Assignment 1 | MAREB|Applied Macroeconomics

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1. Taking Equation 2, describe the role of σe. In particular, derive and explain the economic intuition when:

- $\sigma_e \to \infty$ (linear function)
- $\sigma_e \to 1$ (Cobb-Douglas function)

ANSWER:

Elasticity of substitution asserts how much percentage one input changes if the price of the other input changes for 1 percent.

When $\sigma_e \to \infty$, (with Hopital Rule) it means that $\frac{\sigma_e}{\sigma_e - 1} \to 1$, thus with the following proof, we have a linear function:

$$L = [\theta L_{HS}^{1} + (1-\theta)L_{LS}^{1}]^{1} = \theta L_{HS} + L_{LS} - \theta L_{LS}$$

Which is a linear. Moreover, the level of L can be fixed with substitutable $L_{LS} and\ L_{HS}$.

When $\sigma_e \to 1$, it means that $\frac{\sigma_e}{\sigma_e - 1} \to 0$, thus with the following proof, we have a Cobb-Douglas function:

$$\ln L = \ln\left[\left[\theta L_{HS}^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\theta)L_{LS}^{\frac{\sigma_e - 1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e - 1}}\right]$$

$$\lim_{\frac{\sigma_e}{\sigma_e - 1} \to 0} \ln L = \lim_{\frac{\sigma_e}{\sigma_e - 1} \to 0} \ln \frac{\left[\left[\theta L_{HS}^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\theta)L_{LS}^{\frac{\sigma_e - 1}{\sigma_e}}\right]\right]}{\frac{\sigma_e}{\sigma_e - 1}}$$

$$\to Hospital's\ Rule:$$

$$\lim_{\frac{\sigma_e}{\sigma_e-1}\to 0} \ln L = \frac{\theta L_{HS}^{\frac{\sigma_e-1}{\sigma_e}} lnL_{HS} + (1-\theta) L_{LS}^{\frac{\sigma_e-1}{\sigma_e}} lnL_{LS}}{\theta L_{HS}^{\frac{\sigma_e-1}{\sigma_e}} (1-\theta) L_{LS}^{\frac{\sigma_e-1}{\sigma_e}}}$$

$$= \frac{\theta L_{HS}^{0} lnL_{HS} + (1-\theta) L_{LS}^{0} lnL_{LS}}{\theta L_{HS}^{0}(1-\theta) L_{LS}^{0}} = \theta . lnL_{HS} + (1-\theta). lnL_{LS}$$

$$\to e^{\lim_{\frac{\sigma_e}{\sigma_e-1}\to 0} \ln L} = L_{HS}^{\theta} . L_{LS}^{1-\theta} \ (cobb - Douglas)$$

2. Derive the marginal productivity of capital and the wage of native high skilled workers.

ANSWER: I am not sure how marginal productivity of capital is going to lead us to national high-skilled worker's wage, but:

$$MPK = \frac{\partial Y}{\partial K} = (1-\alpha). A. L^{\alpha}. K^{-\alpha}$$

I believe it was a typo and the results asked for are the following:

Considering the ultimate form of production function, with $\rho_e = \frac{\sigma_e - 1}{\sigma_e}$, $\gamma_e = \frac{\sigma_{e,b} - 1}{\sigma_{e,b}}$:

$$Y = A\left\{\left[\theta\left[\ (\beta L_{HS,N}^{\gamma_{HS}} + (1-\beta) L_{HS,m}^{\gamma_{HS}})^{\frac{1}{\gamma_{HS}}}\right]^{\rho} + (1-\theta)\left[(\beta L_{LS,N}^{\gamma_{LS}} + (1-\beta) L_{LS,n}^{\gamma_{LS}})^{\frac{1}{\gamma_{LS}}}\right]^{\rho}\right]^{\frac{1}{\rho}}\right\}^{\alpha} \cdot K^{1-\alpha} \ (*)$$

(Unfortunately, the power used in calculating L in equation (2) is not clearly defined. When we substitute e with the relative group, the type of $\frac{\sigma_e}{\sigma_e - 1}$ at the end of the equation is not specified. Thus, I am considering it fixed and calculate the MPL_e by hand, and then I use the general formula in the paper to do it again.)

Calculations based on (*):

Obviously, we can consider profit maximizing firm and drive the equillibrium results of marginal production relations and wages or even cost of capital. With the wages equal to marginal productivity of labor with a specific type, we can write:

$$\ln(W_{HSN}) = \ln(MPL_{HSN})$$

$$= \ln\left\{A.\alpha.L^{\alpha-1}.\theta.\rho.L_{HSN}^{\rho-1}.\frac{1}{\gamma_{HS}}.\left[(\beta L_{HS,N}^{\gamma_{HS}} + (1-\beta)L_{HS,m}^{\gamma_{HS}})^{\frac{1}{\gamma_{HS}}-1}\right].\beta.L_{HSN}^{\gamma_{HS}-1}.K^{1-\alpha}\right\}$$

$$= lnA + ln\alpha + (\alpha-1).lnL + ln\theta + \ln\rho + (\rho-1).lnL_{HS} + \ln\left(\frac{1}{\gamma_{HS}}\right) + \ln\left(\frac{L_{HS}}{L_{HS}{}^{\gamma_{HS}}}\right) + ln\beta$$

$$+ \ln(\gamma_{HS}) + (\gamma_{HS}-1).lnL_{HSN} + ln(1-\alpha) - \alpha.lnK$$

Also, for the high skilled Immigrants we have:

$$\ln(W_{HSM}) = \ln(MPL_{HSM})$$

$$= lnA + ln\alpha + (\alpha-1).lnL + ln\theta + \ln\rho + (\rho-1).lnL_{HS} + \ln\left(\frac{1}{\gamma_{HS}}\right) + \ln\left(\frac{L_{HS}}{L_{HS}{}^{\gamma_{HS}}}\right) +$$

$$\ln(1-\beta) + ln(\gamma_{HS}) + (\gamma_{HS}-1).ln(L_{HSM}) + ln(1-\alpha) - \alpha.\ln(K\quad)$$

Using the general formula introduced in Ottaviano and Peri (2012), should reach a similar specification.

3. Compute the difference between the logarithm of the wage of native high skilled workers and migrant high skilled workers.

ANSWER:

Using the formulas generated above:

$$\ln(W_{HSN}) - \ln(W_{HSM})$$

$$= \ln(\beta) - \ln(1-\beta) + (\gamma_{HS} - 1).(lnL_{HSN} - ln(L_{HSM}))$$

Which is similar (not necessarily equal) to the general form without my notations in the paper:

$$\ln\left(\frac{w_{Fkt}}{w_{Dkt}}\right) = \varphi_k + \varphi_t - \frac{1}{\sigma_N}\ln\left(\frac{L_{Fkt}}{L_{Dkt}}\right) + u_{it}, (**)$$

The divergence between the logarithms of wages is defined by elasticity of substitution between high skilled natives and high skilled migrants, raletive productivity of native workers, and the difference of logarithmic form of the size of each labor type (native and migrant). It completely makes sense. Remember that $\gamma_e = \frac{\sigma_{e,b}-1}{\sigma_{e,b}}$, which is not exactly elasticity of substitution between high skilled natives and high skilled migrants, but somehow, a function closer to its inverse. Thus, interpret it more like an inverse

4. Looking at Ottaviano and Peri (2012) paper, briefly describes how they manage to estimate $\sigma_{e,b}$ of individuals with same skills. What they conclude about the degree of substitution between native and immigrant workers?

Answer:

Like in the assignment description, they have considered a general form of nested-CES where the workers types and characteristics are not limited to 4 (like in the assignment). Finally, they drived the specification (**). By running the regression on a sample of US workers, and estimating the coefficient ($-\frac{1}{\sigma_N}$), they estimated the elasticity of substitution between natives and immigrants. Considering Table 2 of the paper, they concluded that although the estimated coefficients are mostly significant and the models ran are robust, the coefficient in absulute value is high ($\frac{1}{\sigma_n}$), meaning that **the elasticity of the substitution between natives and immigrants are very low ($\boldsymbol{\sigma_N}$)**. This result confirms the studies mentioned in the literature, and illustrate that the degree of σ_N is lower than what mentioned in previous research. In a nutshell, immigration, had a mild effect on the native wages.

References

Ottaviano, G. I. and Peri, G. (2012). Rethinking the effect of immigration on wages. *Journal of the European economic association*, 10(1):152–197.