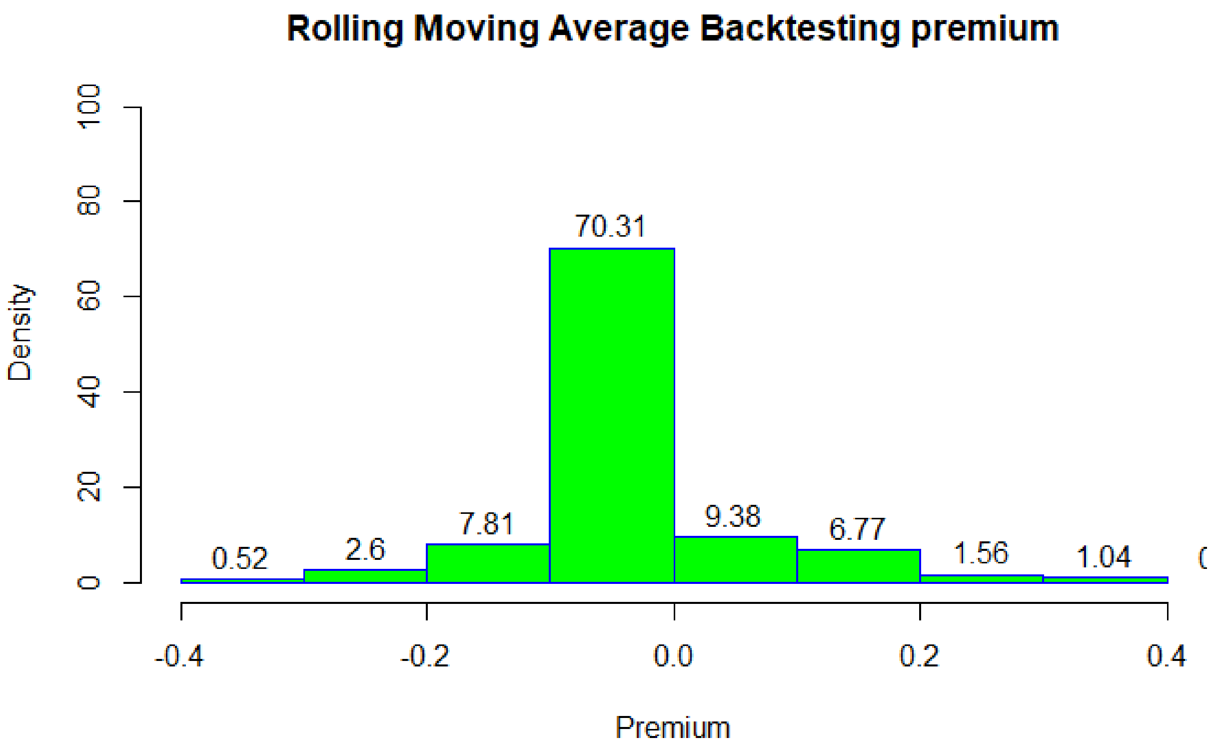
1 Back-testing trading strategies

1.1 Typo in Time Interval:

First Things first, the date interval allocated to me was not compatible with the provided Price data for AMZN (2018-01-02 – 2020-05-21). Considering that it was a typo and you meant the first two years, I changed the time interval to the first two years of the data "2018-01-01" -- "2019-12-31". However, I can easily get the Price data for "2015-01-01" -- "2016-12-31", but I don’t have access to sentiment components database that you provided for the mentioned time interval. Thus, I wanted it to be compatible with the data provided so you can easily replicate my results without any struggle.

1.2 Back-testing with sentiment indicators (Question 1)

Using short window $s \in \{5;\ 10;\ 15;\ 20\}$ and medium/long window $m \in \{25,50,100\}$ with long- short and long-only strategies with window size 254 and 127, I have implemented the rolling moving average testing for all the sentiment indicators Bull, Bear, BBr and NPlog. Looking at the results, I made a histogram to see how the premium (trading strategy Annualized return – Buy and hold Annualized return) is distributed in sample. I chose Annualized return instead of Cumulative Return to account for different combinations of window size. Interestingly, most of the combinations (70.31% of them) lose 0 to 10% and around 3% (left tail) lose a 20-40% compared to the Annualized return. On the other flip of the coin, around 2.6% (right tail) gain 20-40% premium.



The following table shows the combinations with the highest (20-40%) premiums:

Table 1: Rolling MA Back-Test Best Performances of different combinations

BH/ME	Cumulative Return	Annual Return	Annualized Sharpe Ratio	Win	Annualized Volatility	Maximum Drawdown	Max Length Drawdown	s	m	long short	win size	ev	premium
Me	46.70%	46.26%	215.77%	60.72%	21.51%	-14.40%	67.66	10	#	0	254	bull	**37.66%**
BH	8.67%	8.60%	25.93%	54.34%	34.05%	-31.26%	163.27	10	#	0	254	bull	
Me	19.17%	43.03%	215.93%	60.06%	19.69%	-11.25%	49.80	10	#	0	127	bull	**29.29%**
BH	5.65%	13.74%	57.58%	54.65%	30.76%	-20.94%	69.65	10	#	0	127	bull	
Me	41.01%	40.63%	190.23%	60.24%	21.54%	-12.45%	67.55	5	#	0	254	bear	**32.03%**
BH	8.67%	8.60%	25.93%	54.34%	34.05%	-31.26%	163.27	5	#	0	254	bear	
Me	33.33%	33.03%	163.78%	58.52%	20.42%	-13.64%	92.11	5	#	0	254	bear	**24.78%**
BH	8.32%	8.25%	25.29%	54.11%	33.73%	-30.90%	168.48	5	#	0	254	bear	
Me	27.02%	26.78%	133.39%	57.55%	20.78%	-15.73%	122.67	5	#	0	254	bear	**23.07%**
BH	3.75%	3.72%	12.98%	53.36%	33.23%	-29.82%	171.52	5	#	0	254	bear	
Me	31.45%	31.16%	142.88%	58.08%	21.82%	-13.64%	72.34	10	#	0	254	bear	**22.91%**
BH	8.32%	8.25%	25.29%	54.11%	33.73%	-30.90%	168.48	10	#	0	254	bear	
Me	17.47%	39.02%	204.78%	60.47%	20.01%	-11.03%	50.25	5	#	0	127	bear	**25.28%**
BH	5.65%	13.74%	57.58%	54.65%	30.76%	-20.94%	69.65	5	#	0	127	bear	
Me	14.63%	32.08%	175.97%	58.64%	19.77%	-11.69%	56.15	5	#	0	127	bear	**22.63%**
BH	3.75%	9.44%	41.03%	54.07%	31.16%	-21.46%	72.02	5	#	0	127	bear	
Me	12.26%	27.45%	160.99%	57.22%	20.40%	-12.78%	66.63	5	#	0	127	bear	**23.30%**
BH	1.32%	4.15%	19.34%	53.21%	31.88%	-22.26%	78.69	5	#	0	127	bear	
Me	13.80%	30.00%	148.12%	58.11%	20.81%	-12.01%	61.48	10	#	0	127	bear	**20.56%**
BH	3.75%	9.44%	41.03%	54.07%	31.16%	-21.46%	72.02	10	#	0	127	bear	

Fortunately, I can see that only Bull and Bear indicators happens to be in the right tail. Thus, if one wants to gamble on these indicators to gain a high reward considering a very high risk of landing in 0 to -10% loss, he/she can choose Bull or Bear.

On the dark side, one who chooses to use these signals can lose a lot of money compared to the buy and hold strategy. The following table is the worst (Negative 20-40%) performance:

Table 2: Rolling MA Back-Test worst Performances of different combinations

BH/ME	Cumulative Return	Annual Return	Annualized Sharpe Ratio	Win	Annualized Volatility	Maximum Drawdown	Max Length Drawdown	s	m	long short	win size	ev	premium
Me	-24.49%	-24.33%	-95.88%	51.49%	24.97%	-33.57%	203.82	5	25	0	254	bbr	**-32.92%**
BH	8.67%	8.60%	25.93%	54.34%	34.05%	-31.26%	163.27	5	25	0	254	bbr	
Me	-21.13%	-20.99%	-85.72%	51.54%	24.26%	-30.27%	214.03	5	50	0	254	bbr	**-29.24%**
BH	8.32%	8.25%	25.29%	54.11%	33.73%	-30.90%	168.48	5	50	0	254	bbr	
Me	-14.52%	-14.42%	-55.95%	48.86%	24.71%	-27.01%	226.55	10	50	0	254	bbr	**-22.67%**
BH	8.32%	8.25%	25.29%	54.11%	33.73%	-30.90%	168.48	10	50	0	254	bbr	
Me	-19.23%	-19.09%	-104.33%	47.02%	18.26%	-27.27%	232.46	10	100	0	254	bbr	**-22.81%**
BH	3.75%	3.72%	12.98%	53.36%	33.23%	-29.82%	171.52	10	100	0	254	bbr	
Me	-10.23%	-17.70%	-57.79%	51.73%	22.41%	-19.81%	88.13	5	25	0	127	bbr	**-31.44%**
BH	5.65%	13.74%	57.58%	54.65%	30.76%	-20.94%	69.65	5	25	0	127	bbr	
Me	-9.98%	-17.91%	-67.24%	51.39%	22.37%	-19.06%	92.89	5	50	0	127	bbr	**-27.35%**
BH	3.75%	9.44%	41.03%	54.07%	31.16%	-21.46%	72.02	5	50	0	127	bbr	
Me	-7.00%	-12.25%	-36.78%	49.03%	22.46%	-17.74%	97.77	10	50	0	127	bbr	**-21.69%**
BH	3.75%	9.44%	41.03%	54.07%	31.16%	-21.46%	72.02	10	50	0	127	bbr	
Me	-9.92%	-18.32%	-96.31%	47.17%	17.92%	-16.60%	102.44	10	100	0	127	bbr	**-22.46%**
BH	1.32%	4.15%	19.34%	53.21%	31.88%	-22.26%	78.69	10	100	0	127	bbr	
Me	-21.21%	-21.06%	-81.93%	51.10%	25.21%	-30.49%	219.47	5	50	0	254	pnlog	**-29.31%**
BH	8.32%	8.25%	25.29%	54.11%	33.73%	-30.90%	168.48	5	50	0	254	pnlog	
Me	-21.31%	-21.16%	-86.86%	49.90%	24.03%	-27.48%	238.16	5	100	0	254	pnlog	**-24.87%**
BH	3.75%	3.72%	12.98%	53.36%	33.23%	-29.82%	171.52	5	100	0	254	pnlog	
Me	-9.27%	-16.06%	-45.91%	51.22%	22.87%	-18.28%	92.08	5	50	0	127	pnlog	**-25.50%**
BH	3.75%	9.44%	41.03%	54.07%	31.16%	-21.46%	72.02	5	50	0	127	pnlog	
Me	-10.64%	-19.07%	-68.59%	49.91%	22.93%	-18.31%	101.19	5	100	0	127	pnlog	**-23.22%**
BH	1.32%	4.15%	19.34%	53.21%	31.88%	-22.26%	78.69	5	100	0	127	pnlog	

Fortunately, this table gives us the information about the extremely negative performance that can occur when I are using BBr and PNlog indicators.

The question here is why aren't I already implementing these trading strategies with the right tail combinations and get rich? These are the possible reasons:

- **The study can be prone to multiple testing. I used a lot of combinations which can land us on the good side but does not necessarily mean that these combinations will always work. This is the danger of multiple testing.**
- **Even if these combinations work or have a high probability of occurring, I should know that combinations slightly different to the best ones usually end up in 0 to -20% premium interval. This again, shows how unrobust these results are.**
- **There is no evidence that the distribution of premiums is stationary meaning that in the next two years things could change. I tried to be robust considering the possible non-stationarity by using Rolling MA tests, but it would be nice to test the best combinations for the next two years and compare the results.**
- **Another doubt is that how practical these trading strategies are? Is the indicators are available in the same trading time that they are reported? If the timing differs, I could have over estimated the performance depending on how delayed the indicators are.**

1.3 Self-Made Trading Strategy

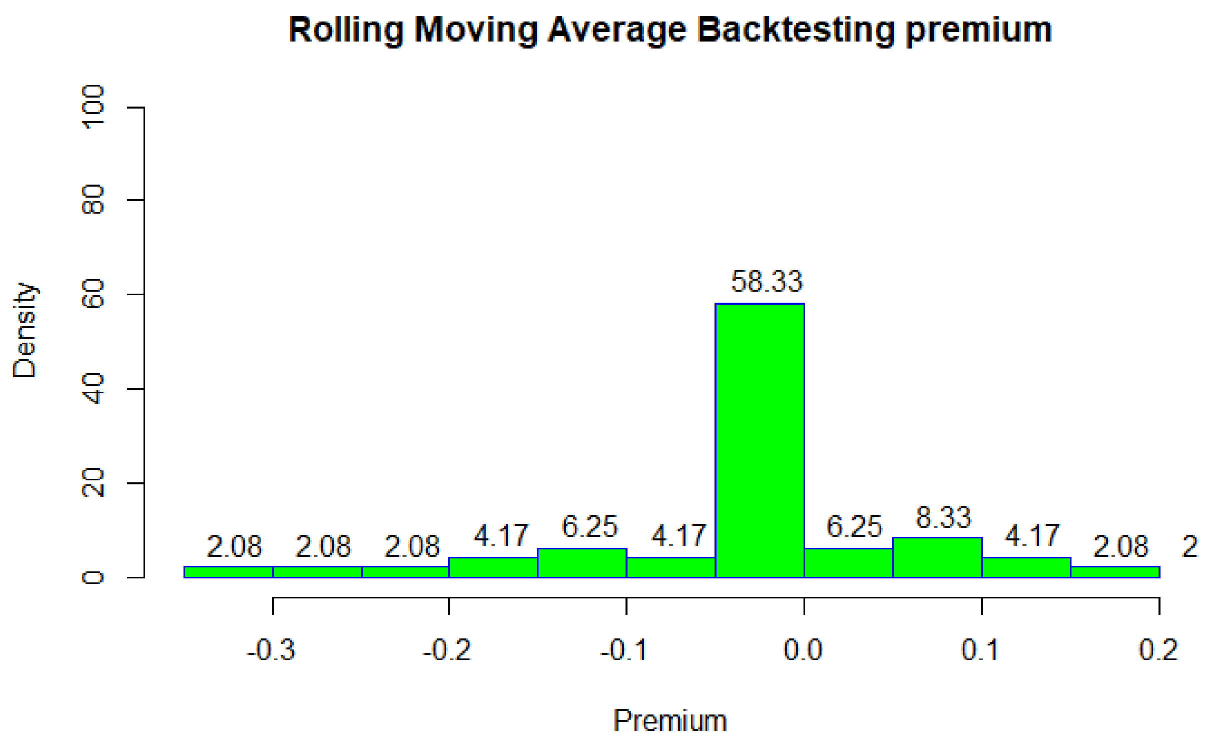
Since the performance of Technical Analysis is studied by Ready (2002), I don't see the point to replicate it. Thus, I decided to make an indicator using compositions and calling it 'Sentimentor' and test it.

$$Bull\ side = \text{positivePartscr} * (1 + \text{certaintyPartscr}) * (1 + \text{finupPartscore})$$

$$Bear\ side = \text{negativePartscr} * (1 + \text{uncertaintyPartscr}) * (1 + \text{findownPartscore})$$

$$\text{Sentimator} = \text{Bull side} - \text{Bear side}$$

Using the Sentimator, I calculated the distribution of the performance:

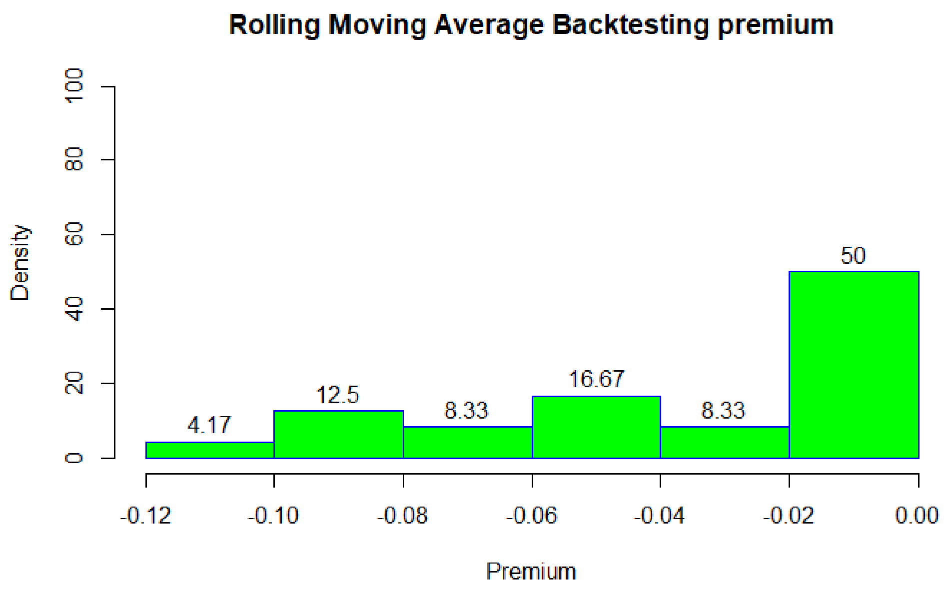


Then, I decided to choose a new strategy to go long only when the indicator is positive and go short (or do nothing depending on long.short parameter) if the indicator s negative. It was total failure with all the combinations resulting negative premium. Then I chose to make the indicator cumulative and test with rolling MA:

$$cum.sentimator_t = cum.sentimator_{t-1} + sentimator_t$$

$$where \quad cum.sentimator_0 = 0$$

Although, with this indicator we have bounded the loss, but the performance is always negative, thus, the cumsum indicator is useless.



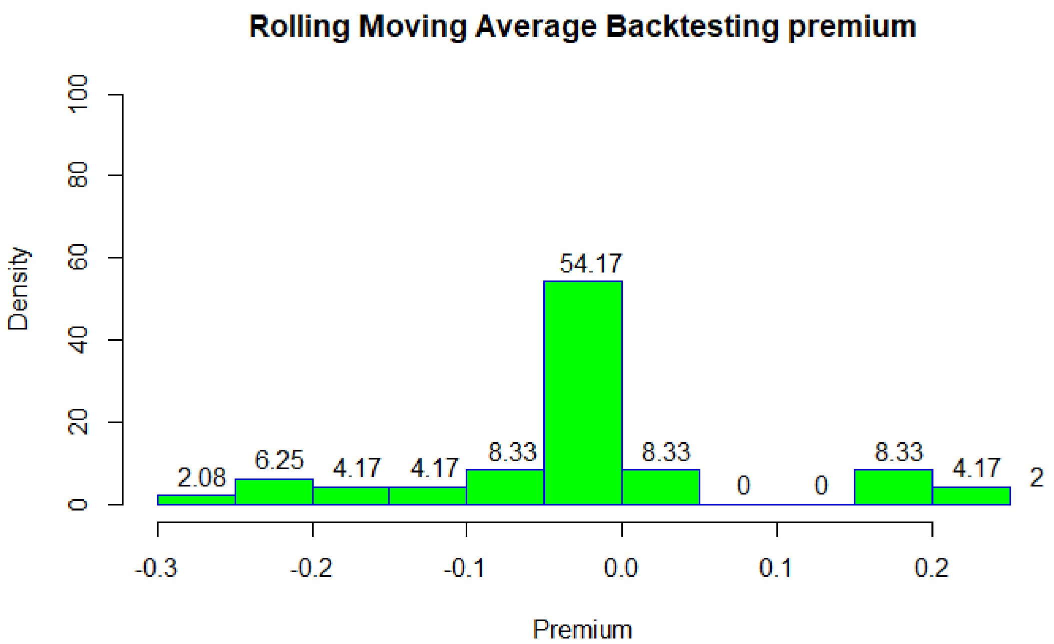
Therefore, I changed the indicator to:

$$sentiment = bull.side + bear.side$$

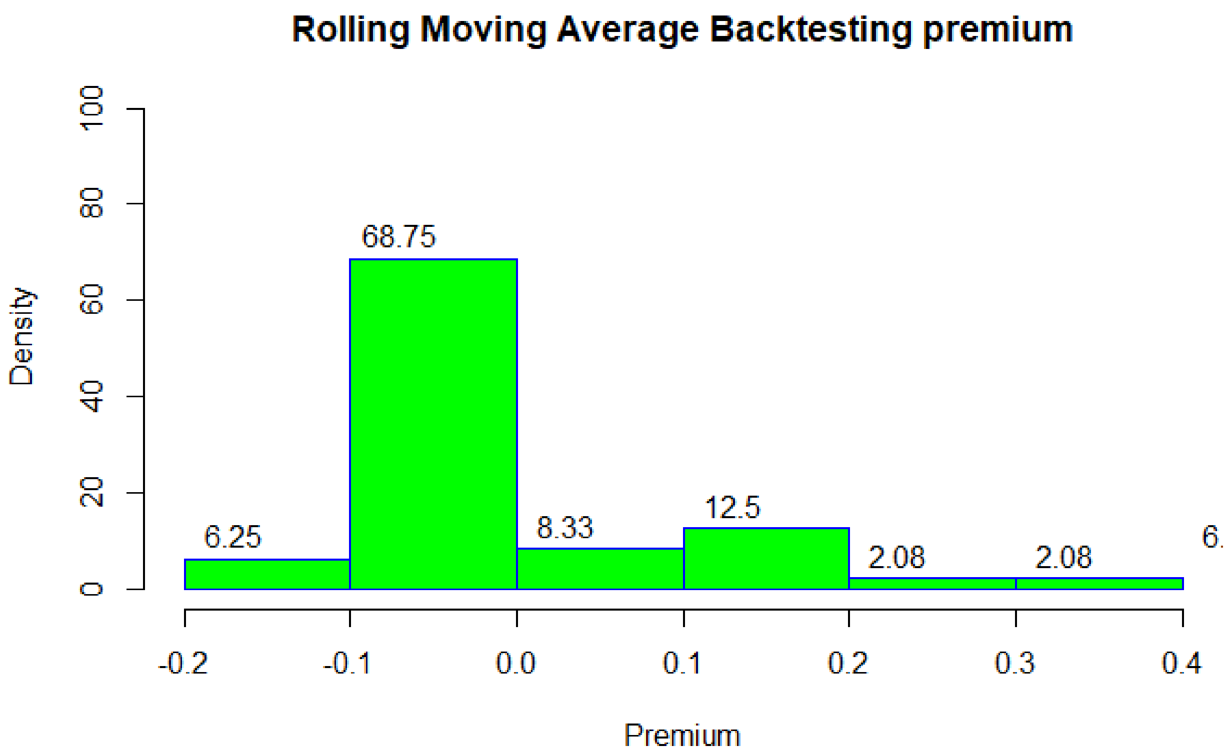
Which will have the same results as:

$$sentiment = (1 + bull.side) * (1 + bear.side)$$

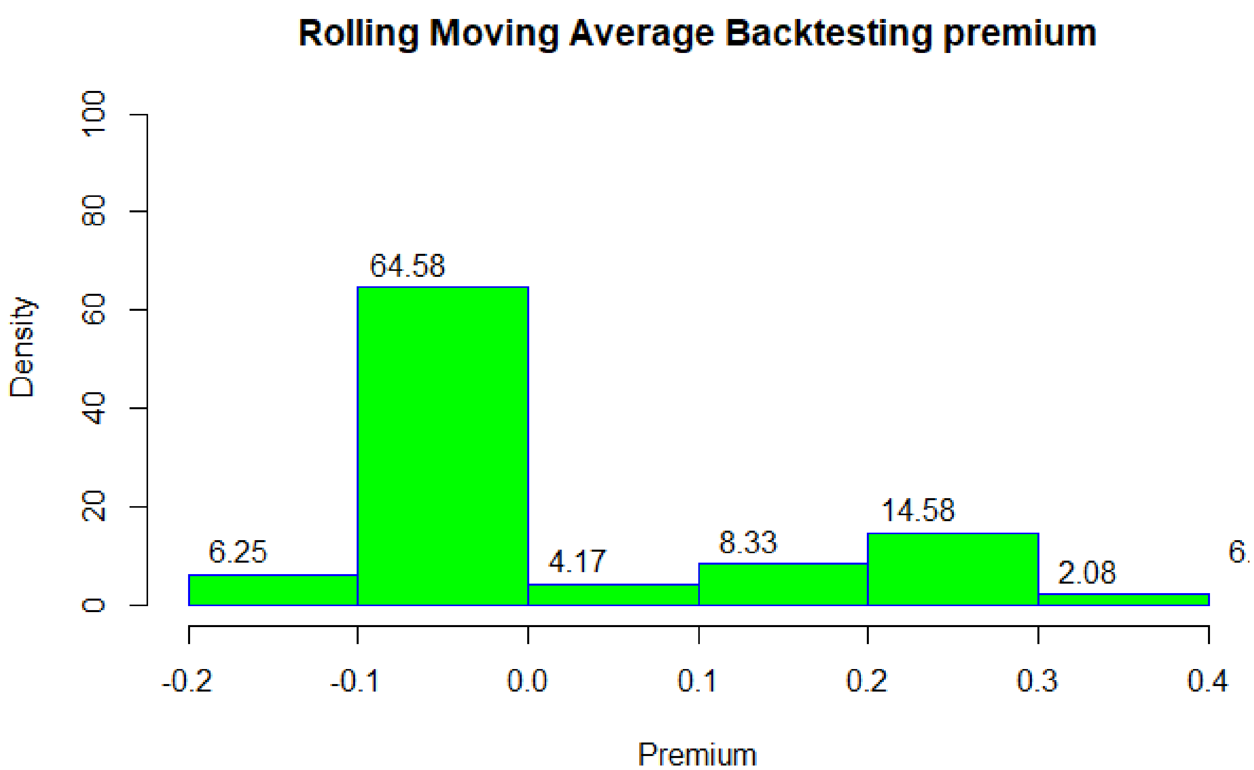
The result showed



If we just put Bull indicator:



We see that it's still better than all of the indicators. Bear indicator has a similar distribution of premium as well:



It seems that Bear indicator has an even better right tail.

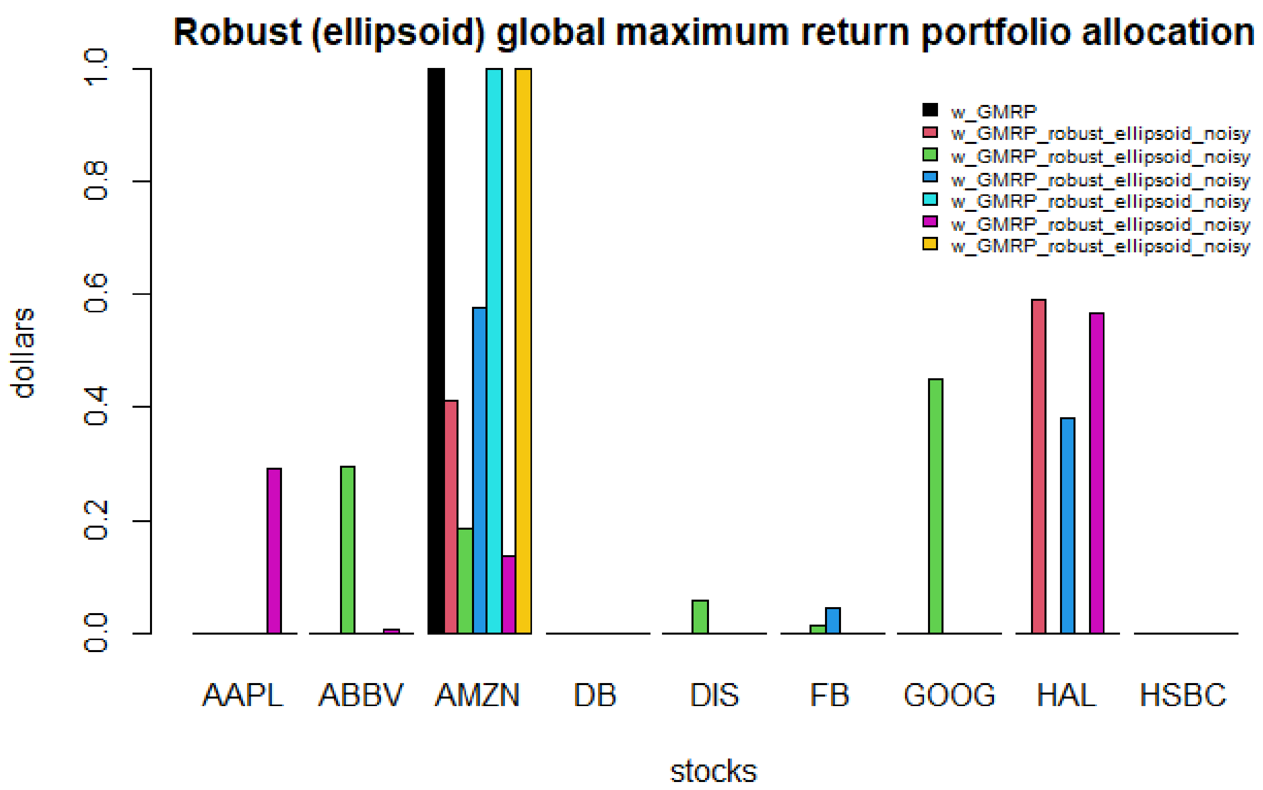
In a nutshell, using these indicators with different combinations of parameters makes us vulnerable to multiple testing bias and we should test the best combinations in the following years to show that they are not prone to multiple testing.

2 Robust (ellipsoid) Global Maximum Return Portfolio

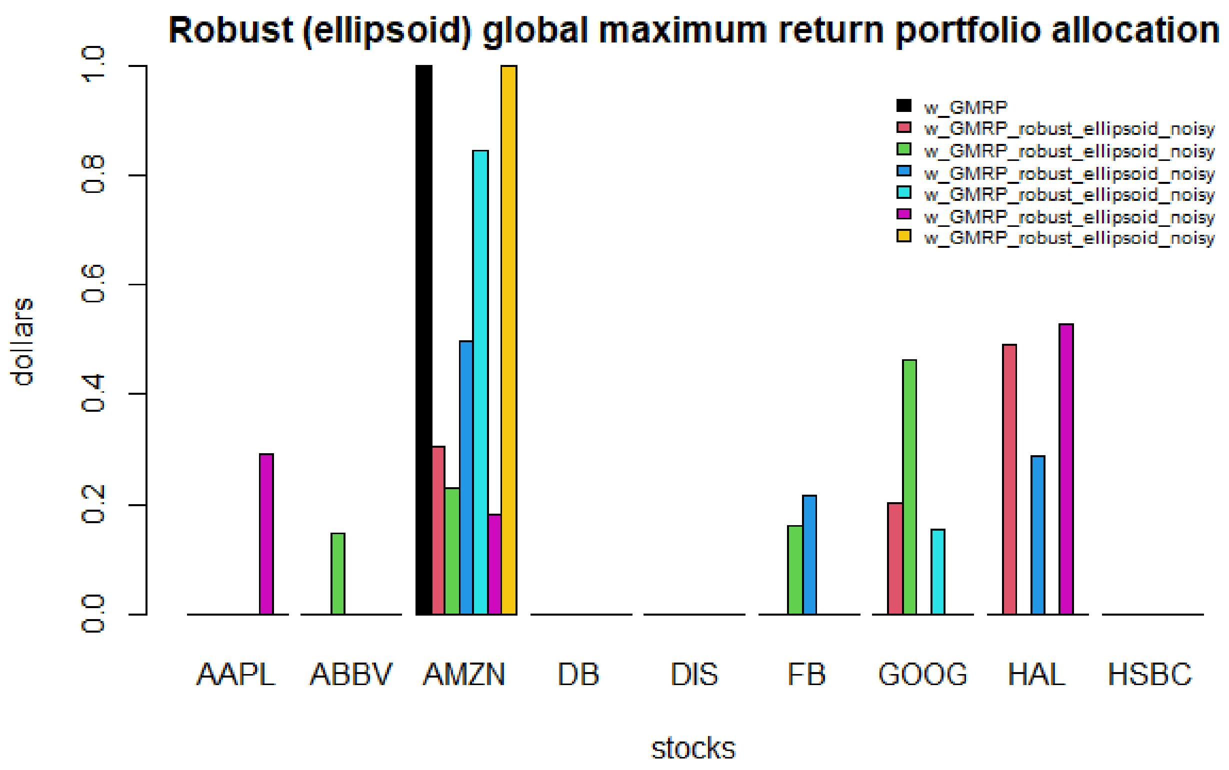
2.1 Choosing 9 stocks

2.2 Using S, FF3, PNlog with different kappas and accounting for sensitivity

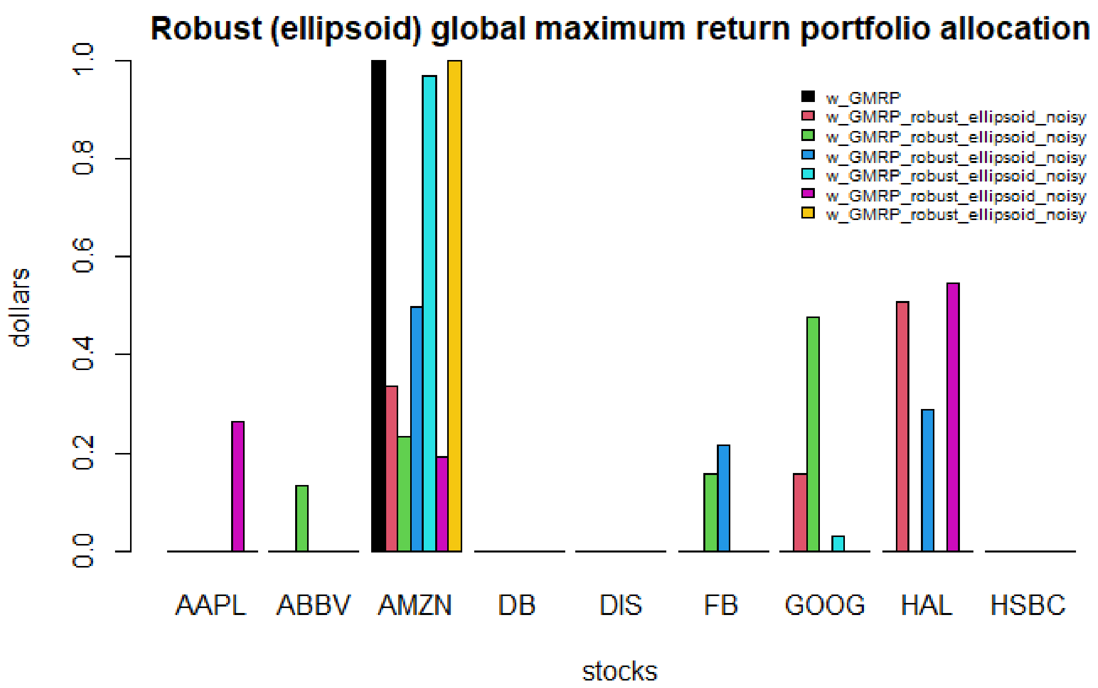
The reason we are implementing this is to see if using perturbation matrix Sigma (S) the Fama-French 3-factors returns 3FF and the Sentiment indicator PNlog can improve diversification and lower the sensitivity or not. First let's see the perturbation matrix Sigma from the stock prices:

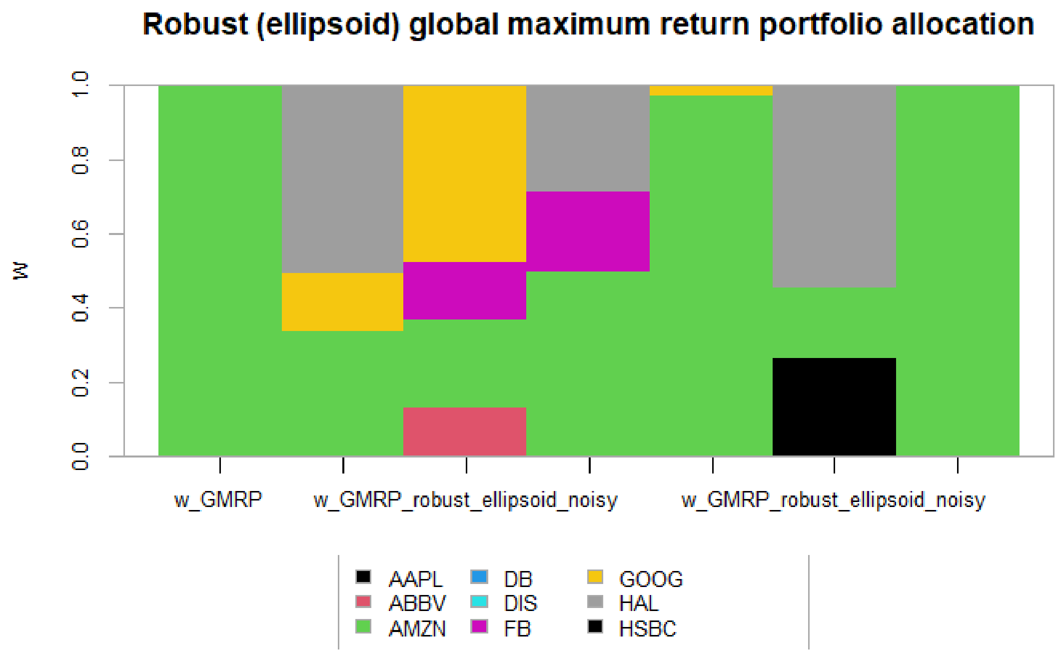


As we see compared to Markowitz's allocation (we saw it in the lecture 8), GMRP is more diversified and less sensitive, but the sensitivity is still too much. Now let's do the same with perturbation matrix Sigma (S) the Fama-French 3-factors returns 3FF and the Sentiment indicator PNlog (FF3 and sentiment):

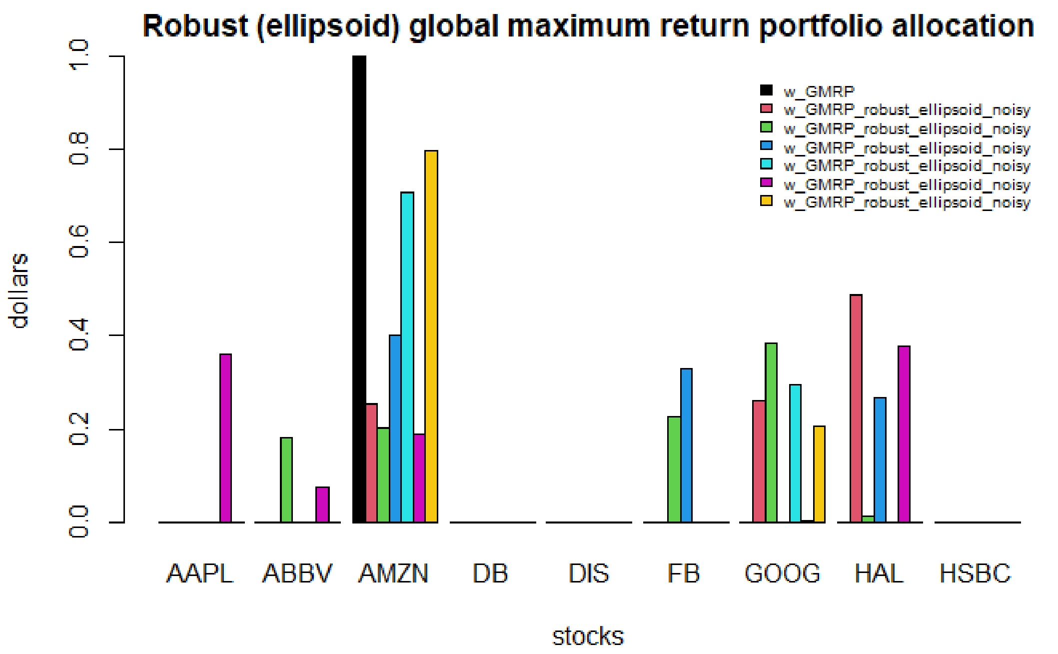


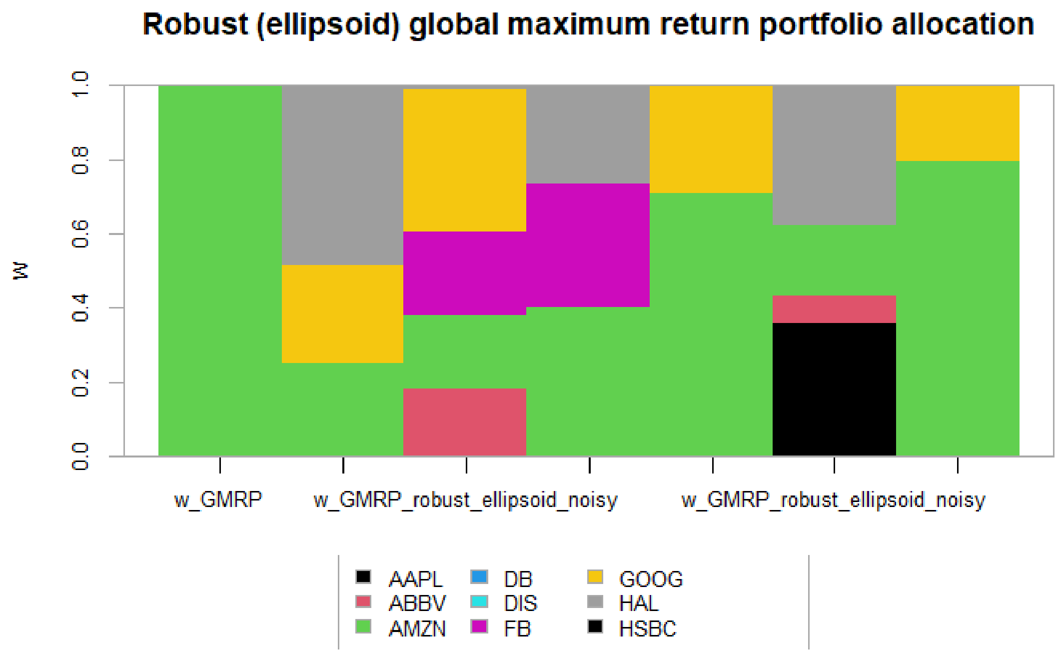
Now to compare let's use FF3:





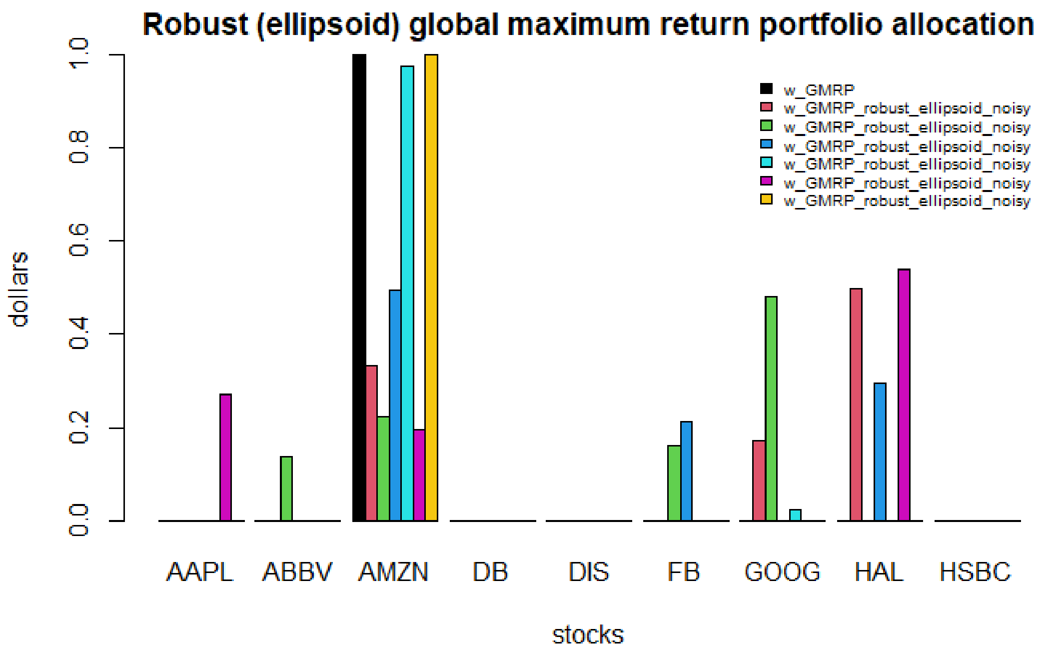
We see that there is no advantage compared to FF3 + sentiment. Now, let's see the allocation using sentiment (SS):

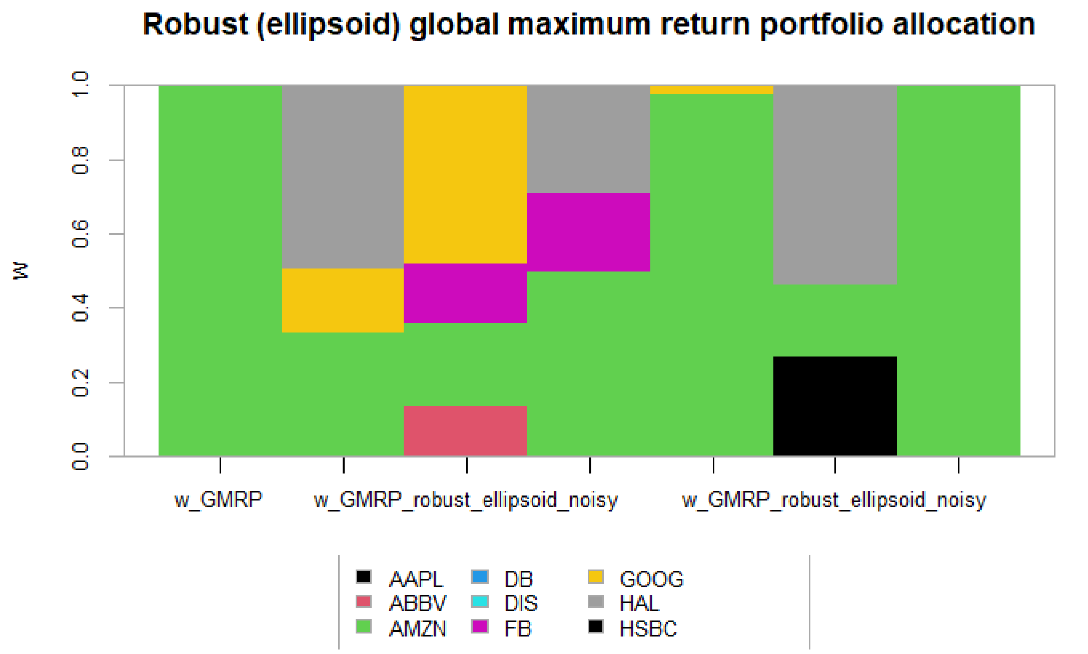




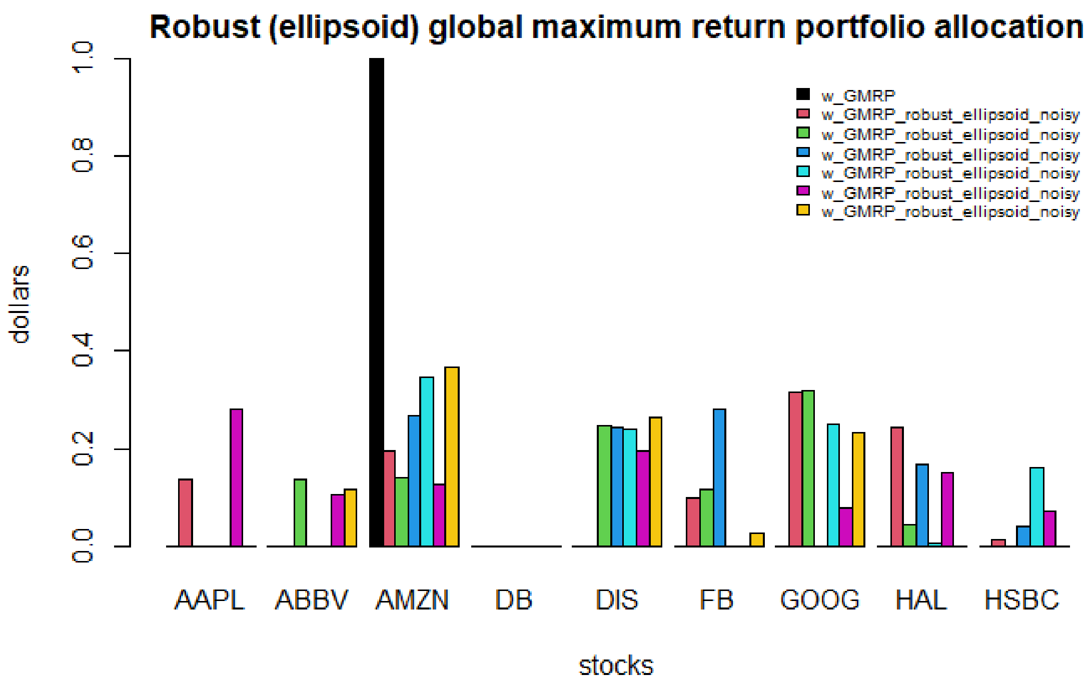
It's less diversified but less sensitive as well.

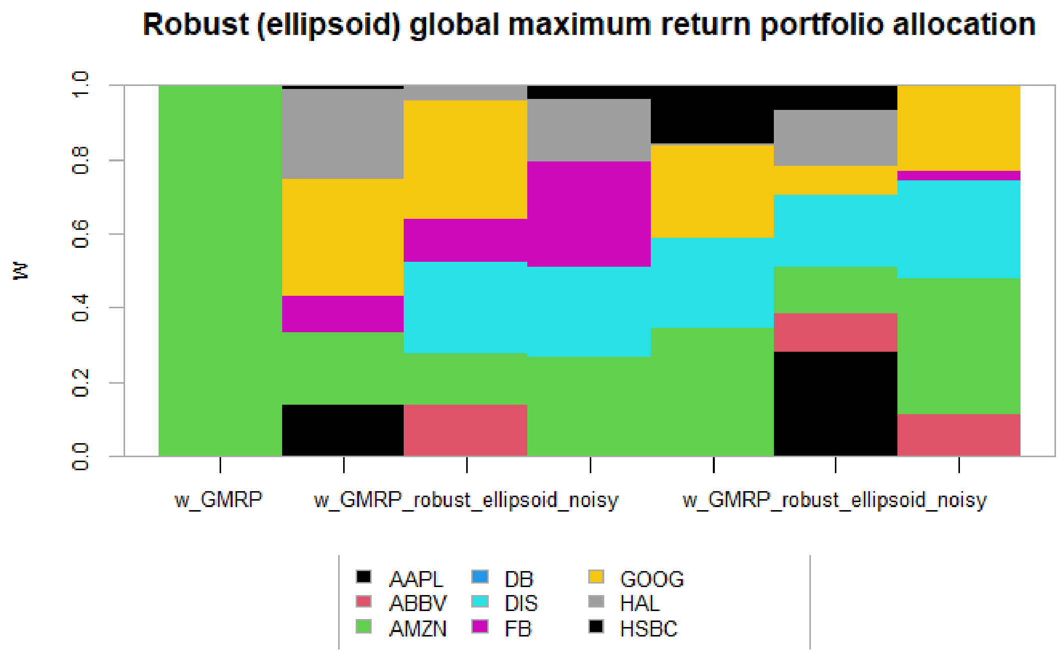
Now let's see the allocation using FF3 and Market index:





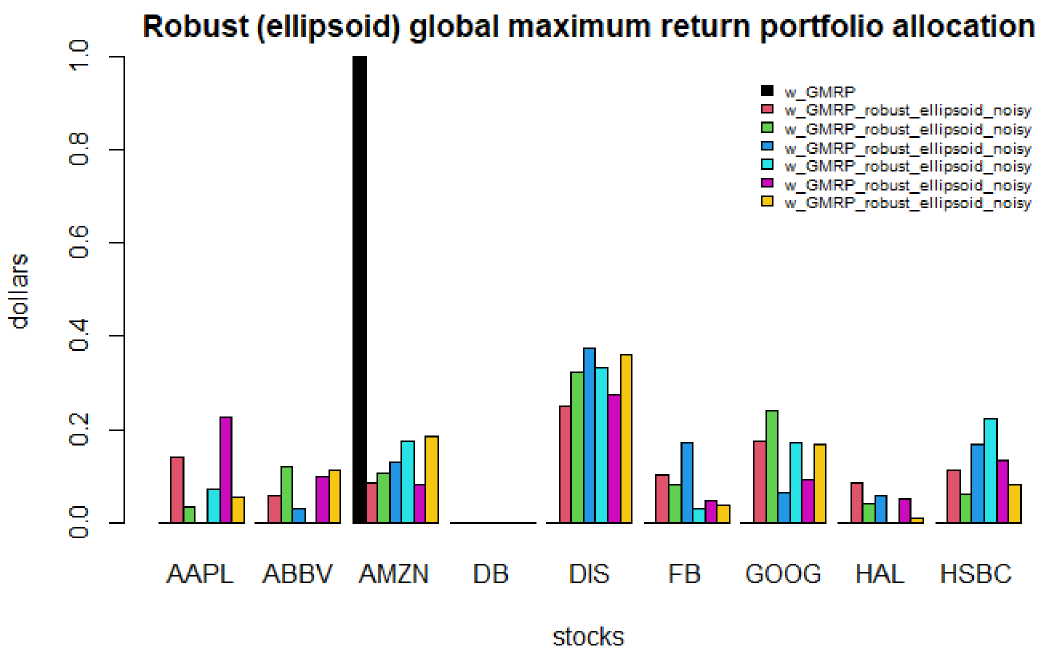
Not much different compared to using FF3 and sentiment. But let's see what happens when we increase kappas. The previous allocations used kappas 0.1, let's increase the kappas to 0.3 and see what happens:

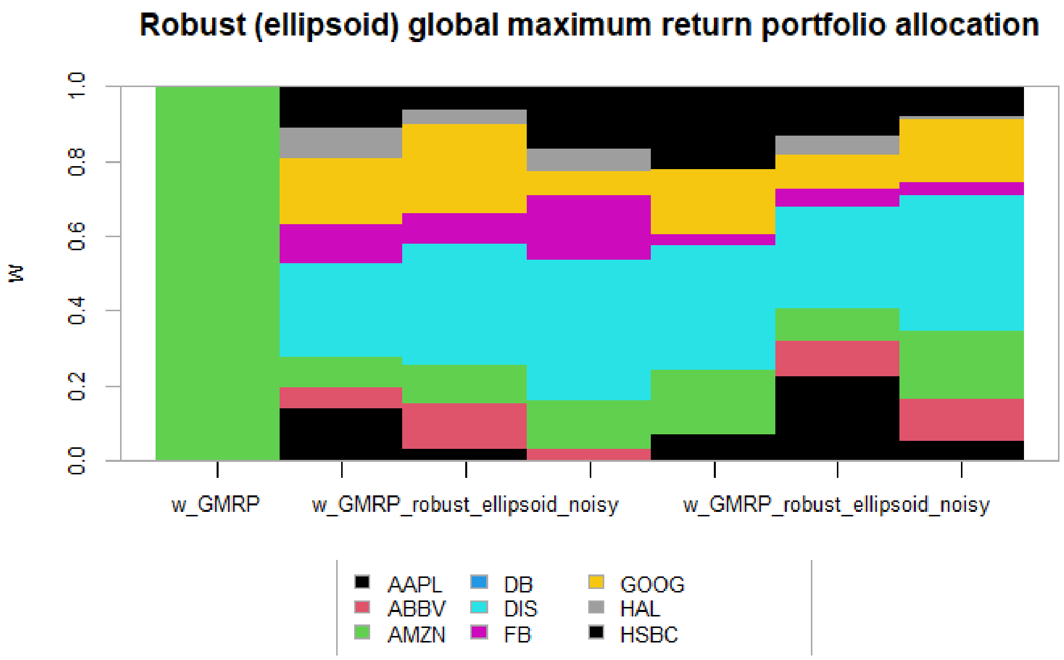




Brilliant, using a higher kappas was fruitful. The diversity is higher and sensitivity is lower.

Let's increase Kappas even more to 0.7:





It helped a little more with diversity but the sensitivity is still there.

3 Proof of variance of portfolio

Prove:

$$Var(\sum_{i=1}^{n} a_i X_i) = \sum_{\{i=1\}}^{n} \sum_{\{j=1\}}^{n} a_i a_j \sigma_i \sigma_j \rho_{ij}$$

$$Var(a_1 X_1 + a_2 X_2) = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + 2 a_1 a_2 \sigma_1 \sigma_2 \rho_{12}$$

$$= a_1 a_1 \sigma_1 \sigma_1 \rho_{11} + a_2 a_2 \sigma_2 \sigma_2 \rho_{22} + a_1 a_2 \sigma_1 \sigma_2 \rho_{12} + a_1 a_2 \sigma_1 \sigma_2 \rho_{12}$$

$$knowing\ that\ \rho_{ii} = 1, thus\ Var(a_1 X_1 + a_2 X_2) = \sum_{\{i=1\}}^{2} \sum_{\{j=1\}}^{2} a_i a_j \sigma_i \sigma_j \rho_{ij}$$

$$Generalizing: Var(\sum_{i=1}^{n} a_i X_i) = \sum_{\{i=1\}}^{n} \sum_{\{j=1\}}^{n} a_i a_j \sigma_i \sigma_j \rho_{ij}$$

4 Mean-Variance approximation to Expected Utility

$$prove: E[u(C)] \approx E(C) - \frac{1}{2}\lambda Var(C), for\ some\ \lambda > 0$$

$$Taylor(2)\ of\ u(C)\ around\ E(C):$$

$$u(C) \approx u\big(E(C)\big) + u'\big(E(C)\big).(C - E(C)) + \frac{u''\big(E(C)\big)}{2}.(C - E(C))^2$$

$$= constant + u'\big(E(C)\big)[C - \frac{1}{2}\lambda.E(C)\big(C - E(C)\big)^2]$$

$$where\ \lambda = \frac{u''(C)}{u'(C)}.then\ we\ take\ expectation:$$

$$E[u(C)] \approx E[C - \frac{1}{2}(C - E(C)^2] \approx E(C) - \frac{1}{2}\lambda Var(C)$$