



The role of bitcoin in well diversified portfolios: A comparative global study

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ABSTRACT

This research explores the effects of adding bitcoin to an optimal portfolio (naïve, long-only, semi-constrained with and without bitcoin shorting) by relying on the mean-CVaR approach. We explore bitcoin's role in portfolios of U.S., European and Chinese assets. We back-test to compare the performance of portfolios with and without bitcoin for each scenario. The results show that by adding bitcoin, the portfolio performance improves; but this is due more to the increase in returns than in the reduction of volatility. In addition, the improvement is linked to bitcoin's performance in 2013. We conclude that bitcoin may have a role in portfolio diversification even though our analysis confirms bitcoin speculative characteristics.

Raj: Wait what's Bitcoin?

Sheldon: "It's a new online currency that's been developed. It's just actual money except you can't see it, hold it or spend it on anything".

From: "The Bitcoin Entanglement" – Big Bang Theory, Season 11, Episode 09

1. Introduction

The Bardi Family used to be very powerful and rich in the fourteenth century. Their activities covered trading as well as banking (they ran one of the largest banks of the late middle age) and, together with the Peruzzi family, were among those who financed King Edward's III war efforts against France. When in 1345 England went bankrupt and was unable to repay them, they were forced out of business. In that very same year, three members of the Bardi family (namely Rubecchio, his uncle Aghinolfo and his cousin Sozzo), having lost almost their entire wealth decided to change their core activities: from bankers to forgers. The suspicious smoke and hammering sounds that could be easily heard when walking around their property, immediately attracted the attention of the local authorities. Due to their lack of expertise as forgers, they were caught at the very beginning of their new endeavour and on 15th October 1345 the local authority started the proceedings against them (Cipolla, 1994). Interestingly enough, the major driver for their very clumsy attempt as forgers was the lack of liquidity and the revaluation of silver and gold that characterised the years between 1333 and 1348 when the explosion of the Black Death completely reshaped

the social and economic fabric around Europe.

Nowadays the link between metal and value of the currency (Metallism) is no longer relevant anymore and even if forging is an illegal activity that has its followers, the maladroitness attempted by the Bardi would not be possible nor effective since the vast majority of currencies no longer come into existence by being printed. Institutions (i.e. the central banks and commercial banks) can expand money and debase it via quantitative easing and lending activity. The result is that the majority of dollars, euros or pounds are created by private banks when they make loans (de Soto, 1995).

In fact, the scepticism about the role of any central authority over the political and economic lives of individuals raises concern about traditional fiat currencies. Nakamoto (2008), the mysterious bitcoin founder, argues that fiat currencies are not a proper medium of exchange because of high transaction costs and the exclusion of a large part of the world population from the banking system. He also argues that fiat currencies do not function well as stores of value, due to the presence of excess inflation. Thus, he proposes Bitcoin as a new tool¹ independent from any central authority, it is able to address these issues by making its supply pre-determined, constant, decreasing, and ultimately finite and thus deflationary. In addition, it aims to be an excellent store of value in the long run (Nakamoto, 2008). Interestingly, the last point raises a key question: if bitcoin is an excellent store of value, should it be included in a portfolio of assets?

Truthfully, ever since the seminal work by Markowitz (1952, 1976), finance stresses the importance of portfolio diversification and a lot of analysis explores the optimal mix of assets that allows for the

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¹ The first ever article on cryptocurrency (Bitcoin) on the Financial Times was published on 6th June 2011 (Alloway, 2011)

maximisation of the return by minimising the risk (i.e. the volatility). Historically the focus was on shares (e.g. Treynor & Black, 1973), bonds (e.g. Barnes & Burnie, 1990), and derivatives (e.g. Galai & Geske, 1984), and spillover effect both in terms of returns and volatility (Hamao, Masulis, & Ng, 1990; Kearney & Daly, 1998; Pyun, Lee, & Nam, 2000). More recently, research explored the link between portfolio diversification and other aspects such as taxes (e.g. Stein, Siegel, Narasimhan, & Appeadu, 2000) and leverage (e.g. Ruban & Melas, 2011). Closer to our research are the works that focus on the role of currencies in portfolio diversification (e.g. Makin, 1978; Pojarliev & Levich, 2011) and the related spillovers (Johnson & Soenen, 2004). The increasing presence of cryptocurrencies has started to attract some interest among academics who wish to assess bitcoin as an asset to include in portfolios. Sadly, very little has been published so far. There are few works that focus on spillovers (Burnie, 2018; Guesmi, Saadi, Abid, & Ftiti, 2018), on the correlation with other currencies (Baumöhl, 2018) or on cryptocurrency volatility (Katsiampa, 2017). In addition, few academics explore cryptocurrencies' liquidity and their investibility (Dyhrberg, Foley, & Svec, 2018; Karalevicius, Degrande, & De Weerd, 2018; Wei, 2018), cryptocurrencies' price formation (Brauneis & Mestel, 2018; Ciaian, Rajcaniova, & Kancs, 2015) or the relationship between cryptocurrencies and other financial assets (Corbet, Meegan, Larkin, Lucey, & Yarovaya, 2018). Finally, very little research most of which is based on portfolios of U.S. assets, has been performed so far on portfolio diversification (Brière, Oosterlinck, & Szafarz, 2015; Carrick, 2016; Wu & Pandey, 2014). All in all, further investigation is very much needed.

Our study explores the bitcoin inclusion in three different geographically-defined portfolios of well diversified assets focusing on its effect on the risk-return. Thus, we examine three portfolios including U.S, European and Chinese assets, respectively. We have two aims: to explore whether bitcoin inclusion improves the performance of the portfolios of assets and whether bitcoin inclusion plays different role in different contexts. Thus, when we select assets to include in our analysis for each geographically defined area (i.e. the U.S., Europe and China), we include those that allow us to measure the performance of the portfolio for that specific area: in the case of U.S. portfolio, we include S & P indices that measure the performance of U.S. firms, in the case of the European portfolio, we include the performance of European firms and in the case of Chinese portfolio we include only Chinese firms. The same applies to bonds. In order to do so we follow the approach of Brière et al. (2015). This approach differentiates our research from previous analysis focused on global portfolios (Bouri, Molnár, Azzí, Roubaud, & Hagfors, 2017).

We use three different portfolio frameworks, namely naïve portfolio, the long-only portfolio, and the semi-constrained portfolio (Eisl, Gasser, & Weinmayer, 2015). We also add a fourth portfolio (semi-constrained portfolio where bitcoin cannot be shorted) since, during the time span covered by our data, it was not easy to take short positions on bitcoin: regulated markets only launched derivatives on bitcoin which enables shorting strategies in late 2017 (Meyer, 2017). In the case of long-only and semi-constrained portfolios (with and without shorting), the weights are calculated using the mean-CVaR optimisation process (Eisl et al., 2015).

Our analysis suggests that bitcoin as an additional asset class in our portfolios tends to generate benefits in terms of increased returns and better performance of the portfolio measured using Risk-Return, Sortino and Omega ratios. In the time window considered (2013–2016), bitcoin generates higher returns but also higher volatility. However, the increased return tends to compensate for the increased volatility so that the performance overall improves. Interestingly, we discovered that the effects of inclusion are very positive and relevant in all the portfolios up to December 2013 while later the inclusion reveals a reduced effect in terms of return. We do not spot any major difference among the three geographical areas considered.

This manuscript is structured as follows: the next section explores

previous research on bitcoin and portfolio diversification. Section 7 discusses the methodology and illustrates the variables used. Section 4 presents the descriptive statistics while Section 5 reviews the separate results in the case of the U.S., European and Chinese assets. Section 6 discusses the results and finally Section 7 concludes.

2. Portfolio diversification and bitcoin

To most ordinary people bitcoin remains a mystery: an intangible, and difficult-to-understand currency with little or no use in the real economy (Garcia, Tessone, Mavrodiev, & Perony, 2014). Nakamoto (2008) in his whitepaper describes bitcoin as a peer-to-peer cash system where a system of nodes (distributed ledger) verifies whether the sender owns the funds they intend to spend and prevents double-spending (Böhme, Nicolas, Edelman, & Moore, 2015; Nakamoto, 2008; Narayanan, Bonneau, Felten, Miller, & Goldfeder, 2017). Thus, bitcoin operates without the need of financial intermediaries so that no central authority controls it (Böhme et al., 2015; Narayanan et al., 2017). The lack of a governance structure implies that there is no obligation to verify a user's identity, there is no prohibition on sales of particular items and payments are irreversible (Böhme et al., 2015). From the financial point of view, bitcoin's total supply and the rate of supply is pre-determined, non-elastic and fully transparent (Nakamoto, 2008). Since the rate of supply is pre-determined and continually decreasing bitcoin is inherently deflationary.

According to Keynesian economics, the demand for money is driven by three determinants: the precautionary demand, transaction demand, speculative demand. These in turn are driven by motives such as the need to save for a rainy day, holding money for daily transactions and the need for taking advantage of investment opportunities (Keynes, 1936).

Is bitcoin suitable as a store of value (i.e. driven by precautionary demand)? Bitcoin's 60 days average volatility over the six-year period (till early 2017) shows a steady decline but it remains much higher than that of gold and the traditional G10 currencies. Furthermore, bitcoin's return distribution exhibits stronger non-normal characteristics and heavier tails (Fry, 2018). Bouri, Molnár, et al. (2017) suggest that bitcoin's distinctive volatility is due to the small size of the market and the trading volume even if Harvey (2017) has found no correlation between volatility and the market capitalisation prior to 2014. However, given the highly volatile nature of bitcoin that makes it unsuitable as a store of value, its demand cannot be driven by the precautionary demand. This evidence leaves two alternative determinants of bitcoin demand: transaction demand (derived from trade transactions) and speculative demand (derived partly from exchange transactions).

Bitcoin transaction data is used by Kristoufek (2015) to show that an increase in trade volume is correlated with bitcoin price, suggesting that bitcoin appreciates in the long run if it is used more for transacting. In fact, the comparison of exchange-traded volume of bitcoin to total transaction volume within the bitcoin network suggests that most users (by volume) treat their bitcoin investments as speculative assets rather than as means of payment (Glaser, Zimmermann, Haferkorn, Christian Weber, & Siering, 2014). Thus, bitcoin is only partially used as a means of exchange. This implies that bitcoin's major driver is the speculative demand (derived partly from exchange transactions) and therefore it has to be considered an investible asset. However, it can be argued that bitcoin does not represent a proper asset class to invest in since it does not have an intrinsic value, neither it does generate any claim on future cash flows like a bond or equity nor has it any claim on future profit like in the case of shares. Nonetheless, this is not a proper justification for arguing that bitcoin is not an investible asset and research suggests that it is an asset (Corbet, Lucey, Urquhart, & Yarovaya, 2018). In fact, bitcoin value changes according to demand and supply (Urquhart, 2017; Wei, 2018). Bitcoin is similar commodities like gold or silver (Dyhrberg, 2016a, 2016b) that even if they do not make any claim on a future cash flow, still have a value linked to their demand and supply

and are perceived as assets that retain value in the long term irrespective of the evolution of the economy. This point is supported by quite a good number of bitcoin long term investors, also known as “hodlers”, bitcoin long term investors (Kaminska, 2017). Bitcoin is also different with respect a piece of art or a rare stamp in the sense that it is characterised by a liquid, on line, electronic and de-personalised market where demand and supply form the price (Brauneis & Mestel, 2018; Dyhrberg et al., 2018). Finally, recent research contests the older evidence that suggests bitcoin has not reached the investment grade status (Cheah & Fry, 2015; Urquhart, 2016) arguing that, in fact, it is now in the mature stage and can be considered a proper asset class (Dyhrberg et al., 2018; Koutmos, 2018; Nadarajah & Chu, 2017). Therefore, if bitcoin can be considered an asset characterised by a speculative demand (derived partly from exchange transactions), it may offer diversification benefits (Corbet, Meegan, et al., 2018; Guesmi et al., 2018). Thus, a related question emerges: is bitcoin an asset that could be included in an optimal portfolio?

In fact, there is limited research on bitcoin (for a systematic review: Corbet, Lucey, et al. (2018)). It moves from its pricing formation explores also whether bitcoin is an investible assets class, the link between bitcoin and other assets and its role as an asset in diversifying and hedge risk. Ciaian et al. (2015) and Kristoufek (2015) point out that the laws of supply and demand, as well as the market consensus among its adopters, play significant part in bitcoin price formation. Polasik, Piotrowska, Wisniewski, Kotkowski, and Lightfoot (2015) using a variety of methodologies, show that the demand factors affect bitcoin price formation significantly more than the supply side. Van Vliet (2018) finds evidence that bitcoin price follows Metcalfe's Law which states that the value of an asset depends on the network that uses it (the value being proportional to the square of the number of users). At the same time, Brauneis and Mestel (2018) suggest that price formation is affected by the liquidity that characterises the cryptocurrency while Urquhart (2017) finds that bitcoin pricing tend to cluster around zeros (that is rounded figures). Interestingly, liquidity is also found to affect cryptocurrencies' returns, with bitcoin that is the biggest gainer, as it is a very liquid cryptocurrency (Wei, 2018). The asset characteristic of cryptocurrencies is also supported by the fact that prices are found to be affected by media sentiment (Karalevicius et al., 2018) in line with previous research that explore sentiment's impact on price of traditional assets (e.g. Bhattacharya & Ritter, 1983; Wisniewski & Moro, 2014).

Cryptocurrencies present high volatility (Katsiampa, 2017) so that they are considered typically high risk investment. Interestingly, cryptocurrencies are mainly traded by private investors and only more recently by professional ones but not by way of algorithms, (Dyhrberg et al., 2018), suggesting that cryptocurrencies in general and bitcoin in particular are investible assets (Phillip, Chan, & Peiris, 2018). Researchers who focus on bitcoin correlation with other assets find that cryptocurrencies have a relatively low correlation with other assets, which might suggest bitcoin as an asset that can help in portfolio diversification (Baumöhl, 2018). This point is also supported by Corbet, Meegan, et al. (2018) who discover relative isolation of cryptocurrencies from other financial assets and by Bouri, Naji, Molnár, and Roubaud (2017) who find that bitcoin is a good hedge against movements in commodities indexes. The potential benefit of including bitcoin in a portfolio is also supported by research that explores spillovers between bitcoin and other assets (Burnie, 2018; Guesmi et al., 2018) since it is found to allow for hedging risk. All in all, the reported research suggests that bitcoin is an asset class that can be included in a well-diversified portfolio.

Actually, little research typically based on portfolios of U.S. assets, has been performed so far on portfolio diversification including bitcoin or cryptocurrencies in general (e.g. Adrianto & Diputra, 2017; Bouri, Molnár, et al., 2017; Bouri, Naji, et al., 2017; Brière et al., 2015; Carrick, 2016; Guesmi et al., 2018; Wu & Pandey, 2014). One of the oldest works by Eisl et al. (2015) show that including bitcoin in an already diversified portfolio of U.S. assets increases both the expected

return and the risk of the portfolios. They suggest a possible allocation of bitcoin in such portfolio to maximise the Sharpe Ratio. Brière et al. (2015) show that by adding bitcoin to an already diversified portfolio of U.S. assets, the Sharpe Ratio (Sharpe, 1963) improves. Bitcoin is found to increase the efficiency of portfolios when tested against other measures, such as the Omega Ratio (Wu & Pandey, 2014) and variations of the Sharpe ratio where VaR and CVaR replace the standard deviation as a measure of risk (Eisl et al., 2015). More recent research provides further support arguing that an optimal mix of bitcoin and U.S. equities can reduce the overall risk of a portfolio (Bouri, Molnár, et al., 2017). Interestingly, similar results are obtained in portfolios that include foreign currencies, commodities, stock and ETF (Adrianto & Diputra, 2017) as well as portfolios including global and emerging market indices (Guesmi et al., 2018).

However, bitcoin's financial characteristics, such as volatility, have evolved considerably since the “crash” of 2013, when some of the earlier properties that made it a safe haven completely disappeared (Kristoufek, 2015). This implies that bitcoin's qualities as a diversifier might also have been affected. More importantly, the quoted empirical studies either adopt the perspective of a U.S. investor or that of a global investor. Incidentally, in different specific markets bitcoin trading activity and the adoption rates are very different. In fact, historical data² from data.ity.org shows that the proportion of bitcoin bought and sold, for instance, in China has steadily grown over the years and as of September 2017 comprises of ~99% of all bitcoin exchange transactions globally, whereas USD transactions have remained relatively stagnant in comparison, amounting to < 1% of global exchange volume.³ Consequently, events happening in the wider Chinese economy directly affect the CNY bitcoin market which in turn can have a significant impact on the USD market (Kristoufek, 2015). A similar situation can be quoted for the European markets and assets. Thus, it is important to explore whether the benefits linked to bitcoin inclusion can be affected by the localisation of the portfolios. Currently no research explores differences in the role of bitcoin in portfolio diversification in European and Chinese assets while the research on U.S. is very much limited.

3. Methodology and data

The nature of this research is primarily explorative. It employs a variation of the established Modern Portfolio Theory (Markowitz, 1952, 1976) to estimate parameters and draw conclusions from historical data. The analysis will focus on the effects of adding bitcoin to an already diversified portfolio, or more precisely the effects that it might have on the return and risk of such portfolios. The analysis requires us to build efficient frontiers of portfolios where bitcoin is present and comparing it to portfolios where ceteris paribus, bitcoin is not present. The efficient frontier comprises of all the possible portfolios which can be constructed from a given pool of assets where the return is maximised given the desired risk. We use terms ‘optimal’ and ‘well-diversified’ interchangeably throughout this work to refer to such portfolios. We follow Eisl et al. (2015) by constructing portfolios in different optimisation contexts, described in more detail below. We then repeat the process but with an asset pool that contains bitcoin. The portfolios are subsequently compared to see if adding bitcoin has any effect on portfolio weights and the portfolio risk-return ratio in respect of the optimisation procedure used.

Scenario 1. Naïve (equal weights) Portfolio ($w_i = \frac{1}{N} \forall i$)

² Accessed on 12/09/2017

³ It is important to point out that in September 2017 China regulators forbade the use of Bitcoin and Bitcoin trade. Even if Chinese population can informally access Bitcoin via foreign platforms, we cannot rule out that, as far as Chinese market findings are concerned, our results can be somehow affected by the new limitations.

The naïve portfolio is constructed so that all assets are allocated equally irrespective of potential effects on the risk-return ratio. In their landmark study, DeMiguel, Garlappi, and Uppal (2007) have shown that a portfolio where asset allocation is calculated using the mean-variance optimisation procedure, performs no better than an equal weight portfolio consisting of the same pool of assets in terms of Sharpe Ratio (Sharpe, 1963, 1964). It would therefore be interesting to see what kind of effect bitcoin might have in this scenario and if such an effect is any different from the one observed in other scenarios. Since no part of the sample data is used to estimate optimal weights, the out-of-sample period equals the entire 60-month sample period.

Scenario 2. Long-Only Portfolio ($w_i \in \mathbb{R}^+ : \sum w_i = 1$)

This optimisation process allows no shorting and effectively limits the individual weights to 100%. The long only portfolio represents a more feasible option for investors given the context; and the asset weights should also be more stable when re-balancing. Since the first 12 months are used to determine portfolio weights, the explored sample period is 12 months shorter than the total sample period.

Scenario 3. Semi-constrained Portfolio ($w_i \in \mathbb{R} : -1 \leq w_i \leq 1 : \sum w_i = 1$)

Here, the optimisation process seeks to maximise the risk-return ratio of a portfolio, without placing any weight-related constraints on assets not allowing leveraging, as such it should yield a better risk-return ratio than other scenarios except for the unconstrained portfolio.

Scenario 4. Semi-constrained Portfolio with No Shorting on bitcoin ($w_{i \neq btc} \in \mathbb{R}, w_{btc} \in \mathbb{R}^+$)

Though hypothetically it is possible to short bitcoin, there is limited evidence at present and it is linked to financial products that are typically over-the-counter. Thus, we estimate a portfolio where all the assets but bitcoin can be shorted. The results of this strategy might therefore be rendered purely theoretical should the optimisation procedure demand that bitcoin is shorted.

It can be argued that it is also possible to build up unconstrained portfolios, that is a portfolio with no restrictions on asset weights, so that both shorting and leveraging are allowed. This type of optimisation is expected to result in extreme long or short positions which might not be implementable in the real world, due to large initial weights in either direction (short or long) and subsequent shifts in weights during rebalancing. In fact, we test this portfolio and we end up with large leverages that cannot be implemented in real life. Thus, we decided not to present these portfolios in the current analysis.

3.1. The mean-CVaR approach

For all scenarios except naïve, we adopt a variation of Markowitz's Modern Portfolio Theory (Markowitz, 1952) as basis for constructing efficient portfolios, where an efficient portfolio is defined as the one that achieves maximum expected return for a desired level of risk. Under the original model, the expected return is simply the weighted average of constituent asset returns and the risk, which is measured by the portfolio variance σ_p^2 , and is a function of the correlations ρ_{ij} of constituent assets, for all asset pairs (i, j) (Markowitz, 1952). The main disadvantage with the mean-variance approach is that it oversimplifies the risk-preferences of investors. Variance is a symmetric measure that incorporates both the upside and the downside volatility, whereas in the real-world, assuming investors are rational, only the downside component is undesirable.

An alternative risk measure proposed in literature is the Value-at-risk (VaR). It is an asymmetric measure that is expressed as a minimum loss value (or percentage) for a given probability and time horizon. VaR's main limitation is that it only estimates the minimum potential loss and does not quantify the amount this threshold could be exceeded by potentially underestimating the tail risk. The Conditional Value at

Risk (CVaR), or as it is also more commonly known, the Expected shortfall (ES) addresses this problem by calculating the expected return (average loss) beyond the VaR threshold (Alexander & Baptista, 2004) and it has been previously used to portfolio optimisation (e.g. Silvapulle & Granger, 2001; Topaloglou, Vladimirov, & Zenios, 2002). Acerbi and Tasche (2002) have shown that CVaR/ES offers a number of advantages over VaR, without giving up any of its original attractions. CVaR/ES is often described as a coherent measure of risk, because it satisfies a set of four desirable properties, namely: Monotonicity, Translation invariance, Homogeneity and Sub-additivity (Artzner, Delbaen, Eber, & Heath, 1999) whereas Variance and VaR do not.

We adopt the Conditional Value at Risk (CVaR) as the measure of portfolio risk which we calculate for each asset at the $\alpha = 5\%$ confidence level.

$$CVaR_p^2 = \sum_i \sum_j w_i w_j CVaR_i CVaR_j \rho_{ik} \quad (1)$$

From which, portfolio risk is derived:

$$CVaR_p = \sqrt{CVaR_p^2} \quad (2)$$

The optimisation procedure for non-naïve scenarios will therefore seek optimal asset weights that maximise the risk-return ratio, provided that the listed constraints for each scenario are met as prescribed.

$$\max \left(\frac{E(R_p)}{CVaR_p} \right) \quad (3)$$

Calculating portfolio risk using this approach requires that we know the CVaR of individual assets used in the optimisation process. There are various methods for calculating the CVaR, the most common being the variance-covariance, stochastic and empirical methods. Skewed distributions make the historical/empirical approach the preferred method for the purpose of our research (note that bitcoin returns exhibit a pronounced positive skew). The main advantage of this method is that it does not make assumptions of normality, since the distribution is inferred from historical data. The downside of this strategy of course is that it assumes that future distributions maintain the same skew over time. Using the 12 month in-sample daily data and for each asset, we calculate monthly moving averages (MA) of daily returns, resulting in 261 data points. CVaR is then calculated as the simple mean of all the observations on and below the Value-at-Risk (VaR) threshold at the $\alpha = 5\%$ confidence interval, or the 5th percentile of the calculated monthly moving average returns:

$$CVaR = \frac{1}{N} \sum_{i=1}^N MA : MA \leq VaR \quad (4)$$

It can be argued that alternative optimisations approaches are available. However, in the case of portfolios with Bitcoin it has been shown that the results for the period up to the end of 2017 are not affected by the portfolio optimisation approach used (Platanakis & Urquhart, 2018).

3.2. Evaluating portfolio performance

We divide back-testing in two parts. The first part assesses whether the initial weights w_i calculated during the optimisation process are robust over the entire sample investment period without any rebalancing. The second part examines the performance of portfolios over the entire sample period but with semi-annual weight rebalancing (eight times in total for the four years period). This technique, with respect to the alternative approach (e.g. Platanakis and Urquhart (2018) use weekly rebalancing), allows us to appreciate the extent of the rebalancing of the portfolio. In fact, frequent rebalancing does not allow us to understand the magnitude of the final adjustment of the portfolio. Since we have only eight rebalances, we do not include the transaction costs

linked to them because of their very reduced impact.

In order to calculate the Risk-Return ratio we calculate the out of the sample return and CVaR for both the portfolio with and without rebalancing. Using these data, we derive and observe monthly Risk-Return (RR) performance defined as:

$$RR_m = \frac{\bar{R}_p - \bar{R}_f}{CVaR_m} \quad (5)$$

The aforementioned approaches have their limitations since they depend on portfolio mean and variance and can have difficulties in providing robust and correct results when returns are not normally distributed (Smetters & Zhang, 2014). Thus, we also estimated the Sortino ratio (Sortino & van der Meer, 1991) that consider the standard deviation of only the downside excess returns, defined as

$$\text{Sortino} = \frac{\bar{R}_p - \bar{R}_f}{DR_p} \quad (6)$$

where DR_p denotes the downside excess return. We also estimate the Omega ratio (Keating & Shadwick, 2002) which represents the ratio between the average gain and the average loss (with target return set to zero following the Bernardo and Ledoit (2000) special case of the Gain-Loss-ratio), defined as

$$\text{Omega} = \frac{\sum_{t=1}^T \max(0, +R_t)}{\sum_{t=1}^T \max(0, -R_t)} \quad (7)$$

The advantage of the Omega ratio is it does not assume Normal distribution.

3.3. Data

3.3.1. The bitcoin price index

We construct the bitcoin USD Price Index (BTCUSD), EUR Price Index (EURBTC) and bitcoin CNY Price Index (BTCCNY) using data from data.bitcoinity.org, which in turn derives its data from the publicly available raw blockchain⁴ data. Historical data for bitcoin starts on 18th July 2010, however, the early period of bitcoin's trade is characterised by very low trading volumes and liquidity which does not significantly improve until 2012. Thus, we choose February 2012 as the start date of the sample period. Our analysis stops on 31st January 2017, covering 60 months of daily returns. We also exclude the observations since 1st February 2017 due to very unusual performance exhibited by bitcoin: it passed 1,000USD on 3rd January 2017 reaching 17,900USD in December 2017. The outstanding performance has been considered by the financial community a typical example of a bubble that, partially, exploded in the first months of 2018. Research suggests that a financial bubble can be defined as “[...] dramatic asset price increases followed by a collapse. Bubbles arise if the price exceeds the asset's fundamental value. This can occur if investors hold the asset because they believe that they can sell it at a higher price to some other investor even though the asset's price exceeds its fundamental value.” (Brunnermeier, 2016). Interestingly, this description matches bitcoin's 2017 behaviour. In addition, not only is bitcoin's 2017 performance characterised by rapidly increasing prices, but during the year press and blogs where full of stories that purport to justify the bubble with people feeling envy and regret they are not a part in the rally. These are further aspects that characterize a bubble (Fox, 2014). Finally, mathematical and empirical analysis also stresses the bubble stage of bitcoin in 2017 (Cheah & Fry, 2015). We argue that the inclusion of such a “bubble” period can introduce a bias in a long period analysis of the benefit of bitcoin inclusion in a portfolio of well diversified assets. In other words, the bitcoin performance in 2017 can generate a portfolio with a large

proportion of bitcoin in it that can largely outperform the portfolio without bitcoin. The real long-term portfolio effect of such “bubble” can be appreciated only by including the subsequent corrections in value that bitcoin has since experienced. However, we believe to be too soon to include the period for the purpose of exploring real long-term effects.

3.3.2. Other asset classes

To allow for creation of a well-diversified portfolio, we sample a broad range of assets available to US, Europe and Chinese investors consisting of equities, fixed income, commodities, real estate, cash equivalents, currencies, and alternative investments. Each of these asset-classes is represented by a liquid investible index, most of which are denominated in the local RMB, EUR or USD currency.

Table 1, shows a detailed overview of assets which will be used in the optimisation process.

The construction of three different portfolios allows us to explore whether bitcoin inclusion has different “local” effects, that is whether it has different impact on portfolio diversification according to whether it interacts with Chinese, U.S. or European assets.

4. Descriptive statistics

The Tables 2A, 2B and 2C below show the descriptive statistics of the excess monthly returns⁵ of assets included in the portfolios used in the optimisation process in the case of the U.S., European and Chinese assets.

In line with the previous studies by Eisl et al. (2015) and Wu and Pandey (2014), bitcoin exhibits large kurtosis and is positively skewed, albeit to a much lesser extent than previously reported (Fig. 1A, B and C).

The correlation coefficients are reported in Tables 3A, 3B and 3C and the evolution of bitcoin with respect other assets in Fig. 2A, B and C.

As far as the U.S. assets are concerned, bitcoin presents correlations with the largest part of the other indices. Interestingly, it shows a fairly high correlation with the Standard and Poor index (positive) while the highest negative correlation is with gold. This is in contrast with earlier works by Eisl et al. (2015) and Wu and Pandey (2014) that do not report significant correlations with any of the U.S. assets indices. Similarly, it is at a variance with the detailed analysis on bitcoin correlation performed by Baumöhl (2018). The difference can be ascribed to a different time window: Eisl et al. (2015) and Wu and Pandey (2014) look at the early stage of bitcoin (that presented quite high volatility and a very high increase in its value); Baumöhl (2018) includes in his analysis 2017, when the huge increase in bitcoin was completely decoupled from the performance of any other asset in the market. In terms of evolution thorough time Fig. 2A, shows the returns of bitcoin and other assets. It is evident how much more “erratic” are bitcoin returns. In addition, bitcoin returns tend to be decoupled with respect to the behaviour of the returns of other assets.

In the case of the European assets, bitcoin shows a relatively high and significant correlation with the hedge fund index (HFRX) and small but statistically significant positive correlations to all the European assets' indices except for the S&P Pan-Europe Developed Sovereign Bond Index, Dow Jones Europe Select Real Estate Securities Index and S &P EURO Futures Index Spot. In terms of evolution thorough time Fig. 2B, show the return of bitcoin and other European assets. The graph is very similar to the bitcoin/U.S. assets one: bitcoin is more “erratic” and decoupled with respect the behaviour of the returns of other assets.

In the case of the Chinese assets, bitcoin shows small but statistically

⁴ The blockchain is the distributed public ledger of all the historical bitcoin transactions.

⁵ The returns used in the analysis are net risk free rate. This explains the differences between values in US and China markets of the same asset class (e.g. bitcoin)

Table 1
Asset classes included in the analysis.

	Name	Mnemonic	Asset class
United States	BTC-USD-Index	btcsud	Cryptocurrency
	S&P U.S. TREASURY BILL INDEX	billus	Money Market
	S&P U.S. TREASURY BOND INDEX	condus	Fixed-income
	DOW JONES EQUAL WEIGHTS	corpus	Fixed-income
	U.S. ISSUED CORPORATE BONDS		
	S&P WCI GOLD (ER)	gold	Gold ETF
	S&P 100	sp100	Equity (large cap)
	S&P 500	sp500	Equity (mid cap)
	S&P 600	spsml	Equity (small cap)
	S&P WCI	wcig	Commodities
	DOW JONES U.S. REAL ESTATE INDEX	resus	Real Estate
	Global hedge fund index	hfrx	Alternative
	Dow Jones FXCM Dollar Index	usdollar	Currency
	BTC	btceur	Cryptocurrency
Europe	S&P Pan-Europe Developed Sovereign Bond Index	condeu	Fixed-income
	S&P Eurozone Investment Grade Corporate Bond Index	corpeu	Fixed-income
	S&P WCI GOLD (ER)	gold	Gold ETF
	S&P EUROPE 350	sp350eu	Equity
	S&P WCI Europe	wcie	Commodities
	Dow Jones Europe Select Real Estate Securities Index	reseu	Real Estate
	Global hedge fund index	hfrx	Alternative
	S&P EURO Futures Index Spot	speuf	Currency
	BTC-CNY-Index	btccny	Cryptocurrency
	S&P CHINA GOVERNMENT BILL INDEX	billcn	Money Market
	S&P CHINA SOVEREIGN BOND INDEX	condcn	Fixed-income
	S&P CHINA CORPORATE BOND INDEX	corpcn	Fixed-income
	S&P WCI GOLD (ER)	gold	Gold ETF
	S&P CHINA A 100 INDEX (RMB)	spc100	Equity (large cap)
China	S&P CHINA A 200 INDEX (RMB)	spc200	Equity (mid cap)
	S&P CHINA A SMALLCAP INDEX (RMB)	spsml	Equity (small cap)
	S&P WCI ASIA	wcia	Commodities
	Guggenheim China Real Estate ETF (TAO)	tao	Real Estate
	Global hedge fund index	hfrx	Alternative
	USDCNY Exchange rate (holding USD as investment)	usdcny	Currency

significant correlations to all the Chinese assets' indices except for the China Mid-Cap, Small-Cap equity indices and the Guggenheim China Real Estate ETF (TAO). The results are in line with the U.S. and European correlations. A significant negative correlation exists between bitcoin and the Chinese government bill index (billcn), and a somewhat weaker but still significant negative correlation with the China Sovereign bond (condcn) and corporate bond (corpcn) indices,

suggesting that bitcoin could be used by some investors in China as a safe haven during events that adversely affect the prices of these assets. In terms of evolution thorough time Fig. 2C, shows the return of bitcoin and other Chinese assets. The graph is very similar to the previous one confirming the fact that bitcoin is also more “erratic” with respect to Chinese assets and its returns; and decoupled with respect to those of other assets.

5. Analysis

In this section, we examine the bitcoin portfolios linked to the different areas: first we explore the role of bitcoin in the case of portfolios of U.S. assets, the European ones and those comprising Chinese assets. The last part of the analysis compares the results exploring similarities and differences in the use of bitcoin in the three areas.

5.1. U.S. assets and bitcoin

The initial weights of the optimisation process (not reported here) show the semi-constrained portfolios with a small weight of bitcoin relative to other assets. When applying semi-annual rebalancing (Table 4) to a long-only portfolio, the weight of bitcoin shows a progressive decline averaging at 5.47% for the duration of the investment period.

Both the semi-constrained/bitcoin long-only portfolios present an average proportion of bitcoin of 2.88% but with proportions between 0.01% to 8.19%. As far as the semi-constrained portfolio is concerned, no shorting emerges in the optimisation process. Thus, the weight of bitcoin in the semi-constrained and semi-constrained (no shorting on bitcoin) is the same. We tried (not reported here) with the unconstrained portfolio (i.e. a portfolio that allows for both shorting and leveraging without any limit). In doing so we obtain large adjustments during the period (from −16.76% to +16.21%) but not as large as in the case of Chinese and European portfolios (see later comments).

In order to have a clearer idea of the performance of a portfolio with bitcoin, we run back-testing: first without any rebalancing, and then with semi-annual rebalancing using the same optimisation technique used to calculate the initial weights (Tables 5A and 5B). (Table 6.)

In portfolios where no rebalancing is applied, the results indicate that adding bitcoin to portfolio of U.S. assets is beneficial in all the cases except in the case of the semi-constrained portfolio where the average return goes down by 6.67%. In terms of risk, volatility increases in both the naïve and long-only portfolio while it decreases in the case of the semi-constrained portfolio. When we look at Risk-Return, Sortino and Omega ratios all the portfolios benefit from bitcoin inclusion. In fact, in the case of naïve and long-only portfolios there is an increase in the return that overcomes the increase in the volatility of the portfolio; in the case of the unconstrained portfolio, performance improvements are linked to the reduction in the volatility of the portfolio of assets.

Semi-annual rebalancing is applied, the results show the positive

Table 2A
Descriptive statistics U.S. assets.

Descriptive statistics of excess monthly returns												
	btcsud	billus	condus	corpus	gold	sp100	sp500	spsml	wcig	resus	hfrx	usdollar
Min	−55.99%	−0.05%	−2.67%	−5.12%	−14.49%	−12.32%	−11.64%	−12.21%	−20.40%	−16.48%	−4.17%	−3.93%
Max	208.31%	0.07%	2.31%	3.86%	12.19%	11.77%	10.55%	17.56%	19.26%	13.34%	2.71%	3.90%
Mean	12.84%	0.00%	0.08%	0.35%	−0.35%	1.25%	1.10%	1.28%	−0.84%	0.58%	0.14%	0.34%
Median	6.51%	−0.01%	0.10%	0.43%	−0.22%	1.58%	1.30%	1.43%	−0.40%	1.03%	0.31%	0.27%
Skewness	2.38	1.11	−0.33	−0.55	−0.06	−0.58	−0.50	0.03	−0.22	−0.45	−0.84	0.06
Kurtosis	7.99	1.76	1.09	1.04	0.05	1.25	1.07	0.68	0.26	0.52	0.91	−0.45
St.Dev.	34.90%	0.02%	0.76%	1.34%	4.54%	3.07%	3.05%	4.14%	6.79%	4.05%	1.12%	1.34%
VaR	24.63%	0.02%	1.16%	2.28%	7.81%	4.53%	4.44%	5.45%	13.17%	6.86%	2.04%	1.71%
C-VaR	32.90%	0.03%	1.70%	3.01%	9.91%	6.49%	6.49%	7.60%	16.29%	8.74%	2.76%	2.24%
Sharpe (C-VaR)	0.39	0.00	0.04	0.11	−0.04	0.19	0.17	0.17	−0.05	0.07	0.05	0.15

Table 2B
Descriptive statistics European assets.

Descriptive statistics									
	btceur	condeu	corpeu	gold	sp350eu	wcie	reseu	hfrx	speuf
Min	−54.15%	−3.99%	−2.83%	−14.88%	−16.18%	−23.55%	−18.11%	−4.57%	−8.71%
Max	190.69%	2.66%	1.46%	11.79%	11.83%	20.47%	12.02%	2.32%	7.14%
Mean	12.76%	0.00%	−0.07%	−0.75%	0.51%	−1.24%	0.19%	−0.26%	−0.67%
Median	6.43%	0.18%	0.07%	−0.62%	0.69%	−0.66%	0.52%	−0.08%	−0.59%
Skewness	2.22	−0.85	−0.83	−0.06	−0.51	−0.25	−0.39	−0.84	−0.27
Kurtosis	7.02	0.90	0.67	0.05	0.85	0.38	0.11	0.91	0.58
St.Dev.	33.82%	1.13%	0.66%	4.54%	3.98%	7.33%	4.74%	1.12%	2.31%
VaR	22.90%	2.28%	1.40%	8.21%	6.85%	14.94%	7.99%	2.43%	4.43%
C-VaR	32.93%	2.94%	1.75%	10.31%	9.32%	18.22%	10.50%	3.15%	6.00%
Sharpe (C-VaR)	0.39	0.00	−0.04	−0.07	0.05	−0.07	0.02	−0.08	−0.11

Table 2C
Descriptive statistics Chinese assets.

Descriptive statistics of excess monthly returns											
	btccny	billcn	condcn	corpcn	gold	spc100	spc200	spcsml	wcia	tao	hfrx
Min	−56.21%	−1.79%	−3.47%	−2.13%	−14.70%	−33.33%	−47.12%	−53.94%	−11.89%	−22.39%	−4.39%
Max	208.10%	1.74%	3.19%	2.95%	11.97%	33.12%	28.94%	35.67%	13.65%	18.02%	2.50%
Mean	12.63%	0.00%	0.09%	0.20%	−0.57%	0.61%	0.61%	1.01%	−0.73%	0.61%	−0.08%
Median	6.29%	−0.03%	0.12%	0.22%	−0.44%	0.57%	1.25%	1.32%	−0.91%	1.22%	0.10%
Skewness	2.38	−0.17	−0.32	−0.06	−0.06	0.31	−0.97	−0.93	0.31	−0.46	−0.84
Kurtosis	7.99	6.26	1.01	1.24	0.05	2.86	3.54	4.50	0.25	0.54	0.91
St.Dev.	34.90%	0.32%	0.88%	0.73%	4.54%	7.77%	9.00%	9.90%	4.07%	6.44%	1.12%
VaR	24.85%	0.34%	1.40%	1.15%	8.03%	11.59%	15.40%	15.52%	7.36%	9.67%	2.25%
C-VaR	33.11%	0.73%	1.99%	1.52%	10.13%	17.59%	24.61%	26.72%	8.50%	14.75%	2.97%
Sharpe (C-VaR)	0.38	0.00	0.04	0.13	−0.06	0.03	0.02	0.04	−0.09	0.04	−0.03

A

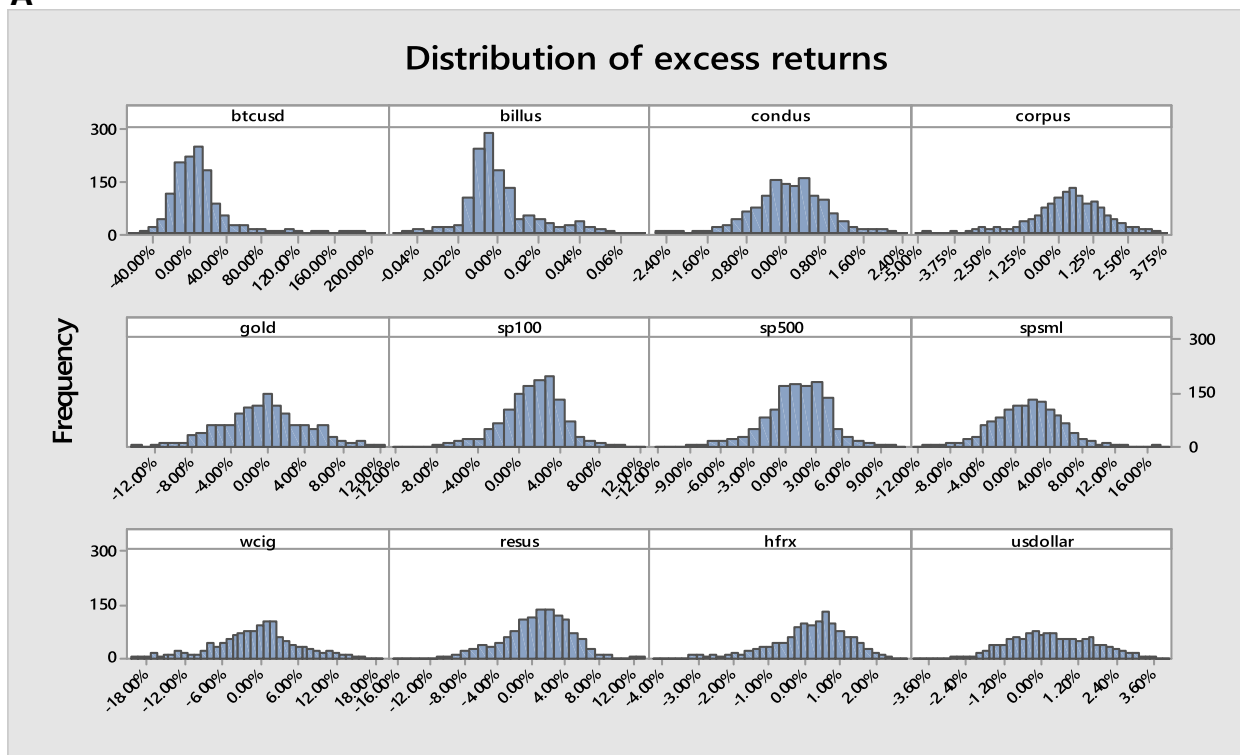


Fig. 1. A U.S. assets distributions of excess returns
B European assets distributions of excess returns
C Chinese assets distributions of excess returns.

B

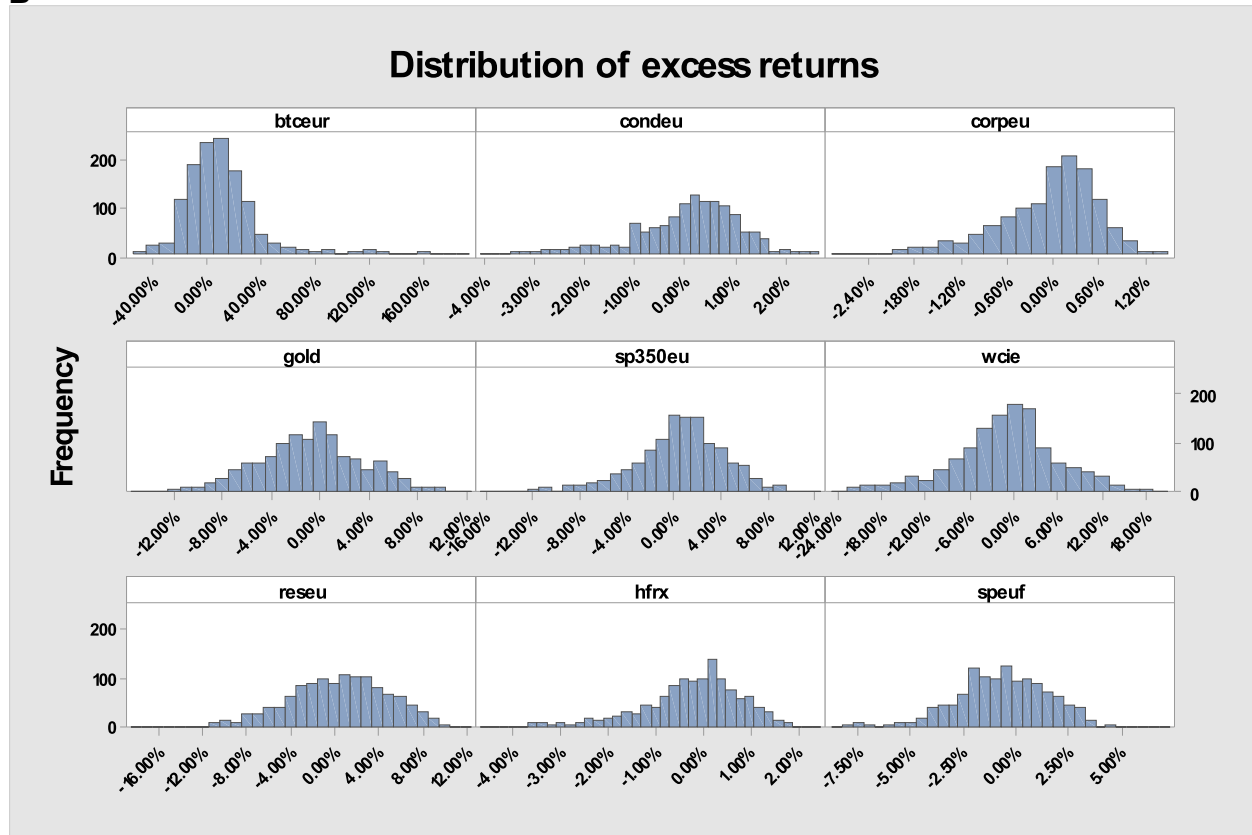


Fig. 1. (continued)

effect on return in the case of the long-only portfolio. In terms of performance the Risk-Return ratio improves in all the portfolios. However, in the case of semi-constrained portfolio the average Omega ratio moves from 3.0031 to 1.8159 and the Sortino ratio worsens from 4.0466 to 1.8454.

We also explore how often the portfolio with bitcoin out-performs the portfolio without bitcoin. In the case of the no rebalancing, the naïve portfolio with bitcoin out-performs that one without bitcoin in 65.70% of the observations, while in the case of long-only and semi-constrained portfolios the percentages decrease to 54.77% and 54.23% respectively. When we look at the portfolio with semi-annual rebalancing, the portfolios with bitcoin outperform those without bitcoin in 54.77 in the case of long portfolio and in 54.23% of the cases in the case of semi-constrained ones. The results suggest that the benefit bitcoin generates relies on good returns in relatively high number of days that compensate for poor returns in a relatively low number of days.

Finally, we explore the performance of the portfolios for subperiods (years) in order to explore whether bitcoin benefit is linked to a specific period (detailed tables not reported here). The analysis suggests that the benefit generated by bitcoin in terms of returns is mainly due to a majority of very good performance in 2013 when the effects of inclusion are significantly positive in naïve and long-only scenarios (4.12% and 6.29% average daily return with respect to 0.57% and 0.10% average return of the portfolio without bitcoin). This result partially supports the earlier findings (Eisl et al., 2015) that suggest that bitcoin improves the Risk-Return ratio in all scenarios if added to an optimal portfolio of western assets. A divergence can be observed in the period after December 2013 when the inclusion reveals marginal effects in terms of the return ratio in all portfolios. In terms of risk (volatility of the portfolio), the results suggest that inclusion of bitcoin increases it in all the portfolios in 2013 and 2016.

5.2. European assets and bitcoin

In the initial optimisation process, only the long-only portfolio results in significant amounts of bitcoin: in the semi-constrained portfolio the weight of bitcoin relative to other assets is relatively small and remains small with semi-annual rebalancing. When applying semi-annual rebalancing to long-only portfolios the weight of bitcoin faces large changes (between 0% and 56.31%). In the semi-constrained portfolio bitcoin has an overall minor yet stable role. We also attempted analysis of the unconstrained portfolio. We end up with rebalancing in terms of bitcoin between -10.18% and 104.97% that cannot be easily implemented.

The back-testing results for the European markets portfolios with and without rebalancing are reported in Tables 7A and 7B.

Without rebalancing (Table 7A), the results in terms of return indicate that adding bitcoin to a portfolio of European assets is beneficial in all the cases. However, all the portfolios show an increase in the volatility, as well. In terms of performance, Sortino and Omega ratio suggest a benefit linked to bitcoin inclusion in all the portfolios strategies while in the case of Risk-Return of the semi-constrained portfolio there is a decrease of the ratio from 0.03 to 0.02. The improvement in the performance (Risk-return, Sortino and Omega ratios) of the majority of the portfolios has to be ascribed to the increase in the returns that compensate for the increase in the riskiness (volatility) of the portfolio. The analysis of the performance of the portfolio with rebalancing (Table 7B) shows that in all the cases the portfolio with bitcoin grants higher returns than those without bitcoin. At the same time all the portfolios face an increase in the volatility. Nevertheless, Risk-Return, Omega and Sortino improves in all cases where bitcoin is added. If anything, findings are stronger than the results we obtain on the portfolio without rebalancing where the improvement in performance (Risk-Return, Sortino and Omega ratio) has to be ascribed to the

C

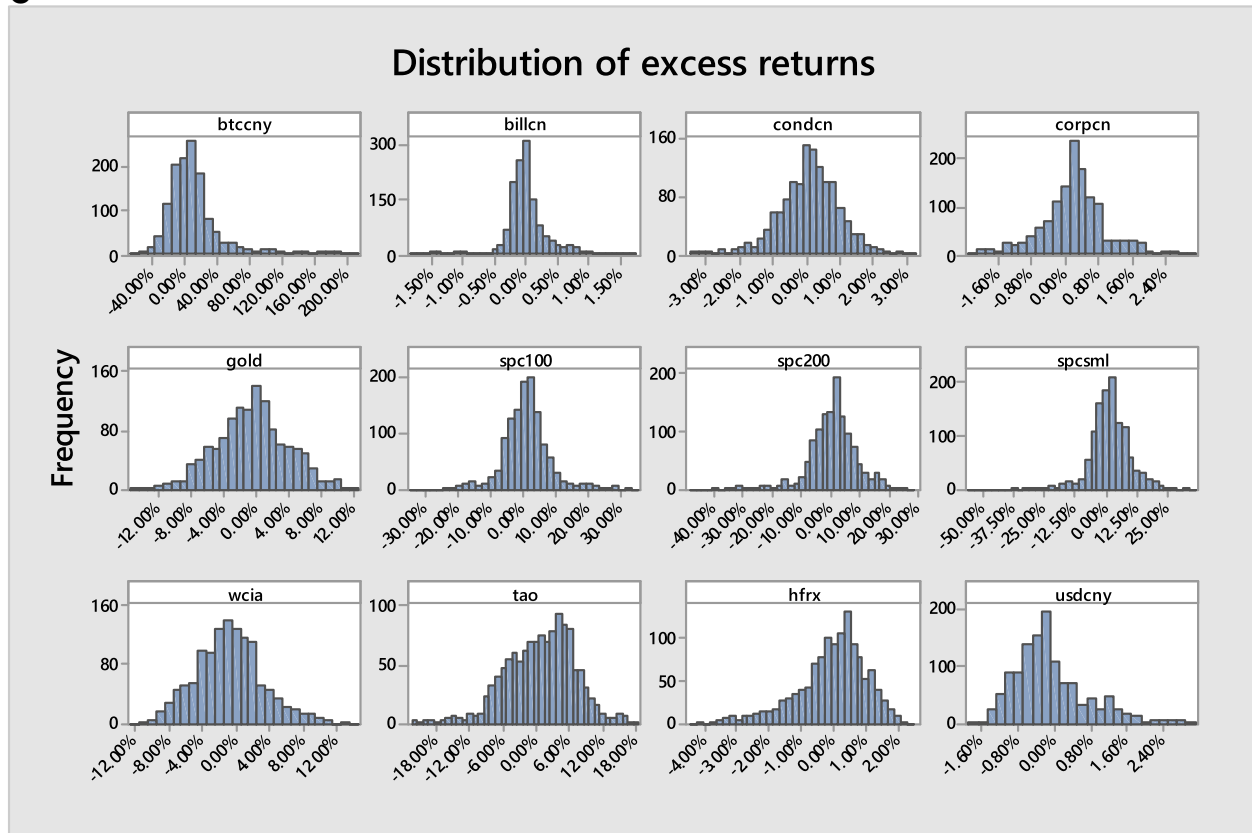


Fig. 1. (continued)

Table 3A
Correlation matrix U.S. assets.

Correlation matrix - coefficients (P-Values)											
	btccusd	billus	condus	corpus	gold	sp100	sp500	spsml	wcig	resus	hfrx
billus	− 0.087 ***										
condus	− 0.051 *	0.245***									
corpus	0.009	0.230***	0.772***								
gold	− 0.183***	0.325***	0.424***	0.364***							
sp100	0.264***	− 0.110***	− 0.322***	0.028	− 0.099***						
sp500	0.244***	− 0.097***	− 0.316***	0.042	− 0.082***	0.990***					
spsml	0.150***	− 0.057**	− 0.407***	− 0.081***	− 0.101***	0.801***	0.850***				
wcig	0.063**	0.273***	− 0.306***	− 0.030	0.117***	0.354***	0.361***	0.387***			
resus	0.006	0.086***	0.335***	0.586***	0.137***	0.522***	0.564***	0.459***	0.007		
hfrx	0.187***	0.035	− 0.289***	0.078**	− 0.092***	0.763***	0.802***	0.760***	0.441***	0.440***	
usdollar	0.058**	− 0.257***	− 0.170***	− 0.233***	− 0.454***	− 0.027	− 0.027	0.014	− 0.338***	− 0.059**	0.044

Table 3B
Correlation matrix European assets.

Correlation matrix - coefficients (P-Values)								
	btceur	condeu	corpeu	gold	sp350eu	wcie	reseu	hfrx
condeu	0.041							
corpeu	0.127 ***	0.793 ***						
gold	−0.191 ***	0.213 ***	0.144 ***					
sp350eu	0.145 ***	0.160 ***	0.394 ***	−0.083 ***				
wcie	0.060 **	−0.254 ***	0.007	0.087 ***	0.291 ***			
reseu	0.021	0.298 ***	0.473 ***	0.074 ***	0.654 ***	0.216 ***		
hfrx	0.215 ***	0.04	0.308 ***	−0.092 ***	0.776 ***	0.445 ***	0.507 ***	
speuf	−0.090	−0.214 ***	−0.192 ***	0.302 ***	−0.316 ***	0.239 ***	0.197 ***	−0.100 ***

Table 3C
Correlation Matrix Chinese Assets.

Correlation matrix - coefficients (P-Values)											
	btccny	billcn	condcn	corpcn	gold	spc100	spc200	spcsml	wcia	tao	hfrx
billcn	−0.357***										
condcn	−0.254***	0.619***									
corpcn	−0.250***	0.683***	0.863***								
gold	−0.183***	0.085***	0.049*	0.009							
spc100	−0.080***	0.051*	−0.060**	−0.121***	0.038						
spc200	−0.057**	0.193***	−0.045	−0.028	−0.015	0.845***					
spcsml	−0.051*	0.241***	−0.030	0.023	−0.009	0.736***	0.974***				
wcia	−0.100***	0.017	0.012	−0.038	0.635***	0.124***	0.050*	0.059**			
tao	−0.046	0.018	0.042	0.042	0.165***	0.496***	0.446***	0.403***	0.221***		
hfrx	0.187***	−0.050*	−0.160***	−0.089***	−0.092***	0.309***	0.361***	0.342***	0.089***	0.486***	
usdcny	−0.094***	−0.019	0.058**	0.071**	−0.055**	−0.157***	−0.213***	−0.202***	−0.075***	−0.403***	−0.283***

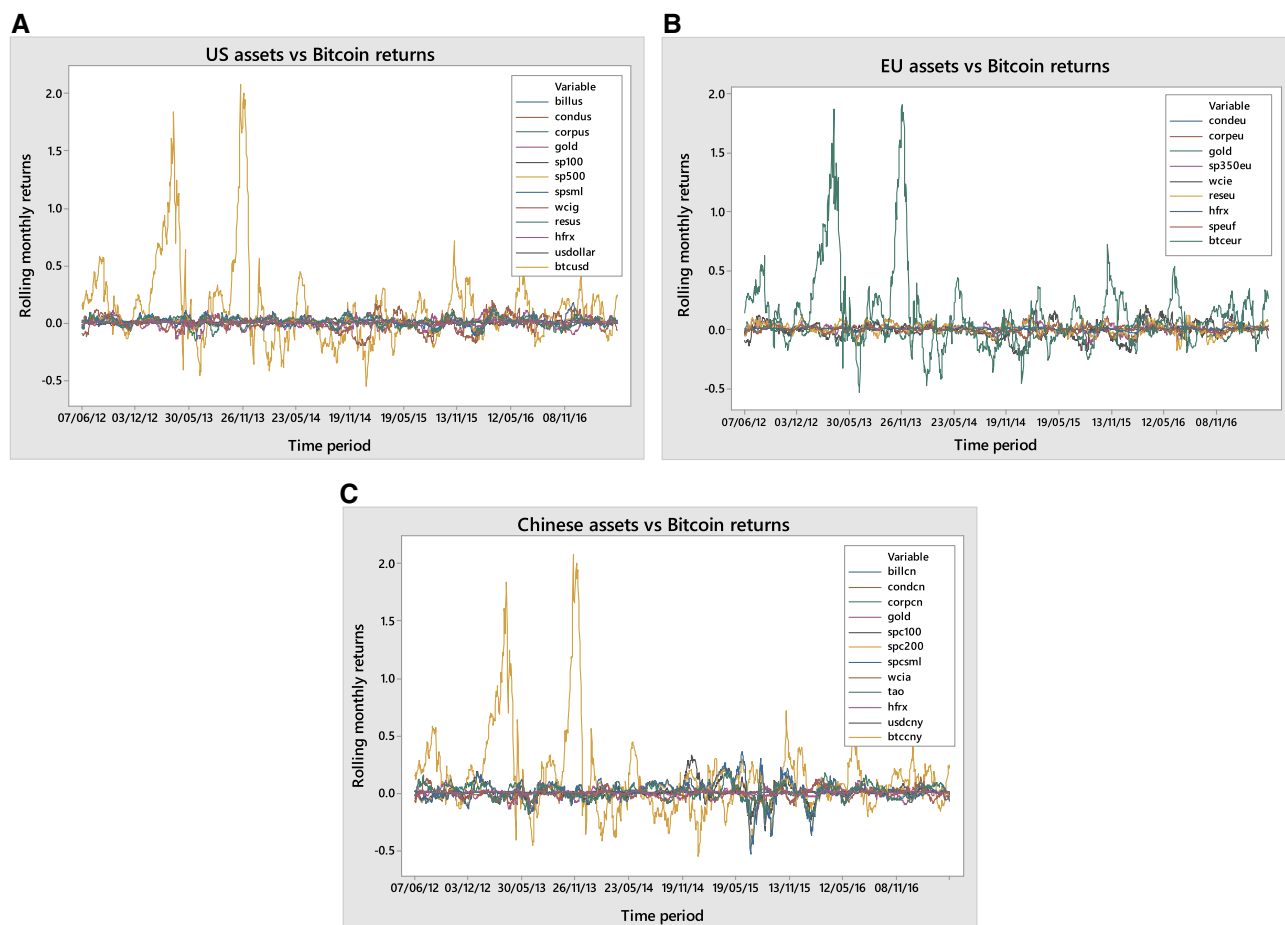


Fig. 2. A U.S. assets and bitcoin returns through time
B European assets and bitcoin returns through time
C Chinese assets and bitcoin returns through time.

increase in the returns that more than compensate for the increase in the risk of the portfolio.

As in the case of the US analysis, we also explore how often the portfolio with bitcoin outperforms the portfolio without bitcoin. Without rebalancing, the naïve portfolio with bitcoin outperforms that without bitcoin in 65.59% of the cases; the long only and the semi-constrained portfolios with bitcoin outperform the portfolio without bitcoin in 63.34% of the case. With rebalancing, the evidence suggests that the portfolio with bitcoin outperforms the one without bitcoin in 54.56% of the cases in the long only strategy, in 33.04% of the cases in the semi-constrained and in 35.05% in the case of semi-constrained

with no shorting on bitcoin. In this case evidence suggests that when rebalancing is included, the superior performance of portfolios that include bitcoin with respect those that do not is linked to a very high returns in a relatively small number of days.

Finally, we explore the performance of the portfolios for subperiods (table not reported here). The inclusion of bitcoin in 2013 increases the performance in all the portfolios (between 2.68% and 40.23%) and the same applies in the case of 2014 even if the benefit is less marked (between 0.68% ad 8.01%). A divergence can be observed in the period after December 2013 where the inclusion reveals marginal effect in terms of the return in any scenario. As far as the volatility is concerned,

Table 4

Portfolio of U.S. assets: bitcoin weights in portfolio with semi-annual rebalancing.

Semi-annual BTC weights			
Portfolio	Long-only	Semi-C.	btc > 1
Jun-13	14.24%	1.30%	1.30%
Dec-13	10.65%	1.80%	1.80%
Jun-14	13.84%	3.70%	3.70%
Dec-14	0.00%	0.01%	0.01%
Jun-15	0.00%	0.74%	0.74%
Dec-15	0.48%	0.47%	0.47%
Jun-16	4.43%	6.85%	6.85%
Dec-16	0.08%	8.19%	8.19%
Bitcoin mean	5.47%	2.88%	2.88%

bitcoin inclusion increases it in naïve portfolio for all the years except in 2016. For other portfolios, bitcoin tends to decrease it. In terms of risk-return, bitcoin inclusion improves it in the case of the naïve portfolio while in the case of other portfolios, the results are mixed with a decreasing benefit as the time goes by. For instance, bitcoin inclusion appears less beneficial in the case of semi-constrained portfolio where the Risk-Return ratio improves only in the second year while in the case of semi-constrained (no shorting) it improves marginally during the same period. Our empirical evidence partially supports earlier findings (Eisl et al., 2015) that suggest that bitcoin improves the return in all scenarios if added to an optimal portfolio of western assets.

5.3. Chinese assets and bitcoin

The initial optimisation process suggests the inclusion of a significant proportion of bitcoin in the long-only portfolio only. The semi-constrained portfolio has a proportion of bitcoin that is relatively small. When applying semi-annual rebalancing (Table 8) to a long-only portfolio, the weight of bitcoin is reduced to a relatively minor proportion.

Turning our attention to the return of the portfolio with Chinese assets, in the case of no rebalancing (Table 9A), both the naïve and the long-only portfolios present improvements with respect the portfolio without bitcoin. At the same time, the semi-constrained portfolio seems to be adversely affected by bitcoin inclusion. However, the semi-constrained portfolios enjoy a reduction in the risk with a 12% decrease in

Table 6

Portfolio of European assets: bitcoin weights in portfolio with semi-annual rebalancing.

Semi-annual BTC weights			
Portfolio	Long-only	Semi-C.	btc > 1
Jun-13	35.46%	11.52%	11.52%
Dec-13	17.53%	6.82%	6.82%
Jun-14	16.25%	1.39%	1.39%
Dec-14	0.28%	0.01%	0.01%
Jun-15	0.00%	−2.02%	0.00%
Dec-15	55.05%	−2.37%	0.00%
Jun-16	26.15%	1.26%	1.26%
Dec-16	56.31%	1.71%	1.71%
Bitcoin Mean	25.88%	2.29%	2.84%

volatility. In terms of portfolio performance, naïve and long only benefit from an improvement on the Risk-Return, Sortino and Omega ratio while in the case of the semi-constrained portfolio the performance outputs mixed results: Risk-Return and Sortino ratio improve while the Omega ratio worsens (even if only marginally). When semi-annual rebalancing is applied, the results show a positive effect on the return of all the portfolio except the semi constrained one that faces a decrease from 0.82% to 0.77%. In terms of performance, the long-only presents an improvement on all the metrics while the semi-constrained faces a worsening in the case of the Sortino ratio. Interestingly, in the case of the semi-constrained portfolio, the improvement in the performance is mainly linked to the reduction in the volatility while in the case of the naïve, the better performance is mainly to be ascribed to the improvement in the return that compensates for the increased volatility. Finally, the semi-constrained portfolio with no shorting on bitcoin faces a worse Omega and Sortino ratio with no change in the Risk-Return one. (Table 9B.)

In terms of comparative performance, the bitcoin portfolios without rebalancing present returns higher than those without bitcoin in between 44.80% of the cases (semi-constrained) and 66.35% of the cases (long-only). When re-balancing is applied, the portfolio with bitcoin outperforms that one without bitcoin in 59.38% of the observations in the case of the long only while the semi-constrained portfolio with bitcoin outperforms the portfolio without bitcoin in 51.34% of the observations.

Table 5A

Portfolio of U.S. assets: performance – no rebalancing.

	Naïve			Long-only			Semi constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	0.29%	0.96%	231.75%	0.19%	0.85%	360.19%	0.82%	0.77%	−6.67%	0.82%	0.77%	−6.67%
std dev	0.0088	0.0196	122.57%	0.0058	0.0291	405.07%	0.0128	0.0103	−19.79%	0.0128	0.0103	−19.79%
Risk-Return (Cvar)	0.0127	0.0176	38.58%	0.0070	0.0155	121.43%	0.9100	1.3500	48.35%	0.9100	1.3500	48.35%
Omega	0.9517	2.4681	159.33%	2.4220	3.4702	43.28%	7.9503	10.7326	35.00%	7.9503	10.7326	35.00%
Sortino	0.5112	1.7165	235.78%	0.7752	1.0140	30.80%	2.3513	2.4556	4.44%	2.3513	2.4556	4.44%

Table 5B

Portfolio of U.S. assets: performance – rebalancing.

	Long-only			SEMI constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	0.19%	0.85%	360.19%	0.58%	0.57%	−2.10%	0.58%	0.57%	−2.10%
std dev	0.0058	0.0291	405.07%	0.0086	0.0098	14.03%	0.0086	0.0098	14.03%
Risk-Return (Cvar)	0.0133	0.0159	19.55%	0.0940	0.0950	1.06%	0.0940	0.0950	1.06%
Omega	1.8171	2.5787	41.91%	3.0031	1.8159	−39.53%	3.0031	1.8159	−39.53%
Sortino	0.6532	0.7541	15.46%	4.0466	1.8454	−54.40%	4.0466	1.8454	−54.40%

Table 7A
Portfolio of European assets: performance – no rebalancing.

	Naïve			Long-only			Semi constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	−0.43%	0.51%	217.30%	−0.01%	2.56%	19,844.81%	0.46%	0.75%	63.64%	0.46%	0.74%	60.84%
std dev	0.0111	0.0246	121.32%	0.0126	0.0695	450.74%	0.0181	0.0236	30.91%	0.0181	0.0236	30.91%
Risk-Return (Cvar)	0.0200	0.0300	50.00%	0.0300	0.0500	66.67%	0.0300	0.0200	−33.33%	0.0300	0.0200	−33.33%
Omega	0.4920	1.7221	250.02%	0.9749	3.6227	271.58%	1.8818	2.7011	43.54%	1.8818	2.7008	43.52%
Sortino	−0.5225	0.5056	196.77%	−0.0166	1.4491	8828.22%	0.5366	1.0070	87.66%	0.5366	1.0068	87.63%

Finally, we explore returns and risk of the portfolios in the sub-periods. In line with evidence from portfolios with European and U.S. assets, during this year, the effects of inclusion are significantly positive in naïve and long-only scenarios (3.54% and 14.96% average daily return with respect 0.45% and −0.13% average return of the portfolio without bitcoin). Divergence can be observed in the period after December 2013 where the inclusion reveals limited effect in terms of the Risk-Return ratio in any scenario: the portfolio with bitcoin overperforms marginally and occasionally it underperforms. As far as volatility is concerned, bitcoin inclusion decreases it in all the portfolios in 2013 while in the following years the result is more varied: in 2014 naïve and long-only portfolio do not benefit from bitcoin inclusion while in 2015 this is only the case for naïve portfolio. Finally, in 2016, only the semi-constrained portfolio gains from a reduction in its volatility for all the years. Also, in this case, our empirical evidence provides partial supports the earlier findings (Eisl et al., 2015) which suggest that bitcoin improves the return in all scenarios if added to an optimal portfolio of western assets even if they are less supportive than in the case of U.S. and European assets.

6. Discussion

The analysis is quite conclusive: (i) the inclusion of bitcoin tends to improve the return of the portfolios in the largest majority of cases; (ii) only in the case of US and Chinese semi-constrained portfolios without rebalancing and in the case of the US semi-constrained portfolio with rebalancing the return of the portfolio including bitcoin decreases. In other words, only in 3 cases out of the 21 different portfolios we examined bitcoin does inclusion worsen portfolio return. At the same time, the portfolios enjoy a reduction in term of volatility only in the case of the China semi-constrained portfolios with and without rebalancing and in the case of the US semi-constrained portfolio with rebalancing. Thus, we can conclude that if bitcoin improves the return, it also increases the riskiness of the portfolios. However, in terms of overall performance, the results tend to support the idea that bitcoin improves the portfolio of diversified assets. In fact, it improves the Risk-Return ratio in all the portfolios but two (EU – semi-constrained with/without shorting in “without rebalancing” case); the Omega ratio in all the cases but that of China semi-constrained (with/without shorting with/without rebalancing) and in the case of US semi-constrained (with/without shorting without rebalancing), that is in 6 cases out of 21 scenarios. Finally, the Sortino ratio improves in all the portfolios without rebalancing and worsens only in the case of US and China semi-constrained (with/without shorting) with rebalancing, that is in 4 cases out of 21. All in all, out of 63 performance measurements, we have 11 cases where the performance worsens (i.e. 17.5% of the cases).

The results in terms of annual (subperiod performance) performance are quite consistent among the three contexts under investigation. More in detail, in the case of the long portfolios, the results are always consistent. In the case of naïve portfolio there is only one case (Chinese portfolio in 2015) where the result is different with respect the other two contexts. A lower level or consistency emerges in the case of

unconstrained portfolios⁶ and in the case of semi-constrained portfolios.⁷ In terms of risk (C-VaR), the results are very consistent among the three contexts. The only investment strategy that presents different results among the three contexts is the semi-constrained portfolio. Thus, we can conclude that there are no major differences in the inclusion of bitcoin in the portfolios of U.S., European of Chinese assets.

An interesting aspect to consider is the evolution of the role of bitcoin through time: there is a difference between the role of bitcoin prior to December 2013 and the subsequent period until the end of 2016. During this early period, the effects of inclusion are significantly positive in all the portfolios. However, this changes at the beginning of 2014 when portfolios with bitcoin underperformed those without in 15 out of 45 cases. We can conclude that the benefit linked to bitcoin's inclusion is mainly linked to a quite specific period (2013) and to the fact that in 2013 bitcoin showed a marked increase in its value.

All in all, our evidence suggests that bitcoin plays a relevant role in portfolios diversification: it is consistent in generating benefits linked to the increase in the returns while it has a more marginal role in decreasing volatility. More in detail, the inclusion of bitcoin consistently generates benefits in the portfolios' performance in naïve and long portfolios both with and without rebalancing. The results are mixed in the case of semi-constrained portfolios. In addition, the overall benefit in the time window considered in the analysis can be mainly ascribed to the result of the high return obtained in 2013 since the benefits of bitcoin in the following years are not systematic and definitely reduced when compared with 2013. However, our results are partially in line with the previous findings and tend to suggest the current speculative nature of bitcoin as an asset class.⁸

So, as far as portfolio managers are concerned, which is the best investment strategy for a portfolio including bitcoin? If rebalancing has to be avoided, the best strategy in terms of absolute returns seems to be

⁶ (Chinese portfolios generate a different result with respect U.S. and European ones in three cases while the U.S. portfolios generate a different result with respect European and Chinese ones in one case)

⁷ (U.S. portfolios generate a different result with respect Chinese and European ones in two cases while the European portfolios generate a different result with respect U.S. and Chinese ones in one case)

⁸ An important caveat must be stressed: the portfolio allocation suggested by our analysis cannot necessarily be implemented. The major problem is linked to the fact that the bitcoin market is not very liquid in the sense that buying and selling large amounts of bitcoin can be problematic. Currently there is a relatively low number of websites where bitcoin can be traded; occasionally they struggle to satisfy investor requests; it is not unusual to find substantial differences in the exchange rate applied by different website (suggesting good possibilities for arbitrage but the low liquidity of the market). This can adversely affect the possibility of implementing the semi-annual adjustments when they imply a large reduction/increase in the amount of bitcoin to include in the portfolio: the investor may struggle to buy the amount of bitcoin that they want to add to their portfolios; similarly, they can discover that by selling the bitcoin the price can drop dramatically. However, more frequent rebalancing (e.g. weekly as suggested by Platanakis and Urquhart (2018) or monthly that implies smaller adjustments) can suggest portfolios that can be implemented even if in those cases the portfolio performance can at least partially compromised by the increase in the transaction costs.

Table 7B

Portfolio of European assets: performance – rebalancing.

	Long-only			Semi constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	− 0.52%	2.44%	569.28%	0.46%	0.76%	62.95%	0.46%	0.82%	76.93%
std dev	0.0176	0.0634	261.07%	0.0086	0.0204	137.87%	0.0086	0.0204	137.19%
Risk-Return (Cvar)	0.0400	0.0300	25.00%	0.0100	0.0100	0.00%	0.0100	0.0100	0.00%
Omega	0.4663	3.0728	558.95%	1.7026	2.8050	64.75%	1.7026	3.0075	76.64%
Sortino	− 0.4044	1.9553	583.55%	1.1433	1.9624	71.64%	1.1433	2.1541	88.42%

Table 8

Portfolio of Chinese assets: bitcoin weights in portfolio with semi-annual rebalancing.

Semi-annual BTC weights			
Portfolio	Long-only	Semi-C.	BTC > 0
Jun-13	34.89%	0.74%	0.74%
Dec-13	9.31%	0.79%	0.79%
Jun-14	4.33%	4.78%	4.78%
Dec-14	0.31%	0.44%	0.44%
Jun-15	0.00%	0.10%	0.10%
Dec-15	0.75%	− 1.35%	2.42%
Jun-16	4.32%	− 2.22%	0.05%
Dec-16	3.42%	0.69%	0.69%
Bitcoin Mean	7.17%	0.50%	1.25%

the naïve one for the US (0.96%) and the long only in the case of EU (2.56%) and China (3.13%). However, if we look at the improvement that can be obtained from bitcoin inclusion, the picture is marginally different since the portfolio that benefits most from bitcoin inclusion is the long one in all the contexts under investigation. When we look at the portfolio with rebalancing, the best strategy is the long one for all the contexts under investigation. All in all, shorting assets as well as shorting bitcoin even when it generates benefit, does not outperform the long only portfolio. The (under)performance of the portfolios where shorting is included compared to the long-only portfolios and especially the naïve portfolio is at least partially in line with the work by [DeMiguel et al. \(2007\)](#) that show that naïve portfolios often perform better than optimised portfolios in general.

Our analysis clearly suggests that portfolios' improved performances compensate for the higher risk incurred because of bitcoin inclusion. However, the increase in the risk of then portfolios raise the question whether authorities should take any action in order to protect investors. In fact, regulators cannot intervene in any way on bitcoin price formation since it is not under the control of any authority by design. However, regulators have the power to set up rules that limit the possibility for any regulated investment tool (e.g. trust funds, insurance policies, pension funds, etc.) to invest in bitcoin (e.g. the case of KodakCoin and the need for Kodak to check the status of individual accredited investors for 40,000 potential investors in order to follow the SEC regulation). They can even limit possibility of trading in bitcoin (something similar to what has been done recently by the Chinese and Korean governments). Thus, the question is: should regulators

intervene?

According to our results, bitcoin (and, possibly, all the cryptocurrencies) seem to be assets that have to be handled with care (along the lines of SEC limitation on KodakCoin investors). However, at this stage we know too little about all the cryptocurrencies and their behaviours: even though the amount of data appears overwhelmingly large (detailed prices and volumes for bitcoin and for many other cryptocurrencies since their inception), the timespan is very much limited to a maximum of 10 years (bitcoin) – with the observations during the early years that are quite meaningless because of the very small volumes. Thus, researchers can hardly derive any final conclusion and further analysis is needed to properly appreciate bitcoin and cryptocurrencies' role; in particular if one considers the 2017 boom/burst evolution.

As far as the regulators are concerned, our analysis suggests two challenges: 1) it is imperative to expand knowledge of this innovative tool as fast as possible in order to evaluate whether any additional regulation is needed and, if this is the case, what kind of rules should be implemented. In fact, if on the one hand too strict a regulation can constrain the expansion of a financial tool that can be an efficient payment tool (if we follow [Nakamoto \(2008\)](#) argument) and an interesting asset to invest in, the lack of it (or a very loose one) can put at risk the savings of many people that can be attracted by the idea of easily obtaining high returns in a very short time without perceiving the real risk incurred in the investment; 2) it is important for regulators to act rapidly and effectively in any decision about bitcoin. Bitcoin and cryptocurrencies in general evolve at a speed that is different to what we are used to (e.g. bitcoin was considered to be an inefficient asset till two years ago ([Urquhart, 2016](#)) but recently academics changed their mind considering it an efficient one ([Wei, 2018](#)) that can be used in portfolio diversification as suggested by our work). The same applies to their popularity. This implies that regulators have to be able to deal with financial tools that evolve very rapidly and become popular even faster.

7. Conclusions

Bitcoin is arguably one of the most important financial innovations in recent times. It has drawn an increasing number of critics and supporters, yet its rise has been exceptional since its inception in the early 2009 when one bitcoin sold for less than a \$0.01 and continued to climb the highs of almost 20,000USD by the fourth quarter of 2017. There is

Table 9A

Portfolio of Chinese assets: no rebalancing.

	Naïve			Long-only			Semi constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	0.34%	0.99%	191.89%	0.13%	3.13%	2226.01%	0.16%	0.14%	− 13.3%	0.16%	0.14%	− 13.3%
std dev	0.0166	0.0192	15.51%	0.0038	0.0687	1705.03%	0.0079	0.0069	− 12.0%	0.0079	0.0069	− 12.0%
Risk-Return (Cvar)	0.0020	0.0020	0.00%	0.0010	0.0050	400.00%	0.0220	0.0300	36.4%	0.0220	0.0300	36.4%
Omega	1.0667	1.4790	38.64%	2.2797	4.2014	84.30%	1.6518	1.6480	− 0.2%	1.6518	1.6480	− 0.2%
Sortino	0.2711	0.8706	221.14%	0.7593	1.9091	151.43%	0.3792	0.4267	12.5%	0.3792	0.4267	12.5%

Table 9B
Portfolio of Chinese assets: with rebalancing.

	Long-only			Semi constrained			Constrained no BTC < 0		
	Base	BTC	Change	Base	BTC	Change	Base	BTC	Change
Performance	0.12%	1.96%	1598.20%	0.82%	0.77%	−6.67%	0.82%	0.85%	3.69%
std dev	0.0169	0.0587	248.15%	0.0128	0.0121	−5.51%	0.0128	0.0121	−5.51%
Risk-Return (Cvar)	0.0020	0.0020	0.00%	0.0029	0.0037	27.59%	0.0010	0.0010	0.00%
Omega	0.7494	2.3893	218.83%	2.2574	2.3038	2.06%	2.2574	1.5766	−30.16%
Sortino	0.0573	0.9282	1521.06%	1.4306	1.2292	−14.08%	1.4306	1.2239	−14.44%

now a rising body of academic literature attempting to explain various value drivers that have contributed to this, such as the genuine transaction demand, speculation, and the in-built deflationary characteristics of bitcoin. There's also a growing body of literature attempting to analyse its other financial characteristics, such as the particularly high volatility, correlations with other more traditional assets and any potential roles that it could play in the financial and investment markets. However, the research remains quite scarce and draws conclusions from the U.S. perspective. There is no previous research that covers the role of bitcoin in portfolios of European and Chinese assets.

Our results show some common themes between the inclusion of bitcoin in a portfolio of U.S., European and Chinese assets: it generates benefits in terms of improvement of the returns, but it tends also to increase volatility. However, portfolio performance in terms of Risk-Return ratio, Sortino ratio and Omega ratio tend to improve. The benefit of bitcoin inclusion is linked to the increase in the return that compensate for the increase in the risk of the portfolio. Interestingly, the positive effect of adding bitcoin to a portfolio is mainly linked to the period before the end of 2013, with marginal benefits thereafter. This finding applies to naïve and long portfolios. Where unleveraged shorting is allowed, effects on the risk-return ratio are more marginal throughout the sample period. The reason behind the relevant role played by bitcoin is linked to the increase in bitcoin in 2013. In fact, after February 2014 bitcoin presents quite a high volatility but with less pronounced increase in its value.

An additional outcome of this research relates to the (under)performance of the portfolios where shorting is present compared to the long-only portfolios and especially the naïve portfolio. This finding is in line with DeMiguel et al. (2007) who show that naïve portfolios often perform better than optimised portfolios in general. However, this does not necessarily explain the entire story since it might be also partially due to the infrequent semi-annual re-balancing we implemented.

However, this research opens to further areas of study. A first aspect that asks for additional investigation is a detailed analysis of the events at the end of 2013 beginning 2014 that affected the performance of the inclusion of Bitcoin. This is an interesting area since a clearer understanding of the role played by bitcoin's high volatility and its increase in value can allow for a better management of bitcoin as an asset. Secondly, this is the first research to investigate the role of bitcoin in China and Europe and one of the few that explore the role in U.S. In actual fact, we do not find any major differences in the role of bitcoin in the contexts under investigation. However, it can be interesting to explore bitcoin's role in portfolios containing fewer liquid assets (such as bonds and shares listed in minor markets). Thirdly, at the beginning of 2017 bitcoin represented the bulk of the total market capitalisation of all tradeable cryptocurrencies. By September 2017, this proportion has fallen⁹ to under 50% (60bn/130bn), mainly due to the faster expansion of alternative cryptocurrencies (e.g. Ether, Litecoin, etc.). This has given rise to a variety of theoretical crypto-indices such as the CRIX¹⁰ and the WorldCoinIndex.¹¹ Future research should seek to explore the

effects of such indices on optimal portfolios for a more comprehensive analysis. An additional area of analysis is to explore the long-term effect of the high increase in value of bitcoin in 2017 and the subsequent decrease and high volatility, similarly to the work by Platanakis and Urquhart (2018): our result suggests that until 2016 the benefit of including bitcoin was mainly due to the improved returns that compensate for the increase in C-VaR. Considering the stellar increase in bitcoin value, this effect can hold in 2017 as well. However, the 2017 “bitcoin bubble” exploded at least partially at the beginning of 2018 with a downward trend in the returns while maintaining a high volatility of its value. Thus, further analysis is needed in order to appreciate the real “2017” bitcoin long term effect on a portfolio of assets. Further, we do not explore in detail the correlation between bitcoin and other assets/currencies (along the lines of the works by Baumöhl (2018) and Burnie (2018) and we do not touch the very relevant topic of spillovers (e.g. the seminal work by Guesmi et al. (2018)). In fact, these are relevant areas to be explored when it turns to manage portfolios of assets that include bitcoin or other cryptocurrencies. Finally, a further area of research is linked to the possibility of using bitcoin to hedge forex and commodities risk (Bouri, Molnár, et al., 2017).

Notwithstanding the limitations, our study indicates that bitcoin is an emerging relevant asset class and it might play a more relevant role in portfolio diversification strategies in U.S., Europe and China than has heretofore been acknowledged.

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⁹ Source: <https://coinmarketcap.com/> (accessed 23 September 2017)

¹⁰ <http://crix.hu-berlin.de/>

¹¹ <https://www.worldcoinindex.com/>

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