Shayan Abbasi NIU: 1596105
Dalia Argudo Barrera NIU: 1627065
Sitti Alisya Masrura NIU: 1615417

9. A firm produces output according to a production function $Q = F(K, L) = \min\{2K, 4L\}$

a) **How much output is produced when $K = 2$ and $L = 3$?**
b) **If the wage rate is $30 per hour and the rental rate on capital is $10 per hour, what is the cost-minimizing input mix for producing 4 units of output?**
c) **How does your answer to part b change if the wage rate decreases to $10 per hour but the rental rate on capital remains at $10 per hour?**

a. $Q(2,3) = \min\{2(2), 4(3)\} = \min\{4, 12\} = 4$
b. $For\ Q = 4\ we\ need\ at\ list\ a\ mix\ of\ K = 2\ and\ L = 1$

$$It\ isn'tasked\ in\ the\ question\ but\ if\ w = 30, r = 10, Cost = rK + wL.$$

$$\rightarrow Cost = 10(2) + 30(1) = 50\$$$

$$which\ is\ the\ minimum\ cost\ for\ producing\ Q = 4.$$

c. In a Leontief production function both inputs need to satisfy the minimum condition being equal to Q so that in a fixed amount of Q, it doesn't matter what are the costs of inputs we cannot substitute them with each other without losing a level of output. Thus, the minimum cost just decreases to 30$ with the same level of inputs ($K = 2, L = 1$).

10. A firm produces output according to the production function Q = F (K, L) = 2K + 4L.

a) **How much output is produced when K = 2 and L = 3?**
b) **If the wage rate is $30 per hour and the rental rate on capital is $10 per hour, what is the cost-minimizing input mix for producing 16 units of output?**
c) **How does your answer to part b change if the wage rate decreases to $10 per hour but the rental rate on capital remains at $10 per hour?**

a. $Q = F(2,3) = 2(2) + 4(3) = 16$
b. $if\ w = 30\$, r = 10\$, for\ Q = 16, Cost = 10K + 30L$

$$\rightarrow Min\ Cost = 10K + 30L$$

$$s.t.\quad Q(K,L) = 16$$

$$\rightarrow to\ solve\ this\ we\ should\ satisfy\ the\ condition \frac{MP_l}{MP_k} = \frac{w}{r},$$

$$However\ we\ have \frac{4}{2} < \frac{30}{10},$$

$$which\ means\ that\ we\ should\ substitute\ more\ L\ to\ K\ (increase\ capital\ and\ decrease\ labor).$$

$$Since\ MP_l\ and\ MP_k\ are\ amazingly\ constant, by\ substituting\ L\ to\ K\ we\ can't\ satisfy\ the\ equality\ condition$$

$$and\ we\ end\ up\ using\ only\ Capital \rightarrow L = 0, K = 8\ and\ Min\ Cost = 10(8) = 80$$

c. Since L and K are perfect substitutable, and 1 unit of L makes more output than 1 unit of K, when they have the same cost, $MRTS > 1 \rightarrow we\ should\ only\ use\ labor \rightarrow for\ Q = 16, we\ use\ L = 4\ with\ the\ minimum\ cost\ of\ 40\$.$

15. You are a manager at Glass Inc.—a mirror and window supplier. Recently, you conducted a study of the production process for your single-side encapsulated window. The results from the study are summarized in the table on the next page, and are based on the 5 units of capital currently available at your plant. Workers are paid $50 per unit, per unit of capital costs are $10, and your encapsulated windows sell for $5 each. Given this information, optimize your human resource and production decisions. Do you anticipate earning a profit or a loss? Explain carefully

Profit-Maximizing Input Usage
To maximize profits, a manager should use inputs at levels at which the marginal benefit equals the marginal cost. More specifically, when the cost of each additional unit of labor is w, the manager should continue to employ labor up to the point where $VMP_L = w$ in the range of diminishing marginal product.

w=50, r=10, p=5

- Given VMPL=w -> 50=50, the profit-maximizing quantity of labor is 5, and the firm should produce 90 units of the output (encapsulated windows).

Labor (L)	Capital (K)	Output (Q)	Wage rate (w)	Capital Cost (r)	Price of Output (p)	Marginal Product of Labor (MP_L)	Average Product of labor (AV_L)	Average Product of capital (AP_K)	Value Marginal Product of labor (VMP_L)
0	0	0							
1	5	10	50	10	5	10	10.00	2	50
2	5	30	50	10	5	20	15.00	6	100
3	5	60	50	10	5	30	20.00	12	150
4	5	80	50	10	5	20	20.00	16	100
5	5	90	50	10	5	10	18.00	18	50
6	5	95	50	10	5	5	15.83	19	25
7	5	95	50	10	5	0	13.57	19	0
8	5	90	50	10	5	-5	11.25	18	-25
9	5	80	50	10	5	-10	8.89	16	-50
10	5	60	50	10	5	-20	6.00	12	-100
11	5	30	50	10	5	-30	2.73	6	-150

- So,
  The Total Cost (TC)= FC+ VC
  FC=r*K=10*5=50
  VC=w*L=50*5=250
  TC=50+250=300

Total Revenues (TR)=p*Q
TR=5*90=450

MAX PROFITS= TR-TC=450-300 =150

- With the optimum level of human resources (5 workers) and the optimum level of production (90 encapsulate windows), Glass Inc. will have maximum profits of $150.