractionally integrated process as defined by Baillie et al. (1996). The FIAPARCH model is expressed as a power transformation of the conditional standard deviation:

$$h_{i,t}^{\delta i/2} = \omega_i + \{1 - (1-\psi_i(L))^{-1}\phi_i(L)(1-L)^{d_{vi}}\}(|\varepsilon_{i,t}| - \gamma_i\varepsilon_{i,t})^{\delta_i} \ \forall\ i = f, b \quad (13)$$

where $h_{i,\ t}$ refers to the conditional variance of $x_{i,\ t}$; ω_i the mean of the process, d_{v_i} the fractional degree of integration of $h_{i,\ t}$; and $\psi_i(L)$ and $\phi_i(L)$ the lag polynomials of respective orders P and K.

3. Data and results

3.1. Descriptive statistics

The data used in our analysis is based on eight variables; stock

[5] This model provides better statistical results than DCC-GARCH.

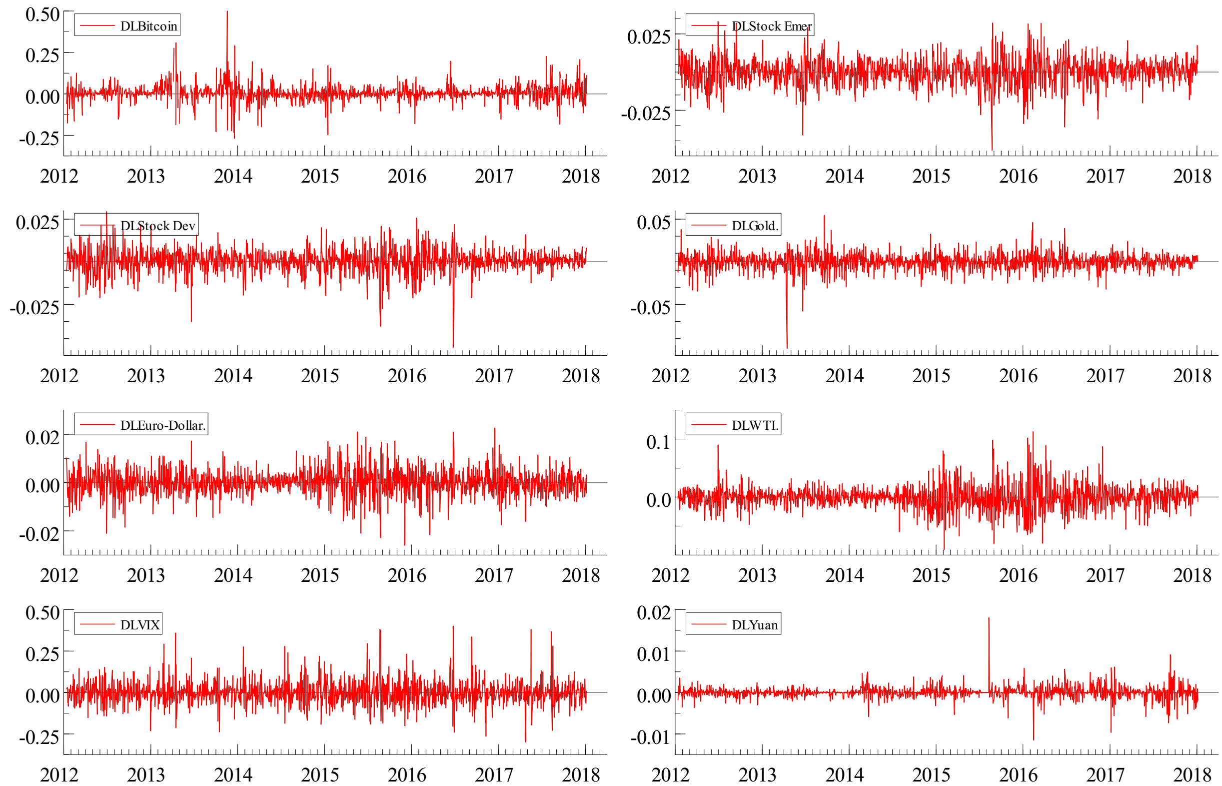

Fig. 1. Returns of selected assets.

markets (MSCI Emerging Markets Index and MSCI Global Market Index), Euro and Chinese exchange rate, gold and oil (gold bullion and West Texas Intermediate (WTI)), Bitcoin (Bitstamp), and the implied volatility index (VIX), as a proxy of the US market volatility. Data are daily and are drawn from DataStream, Eurostat, and the Federal Reserve Bank of St. Louis. The selected period is from 01/01/2012 to 05/01/2018. Our sample period begins in 2012 because Bitcoin price level and volatility were small in 2010 and 2011. In fact, during the first two years since its creation, the parity of Bitcoin with the U.S. dollars was about 0.10. It is only by February 7, 2011, the price of Bitcoin reached $1. However, in unreported results, we find that our findings remain qualitatively unchanged if our sample begins on July 18, 2010 instead of January 01, 2012. The return of each series is calculated as the difference of logarithm of two successive prices, multiplied by 100.

Fig. 1 illustrates the dynamic of the all studies variables. The results show that Bitcoin volatility has a different pattern of the volatility of other financial variables. This first observation might be a source of potential diversification between Bitcoin and financial equities. Table 1 provides summary statistics for all the series, along with Jarque-Bera test for normality. Table 1 indicates that the on average the volatility of Bitcoin is lesser important than all equities assets, expect for the VIX series. Such high level of volatility hinders Bitcoin ability to be used as a medium of exchange. Kurtosis statistics are positive and high for all variables, suggesting fat tails behavior. The WTI, VIX, Yuan currency and Bitcoin (Bitstamp) are skewed to the right, while those of stock markets and gold are skewed to the left. This leptokurtic excess and asymmetry are in line with the Jarque–Bera test results, justifying the rejection of normality.

3.2. Empirical results

Our analysis offers three main set of results. First, we present the estimation results of all competing models used to investigate the volatility transmission between Bitcoin and the selected financial assets. This allows us to provide some insights on the transmission channels between financial environment and Bitcoin. Second, we use the dynamic of volatility transmission to analysis its implication on portfolio diversification. Third, we investigate the hedging effectiveness involving financial assets and Bitcoin.

3.2.1. Volatility cross-effects in selected variables and bitcoin

In this subsection, we investigate the dynamic of volatility transmissions between Bitcoin and financial assets. The estimation results of the benchmark model, DCC-GARCH (1,1), and the competing models are reported in Table 2. All models confirm the significant returns and volatility spillovers (Table 3). However, the results indicate that VARMA (1,1)-DCC-GJR-GARCH is the best-fit model for modeling the joint dynamics of different financial variables and Bitcoin, as evidenced by its log likelihood ratio (Akaike and Schwarz values). Indeed, the results of VARMA (1,1)-DCC-GJR-GARCH estimation (Table 3) show that the autoregressive parameters are significant for all financial assets under consideration. In addition, the one-period lagged financial variables returns are found to significantly affect the current returns of Bitcoin at the 1% level. Moreover, the conditional volatility $({\varepsilon_{t-1}}^{Bitcoin})^2$ of bitcoin returns is significantly affected by unpredicted changes in the returns on different financial variables. Thus, a shock to financial assets, regardless of their signs, implies an increase in the volatility of bitcoin returns.

Table 4 presents the results of the diagnostic test on standardized residuals and squared standardized residuals. The results show that our competing models are flexible enough to capture the dynamics of the conditional return and volatility, as the test statistics highlight significantly the departure from both normality and serial correlations hypotheses.

3.2.2. Optimal portfolio designs and hedging ratios

Our objective here is to determine the optimal portfolio design, that includes fiat currencies, commodities, global stock indices and Bitcoin,

Table 1
Descriptive statistics of returns series.

Statistics/variables	Bitcoin	MSCI Emerging Market Index	MSCI Global Market Index	Gold	Euro-Dollar	WTI	VIX	Yuan
Mean	0.0050	0.0001	0.0003	−0.0001	0.00003	−0.0003	−0.0005	0.00001
Min	−0.2696	−0.0512	−0.0502	−0.1016	−0.0259	−0.0905	−0.2998	−0.0114
Maximum	0.4996	0.0332	0.0293	0.0543	0.0225	0.1128	0.4010	0.0180
Standard deviation	0.0519	0.0086	0.0067	0.0096	0.0053	0.0205	0.0713	0.0014
Skewness	0.4303	−0.2699	−0.5881	−0.7578	−0.0714	0.3526	0.7165	0.9070
Excess kurtosis	10.474	2.1415	4.2825	10.372	1.8947	3.3831	4.2295	21.419
Jarque-Bera	7183.5	317.25	1282.8	7146.3	234.83	776.79	1297.1	30,054
P-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ARCH LM test (1)	66.07***	20.05***	35.71***	11.23***	5.65***	88.65***	56.60***	58.53***
ARCH LM test (5)	35.59***	17.95***	21.25***	4.86**	8.51**	51.06**	22.98**	24.20***
ARCH LM test (10)	20.24***	12.52***	11.66***	2.87**	5.76**	29.87***	11.77***	12.39***
LB-Q (5)	3.78	53.30***	23.84***	2.90	2.07	1.978	8.042	11.75**
LB-Q (10)	6.14	57.64***	26.85***	9.57	10.72	3.192	18.34**	15.34
LB-Q (20)	25.28	63.52***	34.48**	21.01	22.88	8.93	39.09***	30.34
LB-Q^2 (5)	230.52***	119.62***	154.72***	23.89***	46.88***	425.07***	142.02***	114.12***
LB-Q^2 (10)	329.83***	200.47***	196.79***	29.69***	71.35***	692.61***	147.76***	120.70***
LB-Q^2 (20)	453.02***	281.52***	223.67***	35.72***	134.65***	1176.35***	158.23***	126.86***
Observations	1561	1561	1561	1561	1561	1561	1561	1561

Note: *, ** and *** indicate rejection of null hypothesis for normality using JB statistic, for no ARCH effects using ARCH LM test and for no autocorrelation using Ljung-Box Q-statistic test at 1%, 5% and 10%, respectively. WTI and VIX refer to West Texas Intermediate and Implied Volatility Index, respectively.

Table 2
Dynamics of the conditional return and volatility with GARCH-type models.

Model specification	Log-likelihood	SBC	HQ	AIC
VARMA(1,1)-BEKK-AGARCH	41,049.001	−52.457	−52.519	−52.556
VARMA(1,1)-DCC-GARCH	41,103.016	−52.516	−52.583	−52.623
VARMA(1,1)-DCC-EGARCH	40,099.137	−51.230	−51.297	−51.336
VARMA(1,1)-DCC-GJR-GARCH	41,156.684	−52.585	−52.652	−52.691
VARMA(1,1)-DCC-FIAPARCH	4140.0160	−52.550	−52.620	−52.630
VARMA(1,1)-cDCC-GARCH	41,103.053	−52.510	−52.580	−52.613
VARMA(1,1)-cDCC-EGARCH	40,098.303	−51.229	−51.296	−51.335
VARMA(1,1)-cDCC-FIGARCH	41,155.133	−52.583	−52.650	−52.689
VARMA(1,1)-cDCC-GJR-GARCH	41,156.491	−52.580	−52.651	−52.690
VARMA(1,1)-ADCC-GARCH	41,103.582	−52.512	−52.581	−52.622
VARMA(1,1)-ADCC-EGARCH	40,099.607	−51.226	−51.295	−51.336
VARMA(1,1)-ADCC-FIGARCH	41,155.383	−52.579	−52.648	−52.689
VARMA(1,1)-cADCC-GARCH	41,103.180	−52.512	−52.581	−52.622
VARMA(1,1)-cADCC-EGARCH	40,098.344	−51.224	−51.293	−51.334
VARMA(1,1)-cADCC-GJR-GARCH	41,156.514	−52.580	−52.649	−52.690
VARMA(1,1)-cADCC-FIGARCH	41,155.228	−52.579	−52.648	−52.688

Best model is selected based on minimum values of Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC) and Hannan-Quinn (HQ). Note that AIC and SBC are not comparable for different samples.

by taking into account the spillovers between these variables (as shown in previous section). Our strategy consists at considering a hedged portfolio composed of different variables. Specifically, we assume that investors invest in Bitcoin to hedge against any turmoil movement coming from currencies, commodities and stock markets. Thus, we assume that investors aim to minimize the risk of portfolios composed of Bitcoin and a variety of financial assets. Following Kroner and Ng (1998) and we determine the optimal holding weight of Bitcoin in a one-dollar portfolio of (Variable i/Bitcoin) at time t, denoted by $w_t^{i/Bitcoin}$, as follows:

$$w_t^{i/Bitcoin} = \frac{h_t^i - h_t^{i/Bitcoin}}{h_t^i - 2h_t^{i/Bitcoin} + h_t^{Bitcoin}} \tag{14}$$

where h_t^i represent the conditional volatility of stock market returns, currencies and commodities returns. $h_t^{Bitcoin}$ and $h_t^{i/Bitcoin}$ refer to the conditional volatility of the Bitcoin and the conditional covariance between different variables and Bitcoin at time t respectively. All these series are estimated through the four competing models. Under the assumption of absence of short selling, the mean-variance portfolio optimization approach imposes the following constraints on the optimal weight of Bitcoin[6]:

$$w_t^{i/Bitcoin} = \begin{cases} 0 & \text{if } w_t^{i/Bitcoin} < 0 \\ w_t^{i/Bitcoin} & \text{if } 0 \le w_t^{i/Bitcoin} \le 1 \\ 1 & \text{if } w_t^{i/Bitcoin} > 1 \end{cases} \tag{15}$$

Because Bitcoin can be used to reduce risk exposure, investor should take an appropriate position on different markets. More specifically, a long position (buying) of one dollar of Bitcoin must be hedged by a short position (selling) of $\beta_t^{i/Bitcoin}$ dollars on other markets, such as commodities, stocks or currencies. Following Kroner and Sultan (1993), the optimal hedge ratio $\beta_t^{Bitcoin/i}$ can be defined as:

$$\beta_t^{i/Bitcoin} = \frac{h_t^{i/Bitcoin}}{h_t^i} \tag{16}$$

Table 5 displays the average values of optimal weights [$w_t^{i/Bitcoin}$] and hedge ratios [$\beta_t^{i/Bitcoin}$] for different portfolio. The results point out to the three kinds of assets that would be used in hedging strategies. The three financial assets with the highest optimal weights are the VIX, the WTI, and Gold, as their respectively values 0.673, 0.207 and 0.059, based on the Benchmark model (DCC-GJR-GARCH). This finding implies that investor should hold more VIX and WTI than other assets in order to minimize the risk without lowering the expected return of the VIX, WTI and Bitcoin portfolio. Concerning the hedge ratio, all models provide low values ranging from −0.0047 to 0.0412. By taking into consideration the benchmark model (DCC-GJR-GARCH), the results indicate a hedge ratio of 0.0412, the largest one. This result is interpreted as follows: One dollar long (buy) in the Yuan exchange rate market requires investors to go short $0.0412 in the Bitcoin market. Loosely speaking, this result provides evidence that a short position in the Bitcoin market allow hedging the risk investment for the different financial assets.

4. Conclusion

Investors are always looking for alternative investment instruments as part of diversified investment portfolios. Cryptocurrencies represent such an alternative because of their high average return and low

[6] The proportion of wealth that the investor puts on Bitcoin index is $1 - w_t^{gs}$.

Table 3
Estimation results for mean equation ARMA(1,1)-DCC-GJR-GARCH model.

	Cst (M)	AR(1)	MA(1)	Cst (V)	Alpha1	Beta1	Gamma1
Bitcoin	0.0031***	−0.9097***	0.9037***	0.7672 10^4	0.1830***	0.834884***	−0.0668**
	(0.0003)	(0.0000)	(0.0000)	(0.1511)	(0.0007)	(0.0000)	(0.0012)
MSCI Emerging Market Index	0.00005	0.1230	0.0737	0.7211 10^6**	−0.0174**	0.9638***	0.0826***
	(0.8306)	(0.4315)	(0.6357)	(0.0139)	(0.0112)	(0.0000)	(0.0000)
MSCI Global Market Index	0.0003**	0.1209	−0.0473	1.7362 10^6**	0.0254	0.8485***	0.1850***
	(0.0171)	(0.4429)	(0.7519)	(0.0065)	(0.5435)	(0.0000)	(0.0001)
Gold	−0.0001	−0.4005	0.4294	1.0604 10^6	0.0249	0.9524***	0.0231
	(0.5278)	(0.1270)	(0.1256)	(0.5258)	(0.4079)	(0.0000)	(0.3749)
Euro-Dollar	0.00008	0.0468	−0.0799	0.0073 10^6	0.0296***	0.9830***	−0.0258**
	(0.4530)	(0.7416)	(0.5985)	(0.8915)	(0.0000)	(0.0000)	(0.0013)
WTI	−0.0002	0.5481	−0.5746	0.0217 10^4	0.0123***	0.9414***	0.0806***
	(0.4369)	(0.1255)	(0.1005)	(0.1437)	(0.4492)	(0.0000)	(0.0000)
VIX	−0.0006**	0.8932***	−0.9819***	10.8272 10^4***	0.3103***	0.6346***	−0.3535***
	(0.0488)	(0.0000)	(0.0000)	(0.0000)	(0.0004)	(0.0000)	(0.0000)
Yuan	−0.000002	−0.7496***	0.8165***	0.8776 10^6**	0.1245**	0.4032**	0.1468**
	(0.9533)	(0.0000)	(0.0000)	(0.03074)	(0.0105)	(0.0307)	(0.0307)

*, **, and *** indicate rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels, respectively.

Table 4
Residual diagnostics for independent series.

	Bitcoin	MSCI Emerging Market Index	MSCI Global Market Index	Gold	Euro-Dollar	WTI	VIX	Yuan
JB	629.80*	39.33*	101.82*	2895*	110.94*	147.69*	181.90*	1237*
Q (20)	22.99	26.17	11.22	3.04	2.058	1.01	2.09	7.49
Q^2 (20)	9.01	33.42	16.89	10.54	9.98	5.78	8.51	13.32

The values Q (20) and Q^2 (20) are the Ljung-Box statistic to test autocorrelation for order 20 applied to standardized residuals and squared standardized residuals.
* Denotes the significance level of 10%.

Table 5
Optimal weights and hedge ratios for different portfolio.

	MSCI Emerging Market Index	MSCI Global Market Index	Gold	Euro-Dollar	WTI	VIX	Yuan
$W^{i/Bitcoin}$	0,0512	0,0334	0,0597	0,0222	0,2071	0,6735	0,0012
$\beta^{i/Bitcoin}$	−0.088	0,0266	−0.010	0,011	−0.0962	−0.0047	0,0412

This table reports average optimal weight of financial assets and hedge ratios for a financial assets-bitcoin portfolio using conditional variances and covariance estimated from our benchmark model and a competitive volatility-spillover model: ARMA (1,1)-DCC-GJR-GARCH.

correlation with financial assets. However, only a few studies have examined cryptocurrencies as hedging and diversification tools. To fill this gap in the literature, we examine the conditional cross effects and volatility spillover between Bitcoin and a set of financial assets using various model specifications extended from the DCC-GARCH models to identify appropriately the best-fit model.

Our empirical results suggest VARMA (1,1)-DCC-GJR-GARCH as the best model specification to describe the joint dynamics of Bitcoin and different financial assets. We also show that a short position in the Bitcoin market allow hedging the risk investment against all different financial assets. Moreover, we find that hedging strategies involving gold, oil, emerging stock markets and Bitcoin reduce considerably a portfolio's risk (variance), as compared to the risk of a portfolio composed of gold, oil and stocks from emerging stock only. Taken together, our results show that Bitcoin may offer diversification and hedging benefits for investors.

An important question remains: What does the future hold for Bitcoin as a diversifier and a hedging tool? The answer to this question is of great important to investors in the wake of uncertain regulatory environments around cryptocurrencies. Although digital currencies are unregulated in most countries, some major economies have regulation in place and others are studying the potential need to do so. As outlined in a comprehensive study of Bitcoin conducted by the Law Library of the U.S. Congress (2014, p. 1), "there is widespread concern about the Bitcoin system's possible impact on national currencies, its potential for criminal misuse, and the implications of its use for taxation." Moreover, Bitcoin has a dark history of mishaps, especially with the Mt. Gox scandal, where hackers stole hundreds of millions of dollars through Bitcoin transactions. The uncertainties around future regulation and exposure of Bitcoin network to hacking activities can influence the design of optimal diversification and hedging strategies using Bitcoin. It would be interesting that future studies examine the role of Bitcoin to hedge against other financial assets before and after major changes in cryptocurrency regulation around the world.

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