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Artificial Intelligence Assignment - 3

A Golomb Ruler of order M and length L consists of M marks placed at unit intervals (i.e. integer positions) along an imaginary ruler such that the differences in spacing between every pair of marks are all distinct, i.e. no two pairs of marks are the same distance apart. The number of marks on the ruler is its order, and the largest distance between two of its marks is its length.

Backtracking is a general algorithm for finding all (or some) solutions to some computational problems, notably constraint satisfaction problems, that incrementally builds candidates to the solutions, and abandons each partial candidate ("backtracks") as soon as it determines that the candidate cannot possibly be completed to a valid solution.

Our overall implementation logic:

For **backtracking**, every set of domain values are iterated until we find the optimal ruler. An array of marker difference is maintained to facilitate the constraint checking. The first and last values of the markers are always 0 and the length provided respectively. Remaining values are being determined. If a value found in an iteration does lead to a successful golomb ruler, that value is discarded and the search is backtracked to find the next possible candidate value.

For **forward checking** in addition to backtracking, the legal assigned values are being figured out and used to set the next possible markers.

For **constraint propagation**, in addition to backtracking, arc-consistency is being checked and the domain values for every iteration is being minimized to the most optimal set. This ensures minimum check of domain values while trying to find a marker value.

Q. Implement a CSP solution to verify whether or not a Golomb ruler of a fixed length L for M marks exists. If a solution exists for length L find an optimal length ruler, that is one for which no shorter length ruler exists for M marks.

Some Statistics:

Starting with the base case: L=0, M=0

Backtracking only solution:
No shortest possible length exists for 0 markers of length 0
Calculation took 0.00100016593933 seconds.
Similarly with all the three cases there is no solution possible for L=0, below are the running time statistics:-

Backtracking with Forward Checking solution:
No shortest possible length exists for 0 markers of length 0
Calculation took 0.0 seconds.
Backtracking with Constraint Propagation solution:
No shortest possible length exists for 0 markers of length 0
Calculation took 0.0 seconds.

Case L=0,M=1

For this case there can only be one marker i.e.0
Below are results generated:
Backtracking only solution:
No shortest possible length exists for 1 markers of length 0
Calculation took 0.0 seconds.

Backtracking with Forward Checking solution:
No shortest possible length exists for 1 markers of length 0
Calculation took 0.0 seconds.
Backtracking with Constraint Propagation solution:
No shortest possible length exists for 1 markers of length 0
Calculation took 0.0 seconds.

As expected the time taken is also negligible .

Case L=3, M=3
For This case the markers which seem to be obvious are :[0,1,3]
Below are the results statistics:-
Backtracking only solution:
Shortest possible length 3 with Backtracking only is possible for marker positions [0, 1, 3]
Calculation took 0.0 seconds.
Backtracking with Forward Checking solution:

Found a solution with BT+FC [0, 1, 3]
checking for more optimal solutions..........
Shortest possible length using Backtracking with Forward Checking have marker positions [0, 1, 3]
Calculation took 0.000999927520752 seconds.
In this case the solution for L=3,M=3 was [0,1,3], but to another check was applied to be certain if that was the optimal solution against this constraint or there was any other length <L which could satisfy this constraint
Backtracking with Constraint Propagation solution:L
Shortest possible length 3 with Backtracking and Constraint Propagation only is possible for marker positions [0, 1, 3]
Calculation took 0.000999927520752 seconds.

Now for smaller L and M values the time taken is still negligible, within milliseconds, but as soon as we increase the L|M, the time difference is gonna cost and occur significantly

Case L=6,M=4
Backtracking only solution:
Shortest possible length 6 with Backtracking only is possible for marker positions [0, 1, 4, 6]
Calculation took 0.00200009346008 seconds.

Backtracking with Forward Checking solution :
Found a solution with BT+FC : [0, 1, 4, 6]
checking for more optimal solutions..........

Shortest possible length using Backtracking with Forward Checking have marker positions [0, 1, 4, 6]
Calculation took 0.00799989700317 seconds.
Backtracking with Constraint Propagation solution:
Shortest possible length 6 with Backtracking and Constraint Propagation only is possible for marker positions [0, 1, 4, 6]
Calculation took 0.010999917984 seconds.

For length 6 the time difference is inherently visible.

Case L=11, M-5
Backtracking only solution:
Shortest possible length 11 with Backtracking only is possible for marker positions [0, 1, 4, 9, 11]
Calculation took 0.0529999732971 seconds.

Backtracking with Forward Checking solution:
Found a solution with BT+FC : [0, 1, 4, 9, 11]
checking for more optimal solutions..........

Shortest possible length using Backtracking with Forward Checking have marker positions [0, 1, 4, 9, 11]
Calculation took 0.825999975204 seconds.

Backtracking with Constraint Propagation solution:
Shortest possible length 11 with Backtracking and Constraint Propagation only is possible for marker positions [0, 1, 4, 9, 11]
Calculation took 0.871999979019 seconds.

Even small increase in length after certain length can have an exponential increase in time like
Case: L= 17, M=6

Backtracking only solution:
Shortest possible length 17 with Backtracking only is possible for marker positions **[0, 1, 4, 10, 12, 17]**
Calculation took **12.43000030518 seconds.**

Backtracking with Forward Checking solution:

Found a solution with BT+FC [0, 1, 4, 10, 12, 17]
checking for more optimal solutions..........

Shortest possible length using Backtracking with Forward Checking have marker positions **[0, 1, 4, 10, 12, 17]**
Calculation took **11.1109998226 seconds.**

Backtracking with Constraint Propagation solution:
Shortest possible length 17 with Backtracking and Constraint Propagation only is possible for marker positions **[0, 1, 4, 10, 12, 17]**
Calculation took **11.4659998417 seconds.**

Case: L= 20, M=6

Backtracking only solution:
Shortest possible length 17 with Backtracking only is possible for marker positions [0, 1, 4, 10, 12, 17]
Calculation took 0.723000049591 seconds.

Backtracking with Forward Checking solution:

Shortest possible length using Backtracking with Forward Checking have marker positions [0, 1, 4, 10, 12, 17]
Calculation took 12.388999939 seconds.

Backtracking with Constraint Propagation solution:

Shortest possible length 17 with Backtracking and Constraint Propagation only is possible for marker positions [0, 1, 4, 10,12,17].Calculation took 0.548000097275 seconds.

Case: L= 20, M=5

Backtracking only solution:

Shortest possible length 11 with Backtracking only is possible for marker positions [0, 1, 4, 9, 11]
Calculation took 0.0439999103546 seconds.

Backtracking with Forward Checking solution:
Shortest possible length using Backtracking with Forward Checking have marker positions [0, 1, 4, 9, 11] Calculation took 0.317000150681 seconds.

Backtracking with Constraint Propagation solution:

Shortest possible length 11 with Backtracking and Constraint Propagation only is possible for

marker positions [0, 1, 4, 9, 11]
Calculation took 0.0750000476837 seconds.

As we can see in the above statistics that the time required has drastically increase by increasing just 1 marker, from milliseconds the time complexity just shot up to seconds. Even investigating further we will see that for 8 markers the running time is alongs the lines of 100 seconds for backtracking itself.Max values for which the algorithms runs within time span of 1 minute is L=34,M=8.For L=45,M=9 it takes about 203 seconds just to run the backtracking solution.

The solution does give the optimal golomb ruling using all 3 methods if one exists. Thank you!