1. Try to answer these two questions using a Microeconomics textbook (like Varian's Intermediate Microeconomics) and searching the Internet: what is the relationship between price-elasticity and profit maximization? Why may this be important in the sports industries?

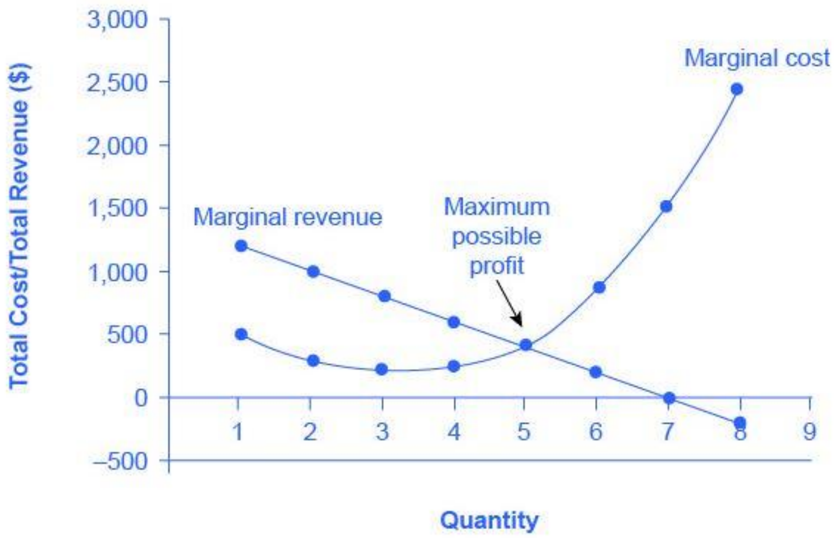
Price elasticity (elasticity of demand) determines how much the demand will change if the price changes for 1%. Knowing that firm's profit is:

$$\pi = total\ revenue - total\ cost$$

To maximize profit, we take a derivative from the profit equation and set it to 0, which leads to having:

$$MR = MC$$

As shown by this graph:



Knowing this, price elasticity can change marginal cost and marginal demand, in a way, tune them until they end up being equal and the firm's profit maximizes. For example, in a competitive market the marginal revenue have the following relation with the price elasticity:

$$MR = P\left(1 - \frac{1}{E_p}\right) \quad , where\ E_p\ is\ absolute\ price\ elasticity$$

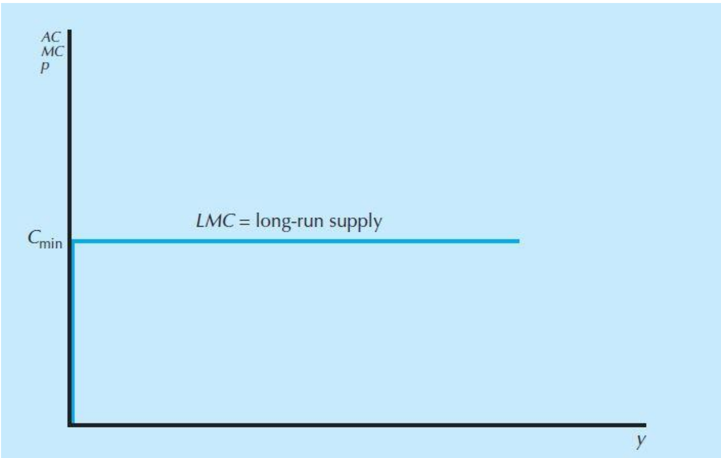
In practice, firms use marginal price for pricing since marginal price is better estimated. **Sports Industry** has two main sectors: 1- entertainment and 2- sports instruments. The entertainment sector has a partially inelastic demand; however, sports tickets are usually sold under the optimal ticket price that maximizes ticket selling profits. This matter was studied and showed that there were non-ticket profits at the stadiums (selling beverages and parking-lot fee and etc.) that benefitted from the low prices for tickets (Ford, 2004). Therefore, the real world price and quantity adjustments could be dependent on other factors rather than profit maximization for one good. Here, the elasticity suggests an optimum price and quantity, but the bundle of goods offered by the sport-oriented firm suggests another, meanwhile, both trying to achieve maximum profit.

2. Using a Microeconomics textbook (like Varian's Intermediate Microeconomics) explain using both the graph of a market in partial equilibrium and the graph of a firm's cost curves, under which conditions the long run supply curve is horizontal and under which conditions it is not?

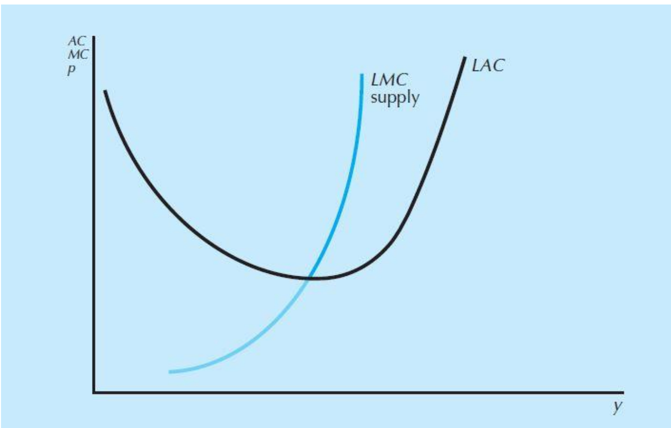
In the short run (assuming competitive market), when the price of a good changes, firm can only respond with changing the quantity or better called the variable costs. However, in the long run, firms can change the operation's size like factory size which is associated with the fixed costs. Thus, the long run supply has a lower slope since the marginal cost is now determined with quantity and fixed-cost related measure (k). The mathematical representation is defined as:

$$p = MC(y, k)$$

Where for the short run k is fixed at a level (k^*). When we have an output function with **constant returns to scale**, firm can produce any level of output with a fixed level of marginal cost and average cost as shown by this graph:



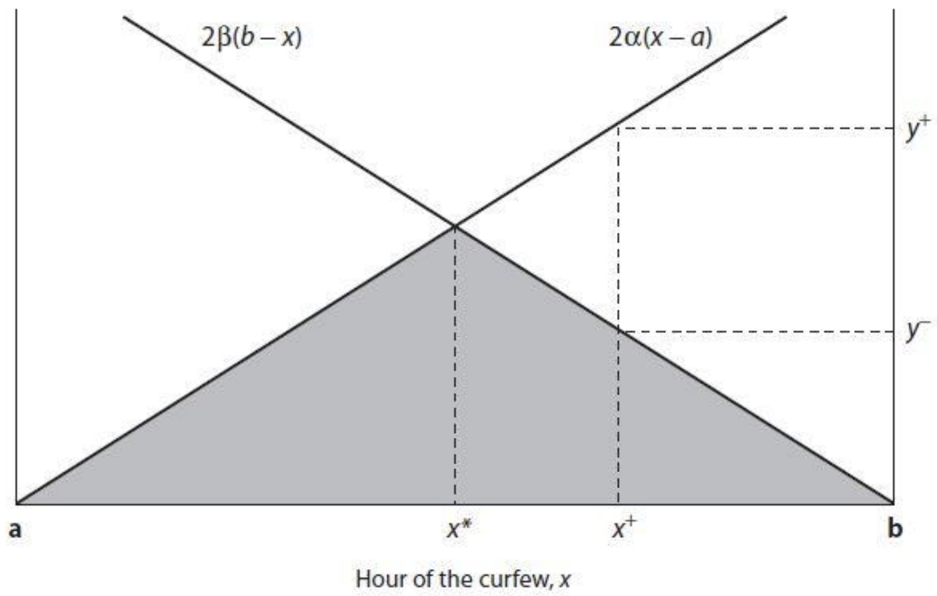
If there is an increasing (decreasing) return to scale, firm will face a decreasing (increasing) marginal and average cost which creates a upward (downward) long run supply curve. The following graphs is for a typical increasing return to scale situation:



The long-run supply curve. The long-run supply curve will be the upward-sloping part of the long-run marginal cost curve that lies above the average cost curve.

3. Read what the book by Samuel Bowles on Microeconomics says about the Coase theorem. Explain using one or two graphs what is the relationship between the result of the theorem and income effects.

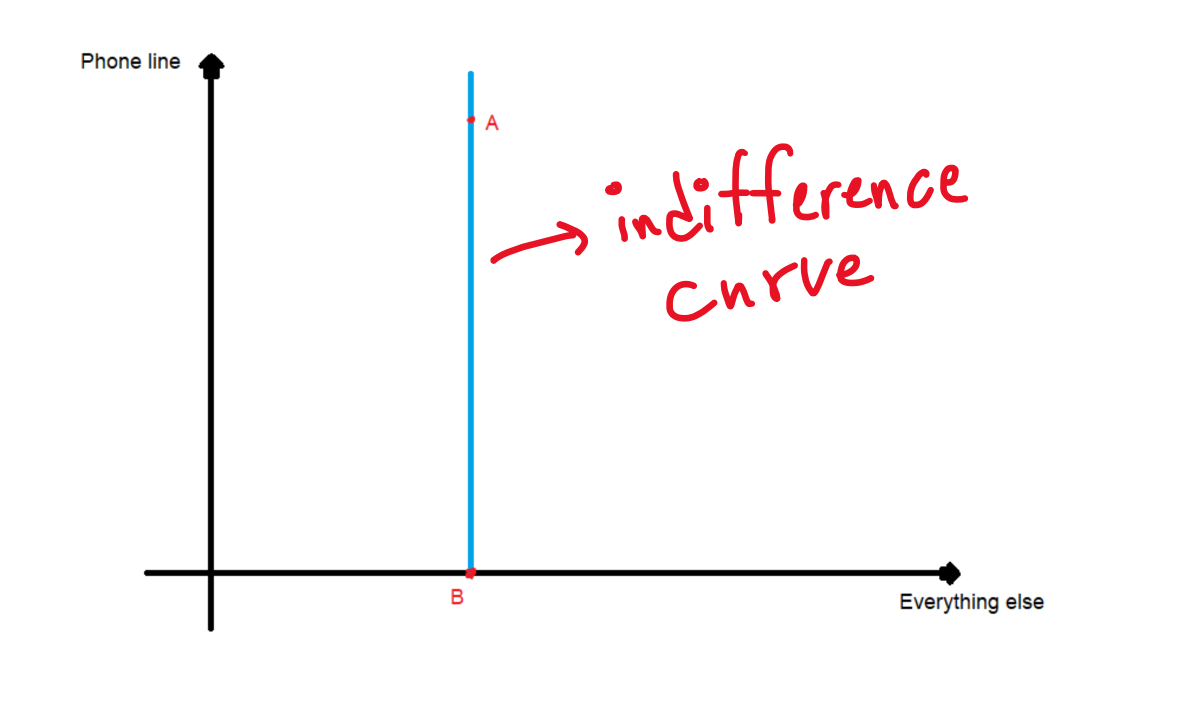
Coase asserts that regardless of where each party initially stands, if the market transactions are costless, they can become into an agreement of exchanging rights that can lead to a optimum social welfare which is Pareto efficient. For example, in the neighbor dispute about curfew hour mentioned in the book, knowing the preferences, the social disutility is the grey area at the x^* point. For a later curfew, A will have a higher marginal cost than B's marginal benefit, so the optimum point remains at x^*. However, Coase asserts that if the marginal cost of later curfew for A is bigger than marginal benefit of a latter curfew, a costless negotiation of rights will lead to a maximized sum of utilities regardless of what was the initial rights assigned. In this example, this could happen as a compensation of y from A to B if there was no curfew (No benefitting rights for A) to be in the x^+ point rather than b. Continuing on the same logic that when the marginal cost for A is bigger than marginal benefit for B, this will lead to the optimum point at x^*.



To illustrate, we can show the Optimal Coase bargaining with the curfew hour on x axis and compensation on the y axis. A and b points are the best positions for A and B. regardless of the initial allocation of rights, Coasean bargaining will lead to x^*(the social optimum). For example, if we are at b point, A is indifferent to lower the curfew hour and compensate B with y' and go to point t. However, point r is better for A and better for B compared to point b. with the same logic, the players will move to the point r and then in multiple steps we reach x^*. Remember that for this to happen we need zero transaction cost and a market that the players can bargain.

4. How would the indifference curves of a person look like if she spends all day talking on the phone, and then she suddenly cancels her phone contract because she could not conceive having the line underutilized.

The only way that she can suddenly change her mind about consuming something and remain on the same indifference curve (not losing utility due to an action), is that the good/services used is neutral. Notice that we are considering that nothing have changed and she is a homo-economicus. In the graph below she is at A Point initially, then she mores to the B point.



5. How does the equilibrium of the penalty kicks game (analyzed by Palacios-Huerta in his article) change if the goal-keeper improves her skills at saving penalty kicks kicked to the natural side of the kicker? Give a numerical example before and after improving her skills.

Imagine that the scoring probabilities are as follow, choosing random draws of left and right:

$i \backslash j$	L	R
L	π_{LL}	π_{LR}
R	π_{RL}	π_{RR}

Also, Kicker (k) and Goal-keeper (g) payoffs are determined by the probabilities of scoring and the probability that the other player choose left or right. g_L is the probability of goaler choose left, and k_L is the probability kicker choses left. Therefore, the payoffs are as follow:

$$p_L^k = g_L\pi_{LL} + (1 - g_L)\pi_{LR},$$
$$p_R^k = g_L\pi_{RL} + (1 - g_L)\pi_{RR}.$$

$$p_L^g = k_L(1 - \pi_{LL}) + (1 - k_L)(1 - \pi_{RL}),$$
$$p_R^g = k_L(1 - \pi_{LR}) + (1 - k_L)(1 - \pi_{RR}).$$

The goal keeper will try to make the kicker indifferent between choosing left or right and vice-versa by choosing g_L *and* k_L. Let's use the measures from the paper as our example and calculate g_L *and* k_L to satisfy Von-Neumann's MinMax theorem.

	g_L	$1-g_L$
k_L	58·30	94·97
$1-k_L$	92·91	69·92

Since the payoffs of choosing left or right must be equal, the probabilities of \choosing left or right should be as follow:

$$P_L^k = g_L * 58.3 + (1-g_L) * 94.97 = g_L * 92.91 + (1-g_L) * 69.92 = P_R^k$$

$$\rightarrow g_L = \frac{25.05}{59.66} = 0.4199$$

$$P_L^g = k_L * (100-58.3) + (1-k_L) * (100-92.91)$$
$$= k_L * (100-94.97) + (1-k_L) * (100-69.92) = P_R^g$$

$$\rightarrow k_L = \frac{121}{314} = 0.3854$$

As confirmed by the paper's Nash Predicted frequencies:

	g_L (%)	$1-g_L$ (%)	k_L (%)	$1-k_L$ (%)
Nash predicted frequencies	41·99	58·01	38·54	61·46
Actual frequencies	42·31	57·69	39·98	60·02

We know that kickers are more probable to kick to their natural side and goal-keepers are intended to pay attention to the kicker's natural side too. Now Imagine that the goal keeper improves catching the natural-side kicks, thus, the probability of scoring at LL (for left-footed) and RR (for right-footed) decreases. Therefore, the distribution of scorings can change into a table like below:

	g	1-g
k	40	94.97
1-k	92.91	50

Thus the payoff calculations change to:

$$P_L^k = g_L * 40 + (1-g_L) * 94.97 = g_L * 92.91 + (1-g_L) * 50 = P_R^k$$

$$\rightarrow g_L = \frac{4497}{9788} = 0.4594$$

$$\underline{P_L^g = k_L * (100-40) + (1-k_L) * (100-92.91)}$$
$$\underline{= k_L * (100-94.97) + (1-k_L) * (100-50) = P_R^g}$$

$$\rightarrow k_L = \frac{4291}{9788} = 0.4384$$

Conclusion: The probability of choosing left increases compared to the natural data used by the paper and assuming that the equilibrium remains intact.

6. What is the relationship between the Cobb-Douglas production function and returns to scale?

(The following questions may require using an advanced textbook, such as Varian's "Microeconomic Analysis")

Cobb-Douglas production function: $Q = A.L^{\beta}.K^{\alpha}$

If we multiply the inputs by k → $A\ (kL)^{\beta}(kK)^{\alpha} = k^{\alpha+\beta}.A.L^{\beta}.K^{\alpha} = k^{\alpha+\beta}.Q$

Thus, the return to scale is $k^{\alpha+\beta}$ and is determined by $\alpha+\beta$. So, it is constant return to scale if $\alpha+\beta = 1$. If $\alpha+\beta < 1$, the production function is decreasing return to scale and if $\alpha+\beta > 1$, it is increasing return to scale.

7. Explain what are the Roy's identity, the Shephard's lemma and the indirect utility function.

Roy's identity states that if $v(p, m)$ is the indirect utility function (maximum utility as a function of price and income), we can calculate the marshallian (ordinary) demand function by calculating the derivative of indirect utility with respect to price, divided by the derivative of indirect utility function in respect to income. The notation below shows the same thing:

$$x_i(\mathbf{p}, m) = -\frac{\frac{\partial v(\mathbf{p},m)}{\partial p_i}}{\frac{\partial v(\mathbf{p},m)}{\partial m}} \text{ for } \quad \mathrm{i} = 1, \cdots, k$$

Where $x_i(p, m)$ is the marshallian demand for good i.

Shephard's lemma states that if we have convex cost function, the minimum cost is a unique bundle of goods. In other words, if the cost function's derivative is decreasing, for a fixed level of utility, we have one bundle of goods that has the minimum cost between all the bundles that provide the same amount of utility. The same idea can be driven in the theory of firm where there is only one bundle of inputs that produces the same output with the minimum cost. The notation can be shown as follow:

$$h_i(\mathbf{p}, u) = \frac{\partial e(\mathbf{p}, u)}{\partial p_i}$$

Where h is the hicksian demand for good i and e is the cost function. Or for a firm:

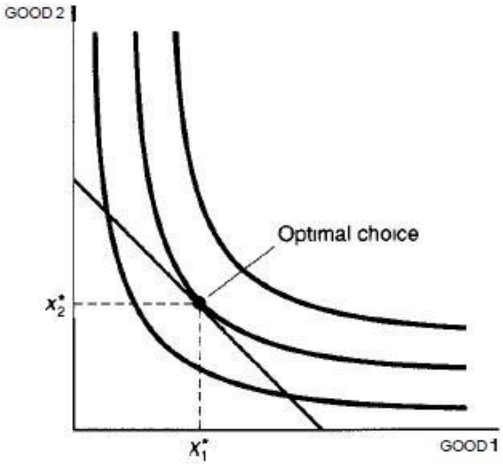
$$x_i(\mathbf{w}, y) = \frac{\partial c(\mathbf{w}, y)}{\partial w_i} \quad i = 1, \cdots, n$$

Where x is the conditional factor demand and c is the cost function with factor prices in w vector and output y.

Indirect utility function is a function stating the maximum utility achievable giving m (income) and p (vector of prices). In other words, in a conditional optimization problem we maximize the utility function considering the budget constraints. The notation is usually shown like this:

$$v(\mathbf{p}, m) = \max u(\mathbf{x})$$
$$\text{such that } px = m$$

And one particular of such optimization looks like this:



8. Explain how three of the most common production functions are particular cases of the Constant Elasticity of Substitution production function.

Constant elasticity of substitution (CES) has a general form of production function and some other famous production functions are special cases of CES. CES has the following form:

$$y = \left[a_1 x_1^{\rho} + a_2 x_2^{\rho}\right]^{\frac{1}{\rho}}$$

Below you can see some of the production functions derived from CES:

1-Linear production function:

If ρ=1, then, $y = a_1 x_1 + a_2 x_2$ which is the linear production function.

2-Cobb-Douglas production function: $q = AK^{\alpha} L^{\beta}$

If $a_1 = a \; and \; a_2 = (1-a) \; and \; x_1 = K \; and \; x_2 = L$, and we take a natural log:

$$\ln q = \ln[aK^{\rho} + (1-a)L^{\rho}]^{\frac{1}{\rho}} = \frac{\ln[aK^{\rho} + (1-a)L^{\rho}]}{\rho}$$

If we take a limit where ρ goes to zero:

$$\lim_{\rho \to 0} \ln q = \lim_{\rho \to 0} \frac{\ln[aK^{\rho} + (1-a)L^{\rho}]}{\rho}$$

L'Hôpital's rule: $\lim_{X \to a} \frac{m(x)}{n(x)} = \lim_{X \to a} \frac{m'(x)}{n'(x)}$

$$\lim_{\rho \to 0} \frac{\ln[aK^{\rho} + (1-a)L^{\rho}]}{\rho}$$

$$\mathrm{m}(\rho) = \ln[aK^{\rho} + (1-a)L^{\rho}]$$

$$\mathrm{n}(\rho) = \rho$$

$$n'(\rho) = 1$$

$$\mathrm{m}'(\rho) = \frac{aK^{\rho} \ln K + (1-a)L^{\rho} \ln L}{aK^{\rho} + (1-a)L^{\rho}}$$

Thus we have the following:

$$\lim_{\rho \to 0} \frac{m'(\rho)}{n'(\rho)} = \lim_{\rho \to 0} \frac{m'(\rho)}{1} = \lim_{\rho \to 0} \frac{aK^{\rho} \ln K + (1-a)L^{\rho} \ln L}{aK^{\rho_4}(1-a)L^{\rho}}$$

Then we simplify:

$$\lim_{\rho \to 0} \frac{aK^{0} \ln K + (1-a)L^{0} lnL}{aK^{0} + (1-a)L^{0}} = \frac{a\ln K + (1-a)\ln L}{a + (1-a)} = a\ln K + (1-a)\ln L$$

$$\lim_{\rho \to 0} \ln q = a\ln K + (1-a)\ln L = \ln K^{a} + \ln L^{1-a} = \ln K^{a} L^{1-a}$$

$$\ln q = \ln K^{a} L^{1-a}$$

$$e^{lnq} = e^{\ln K^{a} L^{1-a}}$$

And finally we can say that:

$$q = A.K^{\alpha}.L^{1-\alpha}$$

3- Leontief production function:

Just like how we proved the cob-douglas function, we can prove the Leontief production function by calculating the limit for $\rho \to \infty$ as follow:

$$\lim_{\rho \to \infty} dQ \Rightarrow \min(aK, (1-a)L)$$

Since we are interested in the limit when $\rho \to \infty$ we can ignore the interval for which $\rho \leq 0$, and treat ρ as strictly positive.

Without loss of generality, assume $K \geq L \Rightarrow (1/K^{\rho}) \leq (1/L^{\rho})$. We also have $K, L > 0$. Then we verify that the following inequality holds:

$$(1-a)^{k/\rho}(1/L^{k}) \leq \gamma Q_k^{-1} \leq (1/L^{k})$$

$$\implies (1-a)^{k/\rho}(1/L^{k}) \leq [a(1/K^{\rho}) + (1-a)(1/L^{\rho})]^{\frac{k}{\rho}} \leq (1/L^{k}) \tag{1}$$

by raising throughout to the ρ/k power to get

$$(1-a)(1/L^{\rho}) \leq a(1/K^{\rho}) + (1-a)(1/L^{\rho}) \leq (1/L^{\rho}) \tag{2}$$

which indeed holds, obviously, given the assumptions. Then go back to the first element of (1) and

$$\lim_{\rho \to \infty} (1-a)^{k/\rho}(1/L^{k}) = (1/L^{k})$$

which sandwiches the middle term in (1) to $(1/L^{k})$, so

$$\lim_{\rho \to \infty} Q_k = \frac{\gamma}{1/L^{k}} = \gamma L^{k} = \gamma\big[\min\{K, L\}\big]^{k} \tag{3}$$

So for $k = 1$ **we obtain the basic Leontief production function.**

9. Explain what is the role played by convexities in the two welfare theorems.

Let's first define these two theorems using the course material, and then, show how convexities play their role.

First Theorem: under some assumptions, a perfectly competitive equilibrium is Pareto efficient (it can be shown in an Edgeworth box in general equilibrium; in a demand-supply graph in partial equilibrium).

When we have convex indifference curves, we can solve the constrained optimization below, and find the pareto efficient points where two agents (players) have tangent indifference curves.

$$\max_{x_A^1, x_A^2, x_B^1, x_B^2} u_A(x_A^1, x_A^2)$$

$$\text{such that } u_B(x_B^1, x_B^2) = \overline{u}$$

$$x_A^1 + x_B^1 = \omega^1$$

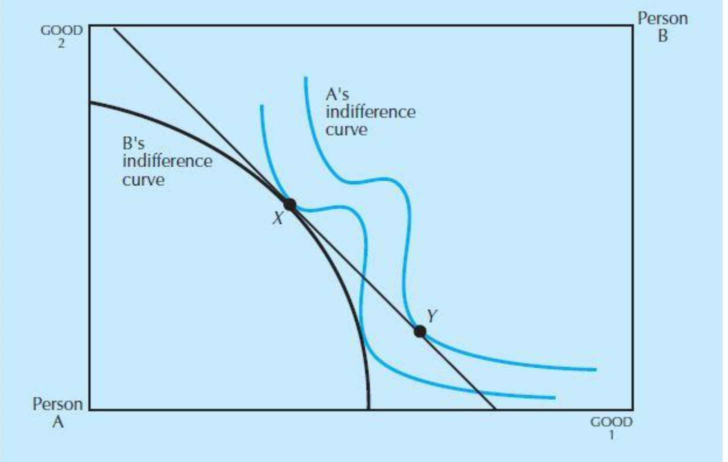
$$x_A^2 + x_B^2 = \omega^2.$$

Leading to MRS for each player to be equal to relative prices which shows a unique point for any set of fixed utilities for both persons.

$$\frac{\partial u_A / \partial x_A^1}{\partial u_A / \partial x_A^2} = \frac{p_1}{p_2}$$

$$\frac{\partial u_B / \partial x_B^1}{\partial u_B / \partial x_B^2} = \frac{p_1}{p_2}.$$

But imagine that we have a non-convex indifference curve like below:



A Pareto efficient allocation that is not an equilibrium. It is possible to find Pareto efficient allocations such as X in this diagram that cannot be achieved by competitive markets if preferences are nonconvex.

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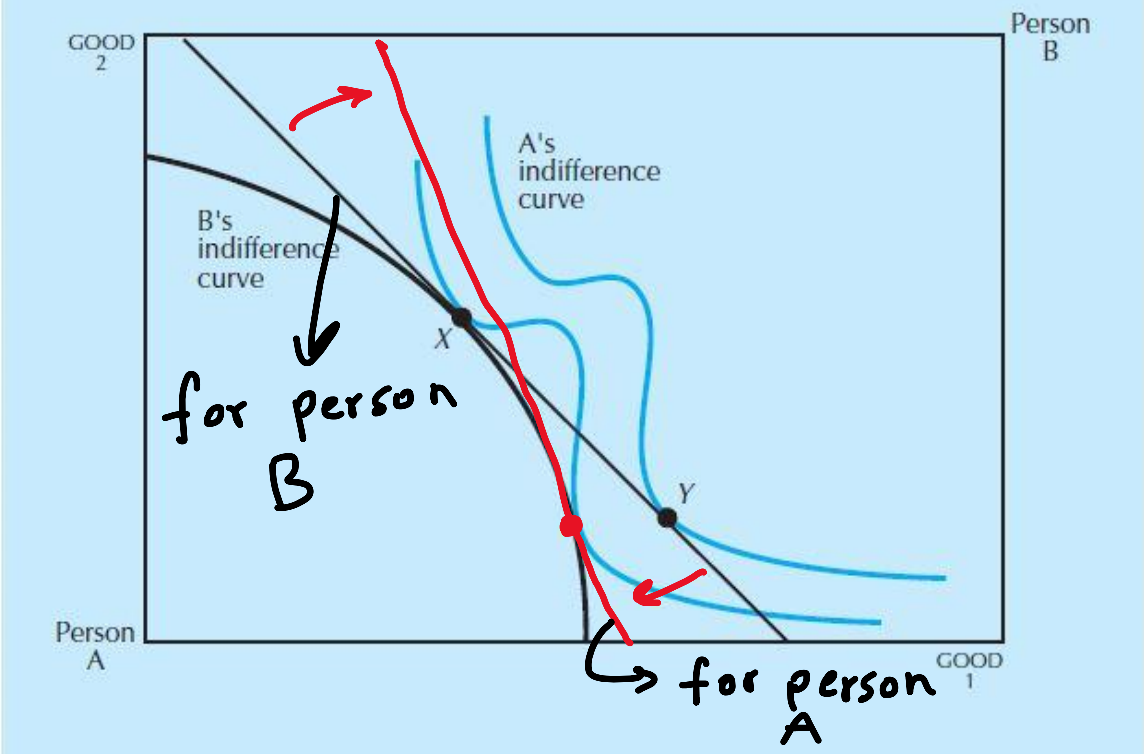
Although it is still pareto efficient, it is not always an equilibrium and we can say that when our indifference curve is not convex, we can have a pareto efficient allocation which is not equilibrium. If all consumers have convex preferences and all firms have convex production possibility sets then Pareto efficient allocation can be achieved and the equilibrium of a complete set of competitive markets are suitable for redistribution of initial endowments.

Second Theorem: under some assumptions, any Pareto-efficient allocation can be achieved through a competitive equilibrium if the initial endowment is previously redistributed with non-distortive taxation. In other words, the second theorem asserts that we can separate the allocation part from the distribution part in a competitive market.

When we have convex preferences and we let the competitive market allocate, we reach an equilibrium. If we subsidize or impose tax on different goods, we can change the relative prices and change the equilibrium to a level of utility that is not as good as the previous ones aggregated. The second theorem implies that changing the relative prices will change the points that $MRS = \frac{p_1}{P_2}$ and this may not be efficient, thus it implies that it's better to distribute income and leave the relative prices alone. If the preferences are not convex and we are not at an equilibrium, price interventions can be justified somehow. How? Just look at this price intervention to the previous example:



A Pareto efficient allocation that is not an equilibrium. It is possible to find Pareto efficient allocations such as X in this diagram that cannot be achieved by competitive markets if preferences are nonconvex.

Here we can have a budget line for one person (or a group of people) and another one for another person.