Question 1: A customer owing a Maruti car right now has got the option to switch over to Maruti Ambassador or Fiat next time with the probability of Vi = (0.2, 0.5, 0.3). Given the transition matrix. Find the probabilities with his fourth purchase.

Answer: Given that Vi = [0.2 0.5 0.3] and

P = 0.4 0.3 0.3
0.2 0.5 0.3
0.25 0.25 0.5

Calculating P^2:

P = 0.4	0.3	0.3		0.4	0.3	0.3
0.2	0.5	0.3	*	0.2	0.5	0.3
0.25	0.25	0.5		0.25	0.25	0.5

Calculating for individual values for matrix multiplication, we get:

P11 = (0.4 * 0.4) + (0.3 * 0.2) + (0.3 * 0.25) = (0.16) + (0.06) + (0.075) = 0.295
P12 = (0.4 * 0.3) + (0.3 * 0.5) + (0.3 * 0.25) = (0.12) + (0.15) + (0.075) = 0.345
P13 = (0.4 * 0.3) + (0.3 * 0.3) + (0.3 * 0.5) = (0.12) + (0.09) + (0.15) = 0.36

P21 = (0.2 * 0.4) + (0.5 * 0.2) + (0.3 * 0.25) = (0.08) + (0.1) + (0.075) = 0.255
P22 = (0.2 * 0.3) + (0.5 * 0.5) + (0.3 * 0.5) = (0.06) + (0.25) + (0.075) = 0.385
P23 = (0.2 * 0.3) + (0.5 * 0.3) + (0.3 * 0.5) = (0.06) + (0.15) + (0.15) = 0.36

P31 = (0.25 * 0.4) + (0.25 * 0.2) + (0.5 * 0.25) = (0.1) + (0.05) + (0.125) = 0.275
P32 = (0.25 * 0.3) + (0.25 * 0.5) + (0.5 * 0.25) = (0.75) + (0.125) + (0.125) = 0.325
P33 = (0.25 * 0.3) + (0.25 * 0.3) + (0.5 * 0.5) = (0.075) + (0.075) + (0.25) = 0.4

Now calculating P^3

0.295	0.345	0.36		0.4	0.3	0.3
0.255	0.385	0.36	*	0.2	0.5	0.3
0.275	0.325	0.4		0.25	0.25	0.5

Calculating for individual values for matrix multiplication, we get:

P11 = (0.295 * 0.4) + (0.345 * 0.2) + (0.36 * 0.25) = (0.413) + (0.069) + (0.09) = 0.572
P12 = (0.295 * 0.3) + (0.345 * 0.5) + (0.36 * 0.25) = (0.0885) + (0.1725) + (0.09) = 0.351
P13 = (0.295 * 0.3) + (0.345 * 0.3) + (0.36 * 0.5) = (0.0885) + (0.1035) + (0.18) = 0.372

P21 = (0.255 * 0.4) + (0.385 * 0.2) + (0.36 * 0.25) = (0.102) + (0.077) + (0.09) = 0.269
P22 = (0.255 * 0.3) + (0.385 * 0.5) + (0.36 * 0.25) = (0.0765) + (0.1925) + (0.09) = 0.359
P23 = (0.255 * 0.3) + (0.385 * 0.3) + (0.36 * 0.5) = (0.0765) + (0.1155) + (0.18) = 0.372

P31 = (0.275 * 0.4) + (0.325 * 0.2) + (0.4 * 0.25) = (0.11) + (0.065) + (0.1) = 0.275
P32 = (0.275 * 0.3) + (0.325 * 0.5) + (0.4 * 0.25) = (0.0825) + (0.1625) + (0.1) = 0.345
P33 = (0.275 * 0.3) + (0.325 * 0.3) + (0.4 * 0.5) = (0.0825) + (0.0975) + (0.2) = 0.38

Now calculating Vi^4 = Vi * P^3:

		0.572 0.351 0.372
0.2 0.5 0.3	*	0.269 0.359 0.372
		0.275 0.345 0.38

Calculating for individual values for matrix multiplication, we get:

P11 = (0.2 * 0.572) + (0.5 * 0.269) + (0.3 * 0.275) = (0.1144) + (0.1345) + (0.825) = 0.3314
P12 = (0.2 * 0.351) + (0.5 * 0.359) + (0.3 * 0.345) = (0.0702) + (0.1795) + (0.1035) = 0.3532
P13 = (0.2 * 0.372) + (0.5 * 0.372) + (0.3 * 0.38) = (0.0744) + (0.186) + (0.114). = 0.3744

Hence the resultant matrix is [0.3314 0.3532 0.3744]. This matrix shows the probabilities.

Question 2: The transition probability matrix of a Markov chain {Xn}, n= 1, 2, having 3 states 1,2 and 3 is given by a matrix. And the initial distribution of P0 is [0.7 0.2 0.1]

Answer 2 (i): Calculating P(X2 = 3)
Calculating P^2:

0.1 0.5 0.4		0.1 0.5 0.4
0.6 0.2 0.2	*	0.6 0.2 0.2
0.3 0.4 0.3		0.3 0.4 0.3

Calculating for individual values for matrix multiplication, we get:

P11 = (0.1 * 0.1) + (0.5 * 0.6) + (0.4 * 0.3) = (0.01) + (0.3) + (0.12) = 0.43
P12 = (0.1 * 0.5) + (0.5 * 0.2) + (0.4 * 0.4) = (0.05) + (0.1) + (0.16) = 0.31
P13 = (0.1 * 0.4) + (0.5 * 0.2) + (0.4 * 0.3) = (0.04) + (0.1) + (0.12) = 0.26

P21 = (0.6 * 0.1) + (0.2 * 0.6) + (0.2 * 0.3) = (0.06) + (0.12) + (0.06) = 0.24
P22 = (0.6 * 0.5) + (0.2 * 0.2) + (0.2 * 0.4) = (0.3) + (0.04) + (0.08) = 0.42
P23 = (0.6 * 0.4) + (0.2 * 0.2) + (0.2 * 0.3) = (0.24) + (0.04) + (0.06) = 0.34

P31 = (0.3 * 0.1) + (0.4 * 0.6) + (0.3 * 0.3) = (0.03) + (0.24) + (0.09) = 0.36
P32 = (0.3 * 0.5) + (0.4 * 0.2) + (0.3 * 0.4) = (0.15) + (0.08) + (0.12) = 0.35
P33 = (0.3 * 0.4) + (0.4 * 0.2) + (0.3 * 0.3) = (0.12) + (0.08) + (0.09) = 0.29

Now calculating:

		0.43 0.31 0.26
0.7 0.2 0.1	*	0.24 0.42 0.34
		0.36 0.35 0.29

Calculating for individual values for matrix multiplication, we get:

P11 = (0.7 * 0.43) + (0.2 * 0.24) + (0.1 * 0.36) = (0.301) + (0.048) + (0.036) = 0.385
P12 = (0.7 * 0.31) + (0.2 * 0.42) + (0.1 * 0.35) = (0.217) + (0.084) + (0.035) = 0.336

$P13 = (0.7 * 0.26) + (0.2 * 0.34) + (0.1 * 0.29) = (0.182) + (0.068) + (0.029) = 0.279$

Hence the resultant matrix is [0.385 0.336 0.279].
$P(X2 = 3)$ is 0.279.

Answer 2 (ii): $P0 = [0.7 \;\; 0.2 \;\; 0.1]$
Hence:
$P(X0 = 1) = 0.7$
$P(X0 = 2) = 0.2$
$P(X0 = 3) = 0.1$

And the transition matrix is given by:

0.1 0.5 0.4
0.6 0.2 0.2
0.3 0.4 0.3

$P11 = 0.1$	$P12 = 0.5$	$P13 = 0.4$
$P21 = 0.6$	$P22 = 0.2$	$P23 = 0.2$
$P31 = 0.3$	$P32 = 0.4$	$P33 = 0.3$

Calculating for $P(X3 = 2, X2 = 3, X1 = 3, X0 = 2)$
$P(X3 = 2, X2 = 3, X1 = 3, X0 = 2)$
$= P(X3 = 2 \mid X2 = 3) . P(X2 = 3 \mid X1 = 3) . P(X1 = 3 \mid X0 = 2) . P(X0 = 2)$
$= P32 . P33 . P23 . P(X0 = 2)$
$= 0.4 * 0.3 * 0.2 * 0.2$
$= 0.0048$

Question 3: Five green balls and 3 white balls are placed in two boxes A and B so that each box contains 4 balls. At each stage, a ball is drawn at random from each box and the two balls are interchanged. Let X(n) denote the number of white balls in box A after the nth draw.

Determine the state space and index set.

If Y(n) is the total number of green balls in box A after the nth draw, then determine the state space and index set for Y(n)

Answer :

<u>State Space and Index Set for X(n)</u>

<u>State Space for X(n)</u>

The variable X(n) represents the number of white balls in the box A after n draws. Initially box A can contain 0, 1, 2, 3 or 4 white balls, so the state space for X(n) is:

SX(n) = {0, 1, 2, 3, 4}

<u>Index Set for X(n)</u>

The index set refers to the set of all possible values of n. Since n represents the number of draws, n can be any non-negative integer:

TX(n) = {0, 1, 2, 3, …}

<u>State Space and Index Set for Y(n)</u>

<u>State Space for X(n)</u>

The variable Y(n) represents the number of green balls in box A after n draws. Initially box A can contain 0, 1, 2, 3, or 4 green balls. Hence the state space for Y(n) is:

SY(n) = {0, 1, 2, 3, 4}

<u>Index Set for Y(n)</u>

The index set refers to the set of all possible values of n. Since n represents the number of draws, n can be any non-negative integer:

TY(n) = {0, 1, 2, 3, …}