IME639: Course Project

Sports Timetabling Problem

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Sports Timetabling Problem

The main purpose of any sports scheduling problem is to find a time slot for each game based on some predefined constraints. The tournament can be of two types i.e., single league or multi-league. There are different ways in which it can be scheduled. It can be scheduled as knockout tournament where the winner of one round advances to the next round and the loser is eliminated. It can also be scheduled as a round robin tournament where each team plays with every other team. The teams can meet any number of times in this type of tournament but the number of games should be same for all the teams.

In this problem, a single league, double round robin tournament is considered. In a double round robin tournament, each team plays the other teams twice. These games are played as home and away i.e., each team is host for one game and visitor in the other. The problem is time constraint and should complete in minimum number of time slots. In this scenario, the total number of slots would be equal to the total number of games played by a particular team. Fig 1 shows a schedule of 6 team and 10 slots in 2RR tournament type.

Round 0	: (1,	3)	(2,	5)	(4,	, 0)
Round 1	: (2,	1)	(3,	4)	(5,	, 0)
Round 2	: (0,	1)	(2,	3)	(4)	, 5)
Round 3	: (1,	2)	(3,	0)	(5)	, 4)
Round 4	: (0,	3)	(4,	1)	(5)	, 2)
Round 5	: (0,	5)	(2,	4)	(3)	, 1)
Round 6	: (1,	0)	(4,	2)	(5)	, 3)
Round 7	: (0,	4)	(1,	5)	(3)	, 2)
Round 8	: (1,	4)	(2,	0)	(3	, 5)
Round 9	: (0,	2)	(4,	3)	(5	, 1)

Figure 1: Sample schedule for n=6 teams and 2*(n-1) = 10 slots

The whole season of a double round robin tournament is split into two equally long intervals. These intervals can be seen as two different 1RR intervals, in which the home- away status of mutual games of two teams alternates between successive intervals. The schedule which follows this property is termed as 'phased'.

Apart from these, there are two other types of constraints i.e., hard constraints and soft constraints. The hard constraints are basic property of timetable and it cannot be violated whereas soft constraints are preferences that should be followed whenever possible. If a soft constraint is violated, a penalty will be added to the objective. The primary goal of the problem is to minimize the total penalties incurred.

Standardized RobinX XML data is used for problem instances. The format enhances problem data sharing and reuse among different users. It minimizes problem specification part which is the hardest task and increases the accessibility.

Constraints:

- 1. Capacity Constraint 1: It is the "place constraint" which stops a team to play a home game or away game in a specific time slot. It's a hard constraint
- Capacity Constraint 2: It is the "top team and bottom team constraint" which stops teams at the bottom to play all initial games against the teams at the top. It's a soft constraint.
- 3. Capacity Constraint 3: It is the limitation on the length of home stands by not allowing successive home breaks. It's a soft constraint.
- 4. Capacity Constraint 4: It puts limitation on the total number of games between top teams or it can even put a limit on the total number of home games per slot. It's a hard constraint.
- 5. Game Constraint 1: It can restrict or schedule a game to a time slot. In real world, these constraints turn out to be helpful when a broadcaster might ask for top game in a particular slot. It's a hard constraint.
- 6. Break Constraint 1: It is used to restrict total number of breaks per team or can even avoid breaks at the starting or closing of the season. It's a hard constraint.
- 7. Break Constraint 2: It is used to restrict the total number of breaks for a subset of teams. It's a hard constraint.
- 8. Fairness Constraint 2: It makes sure that the difference in played home games between any two teams should be smaller than into at any particular time. It's a hard constraint.
- 9. Separation Constraint 1: It makes sure that two games between the same opponent should be separated by at least a particular number of slots. It's a hard constraint.

classmate Sports Timetabling We will organize the sport timetabling using the double round nobin process. Say n= total teams ij ith team plays home gone against the it team (home away) in the Kth glot 0, otherwise where i: 6 }0,1,2,..., n-1} K & {0,1,2,...,2(n-1)} Basic Constrain i) No term plays against itself => xx xiik=0 Vi, K ii) Each team plays exactly once => \(\sum_{j\in K} + \times_{j\in K} \) = 1 a single round iii) Freh team plays exactly (1-1) home games iv Each team plays 1 home and $\Rightarrow \sum_{K} (x_{i'j'K}) = 1$ as 1 away against each other. Objective Junction minimize \(\sigma' \chi_2:\) Jan every soft constrain 2:

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j K	1 mode 2 2 40 5 5 1 (7 + 7) < me~	
	j K	

Scanned by CamScanner

	Game Constraints
	GAz, where Sc {0,1,2,, meeting-size-1}
	meetings $\in \{i_1, i_2, i_2, i_3, i_3, \dots, i_5, i_5\}$ $K \in Slob$
	$\min_{S \in X} \leq \sum_{i \in S} \sum_{j \in K} x_{i,j,k} \leq \max_{i \in S} x_{i,j,k}$
	Brec K Constraint
\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	B81 , where $i = \text{tcmo}$, $j \in \{0,1,2,,n-1\}$ $K = \text{Slob}$, $J \in \{0,1,2,,n-1\}$
	if there is a home break in $(\sum_{i,j,k-2})(\sum_{j}x_{ijk}) = 1$, mode $2 = H$
	If there is a away baseck in (\frac{7}{2}\tilde{1}ik-1)\left(\frac{7}{2}\tilde{1}ik\right)=1, mode 2= A
	if there is a home-away break (\sum_{j' \text{in k+1}} \sum_{j' \text{vik-1}} \sum_{j' \text{vik-2}} \sum_{j' \tex
	so, the constraint would be given as
_	$\frac{\text{Mode 2} = AH}{\text{K} \text{ i}} \sum_{i,j,k=1}^{N} \frac{\sum_{i}^{N} \chi_{i,k}}{\sum_{i}^{N} \chi_{i,k}} \leq \text{intp}$
_	$\sum_{K} \sum_{j} \sum_{k} \chi_{j,K-1} \qquad \chi_{j,K} \leq \text{intp}$
/	marke 2 = A , Si Si Xjik-L Xiik = intp
	ZI ZI S (>Lajk-1 + x jin-2 .) Leik) = intp

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(ii	BRg where i = teams
(j∈ {0,1,2,,n-1}
	l ∈ {0,1,2,, n-1}
	14 E 310th
	$\frac{\sum \sum \sum \left(\chi_{i,1K-1}, \chi_{i,K} + \chi_{i,K-1}, \chi_{i,K}\right) \leq intp}{i j k i } \left(\chi_{i,K-1}, \chi_{i,K} + \chi_{i,K}\right) \leq intp}$
	Faigness Constraints
	FA, where $i \in \text{teams}$, $j \in \{0, 1, 2, \dots, n-1\}$
	1 ∈ tecms 1 K ∈ 2 slots
11 2 2 4	Number of home game of ith team => $\sum_{i}\sum_{j}\sum_{k}x_{ijk}$ after k^{th} time slot
1, 0, 5, 50	So, $-intp \leq \sum \sum \sum x_{ij:k} - \sum \sum x_{ij:k} \leq intp \forall \ (i, e) \in teams \times teams$
Everent or de	
	Seperation Constraint
	SE1, where i, j & teams
	$k \in \{0,1,2,\cdots,2n-1\}$
	Mark to the state of the state
	∑ (K. 22; - K. 26; K) > (min +1), ∀7; j ∈ teams s+ 2; ≠ j
	021
	$\frac{\sum (K \times i_{j:K} - K \times i_{j:K}) \leq -(\min + 1)}{K} \cdot \forall i_{j:j} \in \text{terms } s \nmid i \neq j}$

	classmate
	Date Page
2	Soft Constraint
	let $\sum \sum x_{ijk} \leq intp$
	1 1 16 1316
	be the caosiponding hard constraint then jon the soft constrain, we
	deline
	2 (2777 2)
	Deviction $(di) = mcx (0, \sum \sum \sum x_{ijk} - intp)$
	and you implementation we will use a constraint
i :	1 30
ii)	d > \(\sigma\) \(\sigm
	į į K
3>	Objective
	Objective Junction = \$\frac{7}{7}idiz , i \in soft constraint
	Bi = primary penalty associated with
	soft constraint 2
	cli= cleviction calculated
	So.
	objective: min Frich
)	

Complexity Analysis

- Decision Variables: (6<=(n =no. of teams)<=20)
 - o x[i][j][k] -> Total count: n*n*(2(n-1)) (15,200 in worst case!) ->Type: binary
 - o h[i][k] -> Total count: n*(2(n-1))(760 in worst case)-> Type: binary
 - o a[i][k]-> Total count: n*(2(n-1)) (760 in worst case)-> Type: binary
 - o d[i] -> Total count: no. of soft constraints (O(10,000))-> Type: positive integer
 - o y[i]-> Total count: no. of separation constraints (O(10,000)) -> Type: binary

Hence, Total decision variables are O(10,000) (may reach as high as 1,00,000)

Constraints:

- o Basic constraints: O(10000)
- Capacity constraint : (O(100) such constraints ,each one of them belongs to either of the below constraints)
 - 1 CA1 => 1 constraint
 - 1 CA2 => 1 constraint
 - 1 CA3 =>O(n*n) constraints
 - 1 CA4=>O(n) constraints
- Game Constraints:(O(10))
 - 1GA1=>1 constraint
- Break Constraint:(O(10))
 - 1BR1=>1 constraint
 - 1BR2=>1constraint
- Fairness Constraint: (O(10))
 - 1FA2=>(O(n*n*n))constraints
- Separation constraints: (O(10))
 - 1SE1=>O(n*n)constraints

Hence, total individual constraints can be higher than 1,00,000!

- Evaluation of a single constraint can reach as high as O(n^5), on an average O(n^2)
- From above it is clear that the complexity of program depends on:
 - \circ n
 - o no. of constraints in input file

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- Owing to above factors run time for the test cases with higher constraints or larger n is higher.
- (For eg, "ITC2021_Test5.xml" with n= 16, took around 35 minutes)