Decentralized Portfolio Optimization

Shayan Malekpour

December 20, 2023

Problem Description

Consider a decentralized portfolio optimization problem with M investors and N assets. Each investor aims to maximize their expected portfolio return while considering risk and a global objective related to the average return across all portfolios.

The problem involves the following key components:

- **Investors:** There are M independent investors, each with its own risk profile, investment goals, and constraints.
- **Assets:** There are N financial assets available for investment.
- **Global Objective:** The global objective is to optimize the overall portfolio performance, considering both individual investor objectives and a collective goal related to the average return.
- **Local Decision Variables:** Each investor has a set of decision variables representing the weights assigned to different assets in their portfolio.
- **Local Cost Functions:** The local cost functions for each investor incorporate their individual objectives, risk tolerance, and a coupling term with the global variable representing the average return.

Problem Description: Local Cost Function Components

Expected Portfolio Return ($R_i(W_i)$)

The expected portfolio return for investor i is a linear combination of the weights assigned to different assets in their portfolio:

$$R_i(W_i) = \sum_{j=1}^{N} W_{i,j} \cdot \text{Expected Return of Asset } j$$

Portfolio Risk ($\sigma_i(W_i)$)

The portfolio risk for investor i is represented by the standard deviation of the portfolio return, considering the covariance between assets:

$$\sigma_i(W_i) = \sqrt{\sum_{j=1}^{N} \sum_{k=1}^{N} W_{i,j} \cdot W_{i,k} \cdot \text{Covariance}(j,k)}$$

Coupling Term ($|R_i(W_i) - \bar{R}|$)

The coupling term represents the absolute difference between the expected portfolio return of investor i and the average return across all portfolios:

$$|R_i(W_i) - \bar{R}| = \left| R_i(W_i) - \frac{1}{M} \sum_{j=1}^{M} R_j(W_j) \right|$$

Global Variable ($\bar{R}$)

The global variable represents the average portfolio return across all investors and is updated iteratively:

$$\bar{R} = \frac{1}{M} \sum_{i=1}^{M} R_i(W_i)$$

Primal Decomposition Method

The primal decomposition method is employed to address the decentralized portfolio optimization problem. The key steps are as follows:

1. **Decompose the Global Objective:** Express the global objective as the sum of individual local cost functions:

$$J_{\text{Global}} = \sum_{i=1}^{M} J_i(W_i)$$

2. **Decompose Individual Local Cost Functions:** Express each local cost function for investor i as a combination of their local decision variables and a coupling term involving the global variable:

$$J_i(W_i) = -R_i(W_i) + \lambda \cdot \sigma_i(W_i) + \alpha \cdot |R_i(W_i) - \bar{R}|$$

3. **Optimize Locally:** Each investor independently optimizes their local cost function, focusing on their individual objectives and constraints.

4. **Update Global Variable:** After each iteration, update the global variable (average return) based on individual portfolio returns:

$$\bar{R} = \frac{1}{M} \sum_{i=1}^{M} R_i(W_i)$$

5. **Calculate Coupling Term:** Calculate the coupling term in each local cost function using the updated global variable.

6. **Minimize Local Cost Functions with Respect to Global Variable:** Theoretical calculation to minimize each local cost function with respect to the global variable, ensuring alignment with the collective objective.

7. **Repeat:** Iterate the process until convergence, allowing each investor to refine their portfolio allocation while considering the global performance.

The primal decomposition method allows for a decentralized optimization approach, where each investor independently optimizes their portfolio based on their preferences, while the algorithm ensures coordination through the coupling term and global variable updates.